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TILTING MECHANISMS KINETOSTATIC ANALYSIS TAKING ACCOUNT ON FRICTION FROM KINEMATIC COUPLERS

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ABSTRACT: *The paper introduces a kinetostatic analysis of the plane mechanisms, taking into consideration the frictions within the kinematic couplings. The kinetostatic study is structured on the determination of the positions of the gravity centers of the cinematic elements, their accelerations, the forces and the moments of the inertia forces as well as the reactions inside the couplings, through the method of successive approximations.*
KEYWORDS: *Mechanisms, friction forces, reaction forces*

INTRODUCTION

The kinetostatic analysis of mechanisms is being done in order to determine the forces that are at work in mechanisms, necessary for check the bolts in articulations, for the dimensioning of the component elements of mechanisms, and for verify the power of the electric driving motor. Mechanisms kinetostatic analysis involves making previously kinematic analysis, respectively the knowledge of angular and linear accelerations of components, in order to calculate forces and moments of inertia forces in the mechanism.

The kinetostatic analysis of the mechanisms is achieved by the following steps:

- *Determining the positions of the gravity centers of the component elements;*
- *The calculation of the accelerations for the gravity centers of the component elements;*
- *The calculation of the inertia forces and momentum;*
- *Establishing the charging schemes for the structural groups;*
- *The calculation of the reactions in the kinematic couplings, with the friction forces in the joints.*

For the case study was chosen tilting mechanism with hook, from rolling lines blooming type structure, whose kinematic scheme, with the division into structural Assur groups is shown in Fig. 1.

From fig. 1 is observed that the mechanism is formed from one dyad of aspect 1 BCD (consists of connecting rod BC and crank balancer CDE), crank AB and mechanism hook, which has a free motion and not affect its kinematics.

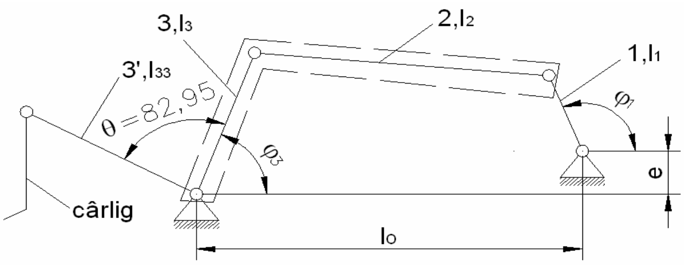
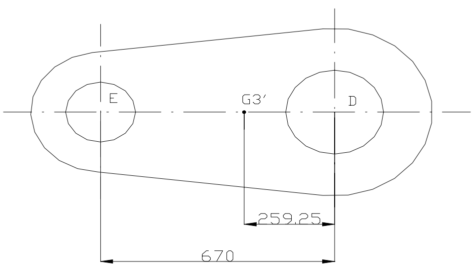


Fig. 1. The kinematic scheme of the tilting mechanism

DETERMINING THE POSITIONS OF THE GRAVITY CENTERS OF THE ELEMENTS

Centers of gravity positions kinematic elements were determined in AutoCAD application, by 2D modeling them, respectively by extracting the desired information. In Fig. 2 shows component DE case of crank arm, along with the data obtained [1].



Area: 602640.9690; Perimeter: 4365.5121
Centroid: X: -259.25; Y: 0
Moments of inertia: X: 9892444384.6087
Y: 97262205215.7966
Product of inertia: XY: -23328777.3881
Radius of gyration: X: 128.1216; Y: 401.7378
Principal moments and X-Y directions about centroid:
I: 9892437865.6838 along [1.0000 -0.0002]
J: 56756916260.0044 along [0.0002 1.0000]

Fig. 2. The position of the gravity center the DE element

According to fig. 2 the position of the gravity center of the DE element with relative to the D joint, is a_{33}=259,25 [mm]. Similarly are determined centers of gravity positions of the other kinematic elements:

- a_3 = 345 [mm] relative to C joint
- a_2 = 1001,5 [mm] relative to C joint
- a_1 = 280,95 [mm] relative to B joint

THE DETERMINATION OF THE ACCELERATIONS OF THE GRAVITY CENTERS OF THE STRUCTURAL GROUP

Knowing the positions of gravity centers of kinematic elements, their accelerations respectively, both in size and direction respectively way, is necessary for calculating that have forces of inertia have application points in the centers.

