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MAGNETIC FIELD EFFECTS ON FLOW PAST AN ACCELERATED ISOTHERMAL VERTICAL PLATE WITH HEAT AND MASS DIFFUSION

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ABSTRACT: *An exact analysis of unsteady flow past an uniformly accelerated infinite isothermal vertical plate, under the action of transversely applied magnetic field has been presented. The plate temperature is raised to T_w and the concentration level near the plate is also raised to C'_w. The dimensionless governing equations are solved using Laplace-transform technique. The velocity profiles, temperature and concentration are studied for different physical parameters like thermal Grashof number, mass Grashof number, Schmidt number, Prandtl number and time. It is observed that the velocity increases with increasing values of thermal Grashof number or mass Grashof number. It is also observed that the velocity increases with decreasing magnetic field parameter.*
KEYWORDS: *accelerated, isothermal, vertical plate, heat transfer, mass diffusion, magnetic field*

INTRODUCTION

MHD plays an important role in agriculture, petroleum industries, geophysics and in astrophysics. Important applications in the study of geological formations, in exploration and thermal recovery of oil,and in the assessment of aquifers,geothermal reservoirs and underground nuclear waste storage sites. MHD flow has application in metrology, solar physics and in motion of earths core. Also it has applications in the field of stellar and planetary magnetospheres, aeronautics, chemical engineering and electronics. In the field of power generation, MHD is receiving considerable attention due to the possibilities it offers for much higher thermal efficiencies in power plants.

Gupta et al [2] studied free convection on flow past an linearly accelerated vertical plate in the presence of viscous dissipative heat using perturbation method. Kafousias and Raptis[3] extended the above problem to include mass transfer effects subjected to variable suction or injection. Free convection effects on flow past an accelerated vertical plate with variable suction and uniform heat flux in the presence of magnetic field was studied by Raptis al[4]. MHD effects on flow past an infinite vertical plate for both the classes of impulse as well as accelerated motion of the plate was studied by Raptis and Singh[5]. Mass transfer effects on flow past an uniformly accelerated vertical plate was studied by Soundalgekar [7]. Again, mass transfer effects on flow past an accelerated vertical plate with uniform heat flux was analyzed by Singh and Singh[6]. Basant Kumar Jha and Ravindra Prasad[1] analyzed mass transfer effects on the flow past an accelerated infinite vertical plate with heat sources.

Hence, it is proposed to study hydromagnetic effects on flow past an uniformly accelerated infinite isothermal vertical plate in the presence of heat and mass transfer. The dimensionless governing equations are solved using the Laplace-transform technique. The solutions are in terms of exponential and complementary error function. Such a study found useful in magnetic control of molten iron flow in the steel industry, liquid metal cooling in nuclear reactors and magnetic suppression of molten semi-conducting materials.

MATHEMATICAL FORMULATION

The unsteady flow of a viscous incompressible fluid past an uniformly accelerated isothermal vertical infinite plate in the presence of magnetic field has been considered. Here the unsteady flow of a viscous incompressible fluid which is initially at rest and surrounds an infinite vertical plate with temperature T_w and concentration C'_w. The x-axis is taken along the plate in the vertically upward direction and the y-axis is taken normal to the plate. At time $t' \le 0$, the plate and fluid are at the same temperature T_w. At time $t' > 0$, the plate is accelerated with a velocity $u = u_0 t'$ in its own plane and the temperature from the plate is raised to T_w and the concentration level near the plate are also raised to

C'_w. A transverse magnetic field of uniform strength B_0 is assumed to be applied normal to the plate. Then under usual Boussinesq's approximation the unsteady flow is governed by the following equations:

$$\frac{\partial u}{\partial t'} = g\beta(T - T_\infty) + g\beta^*(C' - C'_\infty) + \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho}u \tag{1}$$

$$\rho C_p \frac{\partial T}{\partial t'} = k\frac{\partial^2 T}{\partial y^2} \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y^2} \tag{3}$$

With the following initial and boundary conditions:

$$\begin{array}{lllll} & u = 0, & T = T_\infty, & C' = C'_\infty & \text{for all} \quad y, t' \le 0 \\ t' > 0: & u = u_0 t', & T = T_\omega, & C' = C'_\omega & \text{at} \quad y = 0 \\ & u \to 0 & T \to T_\infty, & C' \to C'_\infty & \text{as} \quad y \to \infty \end{array} \tag{4}$$

