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NUMERICAL DISCUSSION OF FINGERING PHENOMENON THROUGH HOMOGENEOUS POROUS MEDIA

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ABSTRACT: *Fingering phenomenon in a double phase flow through homogeneous porous media is recapitulated here for its numerical discussion under a suitable set of initial and boundary conditions. The mathematical model of the phenomenon is formulated. The results are computed and presented graphically for saturation with respect to space coordinates and time. From graphs, it is observed that the saturation has a decreasing tendency with respect to space coordinates whereas it has increasing tendency with respect to time. This type of problems appears particularly in petroleum technology, geophysics, hydrogeology etc.*
KEYWORDS: *Double phase flow, Instability, Porous media, Finite difference method*

INTRODUCTION

It is observed usually that in the case of optimal oil exploitation in secondary recovery process from oil reservoirs, instability of interface may causes because of the viscosity difference of the two flowing phases, effects on the improvement of recovery. In contract, stable displacement of the interface yielding relatively high oil recoveries. The present paper numerically computed the above problem in a homogeneous porous media.

It is well known physical fact that fingering phenomenon [1] arise because of the difference in the viscosities of the two flowing phases. It has gained much importance in various fields like petroleum technology, geophysics, hydrogeology etc. and many authors have received considerable attention by various aspects; for example [2-6].

A finite difference method has been employed to solve the governing nonlinear partial differential equation with suitable set of initial and boundary conditions and the numerical results obtained indicates that the stabilization of fingers is possible in the investigated case.

THE PROBLEM AND ITS MATHEMATICAL MODEL

We consider here that there is a uniform water injection into an oil saturated porous medium of homogeneous physical characteristics, such that the injected water cuts through the oil formation and give rise to protuberance. This furnishes a problem of well developed fingers flow (Figure 1)

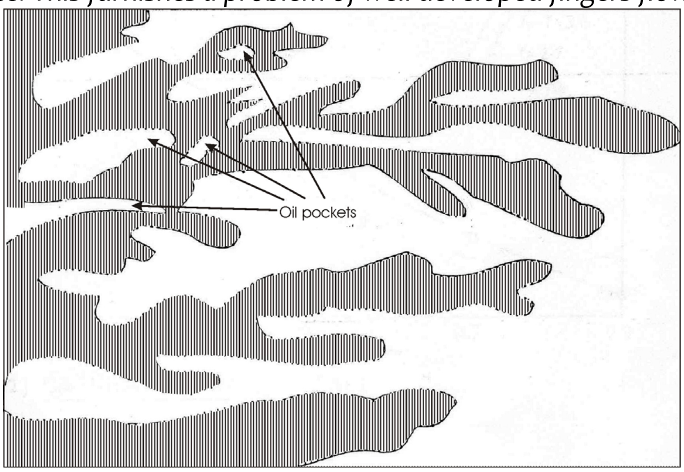

Figure 1. Fingering process in porous medium oil occupies unshaded area and water occupies shaded area

For the mathematical formulation, we assumed that the Darcy's law is valid for the investigated flow system and assumed further that the macroscopic behavior of fingers is governed by a statistical treatment [2]. In statistical treatment, the saturation of the injected fluid is defined as the average cross sectional area occupied by it at the level x, and thus the saturation of the displacing fluid in the porous medium represents the average cross sectional area occupied by the fingers.

The basic flow equations governing the phenomenon[4] are

$$V_w = -\frac{K_w}{\delta_w} K\left(\frac{\partial p_w}{\partial x} + \rho_w g \sin\theta\right), \; V_o = -\frac{K_o}{\delta_o} K\left(\frac{\partial p_o}{\partial x} + \rho_o g \sin\theta\right), \tag{1}$$

$$P\frac{\partial S_w}{\partial t} + \frac{\partial V_w}{\partial x} = 0, \quad P\frac{\partial S_o}{\partial t} + \frac{\partial V_o}{\partial x} = 0, \tag{2}$$

$$S_w + S_o = 1, \quad p_c = p_o - p_w . \tag{3}$$

Combining equations (1) to (3), using the relations

$$p_o = \bar{P} + \frac{1}{2}p_c, \; K_w = S_w, \; K_o = 1 - S_w, \; p_c = -\beta S_w,$$

due to[7] yields

$$P\frac{\partial S_w}{\partial t} = \frac{\beta K}{2\delta_w}\frac{\partial}{\partial x}\left[S_w \frac{\partial S_w}{\partial x}\right] + \frac{K\rho_w g \sin\theta}{\delta_w}\frac{\partial S_w}{\partial_x}. \tag{4}$$

This is nonlinear partial differential equation governing the phenomenon.