The acceleration of the gravity center of elements is determined according to the polygon of accelerations. For example Fig. 3 shows the crank arm case with two of its components DE and CD.

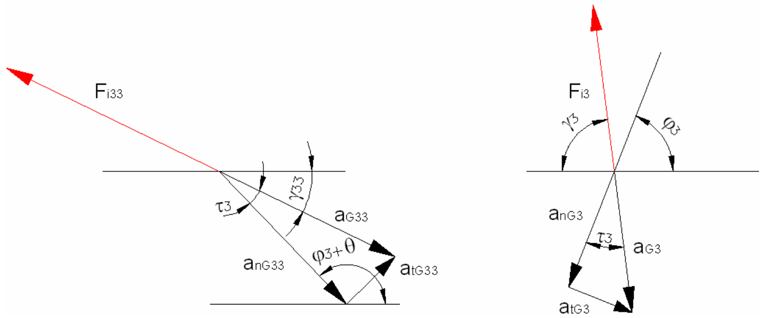


Fig. 3. The acceleration of the gravity center of CDE element

According to fig. 3 can be determined the following parameters:

- The angle between accelerations components of gravity centers G_3 şi G_{33}

$$\tau_3 = \arctan\frac{\varepsilon_3}{\omega_3^2} \tag{1}$$

- The direction of inertia forces F_{i33}, F_{i3} with respect to the horizontal

$$\gamma_{33} = \pi - \phi_3 - \tau_3 - \theta; \quad \gamma_{33} = \pi - \phi_3 - \tau_3 \tag{2}$$

- The accelerations of gravity centers G_{33}' G_3

$$a_{G33} = a_{33}\cdot\sqrt{\omega_3^4+\varepsilon_3^2}; \quad a_{G3} = (l_3 - a_3)\cdot\sqrt{\omega_3^4+\varepsilon_3^2} \tag{3}$$

Accelerations of other centers of gravity, respectively the directions of inertia forces from the horizontal are determined similarly.

THE CALCULATION OF INERTIA FORCES AND MOMENTUMS OF INERTIA FORCES THAT WORK ON KINEMATIC ELEMENTS

The inertia forces that work on the elements of the mechanisms can be reduced to the inertia force applied on the gravity center of the elements and the momentum of the inertia force. Their values are calculated according to the relations given below (on condition of knowing the masses of the elements and the linear and angular accelerations calculated within the kinematic analysis).

$$\bar{F}_{ic} = -m_c\cdot\bar{a}_E; \quad \bar{F}_{i33} = -m_{33}\cdot\bar{a}_{G33}; \quad \bar{F}_{i3} = -m_3\cdot\bar{a}_{G3}; \quad \bar{F}_{i2} = -m_2\cdot\bar{a}_{G2} \tag{4}$$

$$\begin{cases} J_{G2} = m_2\dfrac{l_2^2}{12} \\ J_{G3} = m_3\dfrac{l_3^2}{12} \\ J_{G33} = m_{33}\dfrac{l_{33}^2}{12} \end{cases} \Rightarrow \begin{cases} \overline{M}_{i2} = -J_{G2}\cdot\bar{\varepsilon}_2 \\ \overline{M}_{i3} = -J_{G3}\cdot\bar{\varepsilon}_3 \\ \overline{M}_{i33} = -J_{G33}\cdot\bar{\varepsilon}_{33} \end{cases} \tag{5}$$

where: J_{Gi} is the moment of inertia of element i relative to an axis through the center of gravity and m_i are the masses of kinematic elements.