On introducing the following non-dimensional quantities:

$$\begin{gathered} U = \frac{u}{(\nu u_0)^{\frac{1}{3}}}, t = t'\left(\frac{u_0^2}{\nu}\right)^{\frac{1}{3}}, Y = y\left(\frac{u_0}{\nu^2}\right)^{\frac{1}{3}}, \\ \theta = \frac{T - T_\infty}{T_\omega - T_\infty}, Gr = \frac{g\beta(T_\omega - T_\infty)}{u_0}, \\ C = \frac{C' - C'_\infty}{C'_\omega - C'_\infty}, Gc = \frac{g\beta^*(C'_\omega - C'_\infty)}{u_0} \\ M = \frac{\sigma B_0^2}{\rho}\left(\frac{\nu}{u_0^2}\right)^{\frac{1}{3}}, Pr = \frac{\mu C_p}{k}, Sc = \frac{\nu}{D} \end{gathered} \tag{5}$$

in equations (1) to (4), leads to

$$\frac{\partial U}{\partial t} = Gr\,\theta + Gc\,C + \frac{\partial^2 U}{\partial Y^2} - M\,U \tag{6}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial Y^2} \tag{7}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2} \tag{8}$$

The initial and boundary conditions in non-dimensional quantities are

$$\begin{array}{lllll} & U = 0, & \theta = 0, & C = 0 & \text{for all} \quad Y, t \le 0 \\ t > 0: & U = t, & \theta = 1, & C = 1 & \text{at} \quad Y = 0 \\ & U \to 0, & \theta \to 0, & C \to 0 & \text{as} \quad Y \to \infty \end{array} \tag{9}$$

The dimensionless governing equations (6) to (8), subject to the initial and boundary conditions (9), are solved by the usual Laplace-transform technique and the solutions are derived as follows:

$$\theta = erfc(\eta\sqrt{Pr}) \tag{10}$$

$$C = erfc(\eta\sqrt{Sc}) \tag{11}$$

$$\begin{aligned} U = {} & \left(\frac{t}{2} + c + d\right)\left[exp(2\eta\sqrt{Mt})erfc(\eta + \sqrt{Mt}) + exp(-2\eta\sqrt{Mt})erfc(\eta - \sqrt{Mt})\right] \\ & - \frac{\eta\sqrt{t}}{2\sqrt{M}}\left[exp(-2\eta\sqrt{Mt})erfc(\eta - \sqrt{Mt}) - exp(2\eta\sqrt{Mt})erfc(\eta + \sqrt{Mt})\right] \\ & - 2c\,erfc(\sqrt{Pr}) - c\,exp(at)\left[\begin{array}{l} exp(2\eta\sqrt{(M+a)t})erfc(\eta + \sqrt{(M+a)t}) \\ + exp(-2\eta\sqrt{(M+a)t})erfc(\eta - \sqrt{(M+a)t}) \end{array}\right] \end{aligned}$$

$$-2d\,erfc(\eta\sqrt{Sc})-d\exp(bt)\begin{bmatrix}\exp(2\eta\sqrt{(M+b)t})erfc(\eta+\sqrt{(M+b)t})\\+\exp(-2\eta\sqrt{(M+b)t})erfc(\eta-\sqrt{(M+b)t})\end{bmatrix}$$

$$+c\exp(at)\begin{bmatrix}\exp(2\eta\sqrt{aPr\,t})erfc(\eta\sqrt{Pr}+\sqrt{at})\\+\exp(-2\eta\sqrt{aPr\,t})erfc(\eta\sqrt{Pr}-\sqrt{at})\end{bmatrix}+d\exp(bt)\begin{bmatrix}\exp(2\eta\sqrt{bSct})erfc(\eta\sqrt{Sc}+\sqrt{bt})\\+\exp(-2\eta\sqrt{bSct})erfc(\eta\sqrt{Sc}-\sqrt{bt})\end{bmatrix} \quad (12)$$

where, $a=\dfrac{M}{Pr-1}, b=\dfrac{M}{Sc-1}, c=\dfrac{Gr}{2a(1-Pr)}, d=\dfrac{Gc}{2b(1-Sc)}$ and $\eta=Y/2\sqrt{t}$.

RESULTS AND DISCUSSION

For physical understanding of the problem, numerical computations are carried out for different physical parameters Gr, Gc, Sc, Pr, M and t upon the nature of the flow and transport. The value of the Schmidt number Sc is taken to be 2.01 which corresponds to water-vapor. The numerical values of the velocity, temperature and concentration are computed for different physical parameters like Prandtl number, thermal Grashof number, mass Grashof number, Schmidt number and time.