A suitable set of initial and boundary conditions becomes

$$S_w(0, t) = 1, \; S_w(x, 0) = 0, \; S_w(L, t) = 0. \tag{5}$$

By setting

$$X = \frac{x}{L}, \; T = \frac{\beta K t}{2\delta_w L^2 P},$$

equations (4) and (5) becomes

$$\frac{\partial S_w}{\partial T} - \frac{\partial}{\partial X}\left(S_w \frac{\partial S_w}{\partial X}\right) + C\frac{\partial S_w}{\partial X} = 0, \tag{6}$$

where

$$C = -\frac{2L\rho_w g \sin\theta}{\beta}$$

and

$$S_w(0, T) = 1, \; S_w(X, 0) = 0, \; S_w(1, T) = 0. \tag{7}$$

The inclination of the considered porous medium depends upon the value of θ with the horizontal. If θ is positive or negative, then inclined porous medium obtained otherwise, we have flat porous medium. For our particular interest we have solved the problem with flat porous medium, i.e. for $\theta = 0$.

FINITE DIFFERENCE DISCRETIZATION

The discretization for equations (6) with conditions (7) obtained by Crank Nicolson technique as:

For $i = 1$,

$$-\left[\frac{1}{(\Delta X)^2}\left(S_{w_{2,n+1/2}} + S_{w_{1,n+1/2}} + 4\right) + \frac{4}{\Delta T}\right] S_{w_{1,n+1}} + \frac{1}{(\Delta X)^2}\left(S_{w_{2,n+1/2}} + S_{w_{1,n+1/2}}\right) S_{w_{2,n+1}}$$

$$= -\left[\frac{4}{\Delta T} - \frac{1}{(\Delta X)^2}\left(S_{w_{2,n+1/2}} + S_{w_{1,n+1/2}} + 4\right)\right] S_{w_{1,n}} + \frac{1}{(\Delta X)^2}\left(S_{w_{2,n+1/2}} + S_{w_{1,n+1/2}}\right) S_{w_{2,n}} - \frac{8}{(\Delta X)^2}$$

with

$$S_{w_{1,n+1/2}} = S_{w_{1,n}} + \frac{\Delta T}{2}\left[\left(\frac{S_{w_{2,n}} - 3S_{w_{1,n}} + 2}{(\Delta X)^2}\right) S_{w_{1,n}} + \left(\frac{S_{w_{2,n}} + S_{w_{1,n}} - 2}{2(\Delta X)}\right)^2\right]$$

For $2 < i < R - 1$,

$$\frac{1}{(\Delta X)^2}\left(S_{w_{i-1,n+1/2}} + S_{w_{i,n+1/2}}\right) S_{w_{i-1,n+1}} - \left[\frac{1}{(\Delta X)^2}\left(S_{w_{i+1,n+1/2}} + S_{w_{i-1,n+1/2}} + 2S_{w_{i,n+1/2}}\right) + \frac{4}{\Delta T}\right] S_{w_{i,n+1}}$$

$$+ \frac{1}{(\Delta X)^2}\left(S_{w_{i+1,n+1/2}} + S_{w_{i,n+1/2}}\right) S_{w_{i+1,n+1}} = -\frac{1}{(\Delta X)^2}\left(S_{w_{i-1,n+1/2}} + S_{w_{i,n+1/2}}\right) S_{w_{i-1,n}}$$

$$-\left[\frac{4}{\Delta T}-\frac{1}{\Delta X^2}\left(S_{w_{i+1,n+1/2}}+S_{w_{i-1,n+1/2}}+2S_{w_{i,n+1/2}}\right)\right]S_{w_{i,n}}-\frac{1}{(\Delta X)^2}\left(S_{w_{i+1,n+1/2}}+S_{w_{i,n+1/2}}\right)S_{w_{i+1,n}}$$

with

$$S_{w_{i,n+1/2}}=S_{w_{i,n}}+\frac{\Delta T}{2}\left[\left(\frac{S_{w_{i-1,n}}-2S_{w_{i,n}}+S_{w_{i+1,n}}}{(\Delta X)^2}\right)S_{w_{i,n}}+\left(\frac{S_{w_{i+1,n}}-S_{w_{i-1,n}}}{2(\Delta X)}\right)^2\right]$$