Figure 1. illustrates the effects of the Magnetic field parameter on the velocity when $(M=2,5,10)$, $Gr=Gc=5, Pr=7$ and $t=0.4$. It is observed that the velocity increases with decreasing values of the magnetic field parameter. This shows that the increase in the magnetic field parameter leads to a fall in the velocity. This agrees with the expectations, since the magnetic field exerts a retarding force on the free convective flow.

The velocity profiles for different $(t=0.2,0.4,0.6), M=2, Gr=Gc=5, Pr=7$ are studied and presented in figure 2. It is observed that the velocity increases with increasing values of t. Figure 3. demonstrates the effects of different thermal Grashof number $(Gr=2,5)$, mass Grashof number $(Gc=2,5)$ and $M=2$ on the velocity at time $t=0.4$. It is observed that the velocity increases with increasing values of the thermal Grashof number or mass Grashof number.

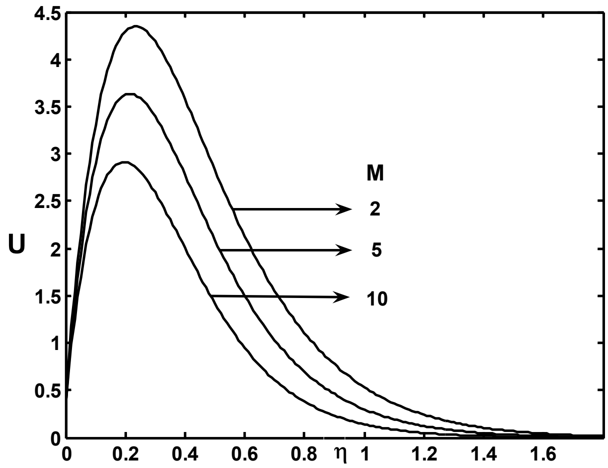

Figure 1. Velocity profiles for different values of M

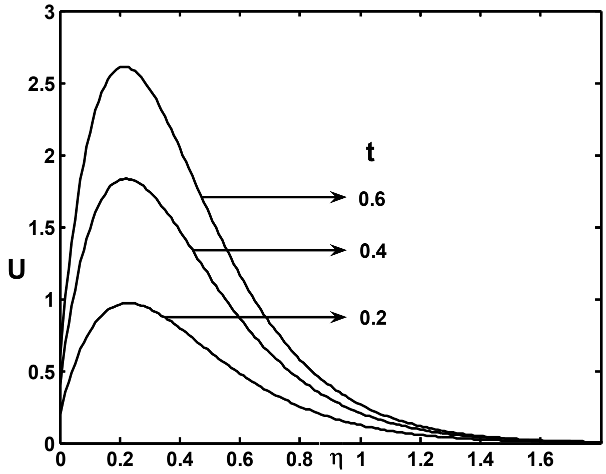

Figure 2. Velocity profiles for different values of t

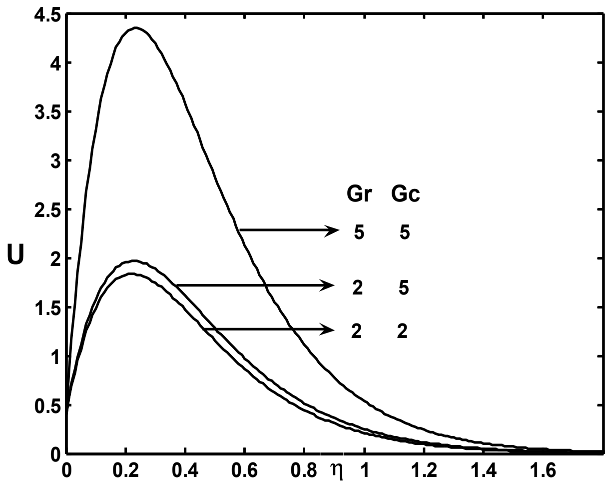

Figure 3. Velocity profiles for different values of Gr and Gc

CONCLUSIONS

The theoretical solution of flow past an uniformly accelerated infinite isothermal vertical plate in the presence of variable mass diffusion have been studied. The dimensionless governing equations are solved by the usual Laplace-transform technique. The effect of different parameters like thermal Grashof number, mass Grashof number and t are studied graphically. It is observed that the velocity increases with increasing values of Gr, Gc and t. But the trend is just reversed with respect to the magnetic field parameter.

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