For $i = R$,

$$\frac{1}{(\Delta X)^2}\left(S_{w_{R-1,n+1/2}}+S_{w_{R,n+1/2}}\right)S_{w_{R-1,n+1}}-\left[\frac{1}{(\Delta X)^2}\left(S_{w_{R-1,n+1/2}}+S_{w_{R,n+1/2}}\right)+\frac{4}{\Delta T}\right]S_{w_{R,n+1}}$$

$$=-\frac{1}{(\Delta X)^2}\left(S_{w_{R-1,n+1/2}}+S_{w_{R,n+1/2}}\right)S_{w_{R-1,n}}-\left[\frac{4}{\Delta T}-\frac{1}{(\Delta X)^2}\left(S_{w_{R-1,n+1/2}}+S_{w_{R,n+1/2}}\right)\right]S_{w_{R,n}}$$

with

$$S_{w_{R,n+1/2}}=S_{w_{R,n}}+\frac{\Delta T}{2}\left[\left(\frac{S_{w_{R-1,n}}-S_{w_{R,n}}}{(\Delta X)^2}\right)S_{w_{R,n}}+\left(\frac{S_{w_{R,n}}+S_{w_{R-1,n}}}{2(\Delta X)}\right)^2\right].$$

RESULT, DISCUSSION AND CONCLUSIONS

A computer program has been prepared for the above scheme and the graphs obtained from numerical results are as shown in Figure 2 and 3 for $\Delta T = 0.09$.

From the graphs it is observed that the saturation has a decreasing tendency along the space coordinates whereas it has increasing tendency along the time. Also, from Figure 2, it is seen that for T=0.9, S_w decreases as X increases in a negative exponential way, thus for larger value of X; that is, as $X \to \infty$, $S_w \to 0$. The graph for T=1.8 indicates decrease in saturation and a linear relationship between S_w and X.

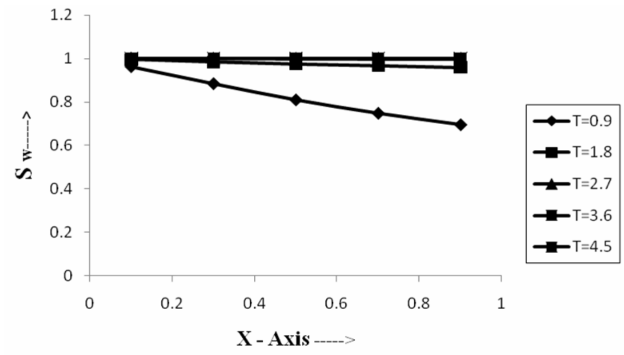

Figure 2. Saturation with respect to space

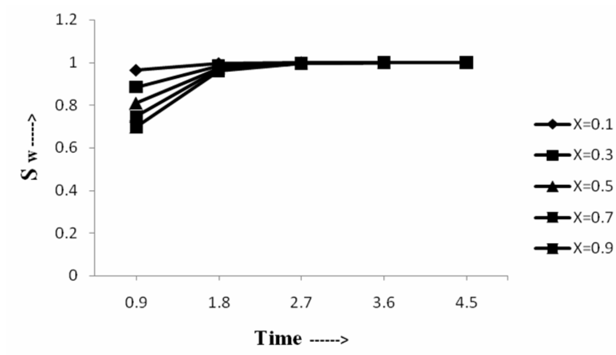

Figure 3. Saturation with respect to time

From Figure 3, it is observed that for $0.9 \le T \le 1.8$ and for all values of X there is a increase in saturation and a linear relationship between S_w and T. After T=1.8, the values of the saturation is slightly decreased for X=0.1 and then it goes on decreasing as we move from X=0.1 to X=0.9. Also, after T=1.8 and for all values of X, the increase in S_w is linearly related with T in the same ratio. After T=2.7 it is seen that for all values of X the saturation tends to 1. Therefore, we can say that the porous medium begins to become fully saturated after time T=2.7 and which is fully saturated approximately at time T=3.6.

The peaks in the graph at time T=1.8 and for all X indicates that the saturation of the injected liquid (water) is nearing its maximum value in oil saturated porous medium, this appears may be on account of the fact that almost full saturation is attained in most of the porous. Thus, the graphical behavior is significant.

Since the saturation has been defined as the average cross sectional area occupied by the fingers (Section 2), $S_w \to 0$ (decreasing tendency of saturation) along the space coordinate may be considered as a criterion for investigating the stabilization of fingers and the graph of the numerical results shows the decreasing nature of the saturation along the space coordinates. Thus, on the basis of the present study, it has been concluded that under the consideration of the specific boundary conditions, the stabilization of fingers is truly possible.

NOTATIONS

V_w, V_o Seepage velocity of water and oil, m/s
K Permeability of the porous medium, m^2
K_w, K_o Relative permeability of water and oil
p_w, p_o Pressure of water and oil, kg/m^2
P Porosity of the medium
t Time, sec
x Linear coordinate
β Capillary pressure coefficient
δ_w, δ_o Viscosity of water and oil, kg/ms
S_w, S_o Saturation of water, oil
L Length of the porous medium
p_c Capillary pressure, kg/ms^2
$\overline{P}$ Mean pressure
θ Inclination of the porous medium
g Acceleration due to gravity
ρ_n Density of Water

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