THE CALCULATION OF THE REACTION FORCES IN THE CINEMATIC JOINTS

Reaction forces from kinematic couplings calculation mechanism, taking into account friction from joints is made by the method of successive approximations and loading schemes based on kinematic groups. In this case the reactions will not pass through the center joints, but will be tangent to the circle friction. In calculations however was considered that the reactions pass through the center joints, but will be taken into account friction moments that they have towards their center.

Method of successive approximations [4], consists in determining in a first phase reaction forces without taking into account friction, with their value will be calculated moments friction around *the joints at first approximation. Thus with reactions obtained at approximation j will calculating the moment's friction at j +1 approximation, the number of approximations shall be chosen so that the difference results from two successive approximations to fit into fixed error in advance. Dyads load schemes BCD and of the crank AB are presented in Fig. 4 and Fig. 5.*

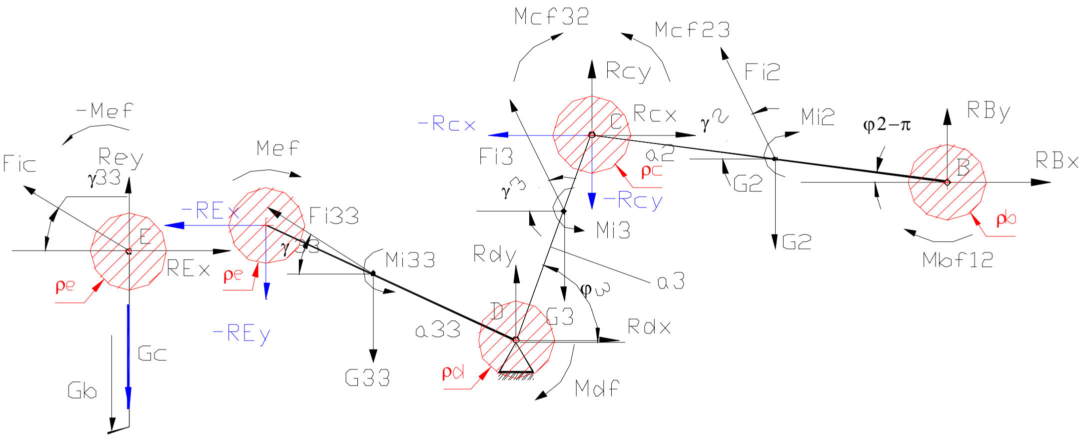


Fig. 4. BCD dyad charging scheme

With the kinematic joints rays and friction coefficient from joints known (μ = 0.1) can determine the friction circles rays, respectively the friction moments at about j+1.

$$r_e = 90\ [mm] \Rightarrow \rho_e=\mu r_e;\quad r_d = 130\ [mm] \Rightarrow \rho_d=\mu r_d;\ r_c = 112{,}5\ [mm]$$
$$\Rightarrow \rho_c=\mu r_c\ \ r_b = 100\ [mm] \Rightarrow \rho_b=\mu r_b;\ r_a = 100\ [mm] \Rightarrow \rho_a=\mu r_a \tag{6}$$

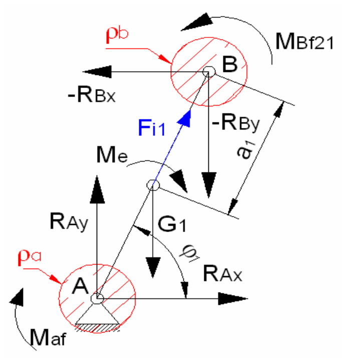


Fig. 5. AB crank charging scheme

$$\begin{aligned}
M_{ef}^{j+1} &= \mu \cdot r_e \cdot R_E^j \cdot sign(\omega_3)\\
M_{df}^{j+1} &= \mu \cdot r_d \cdot R_D^j \cdot sign(\omega_3)\\
M_{cf23}^{j+1} &= \mu \cdot r_c \cdot R_C^j \cdot sign(\omega_2-\omega_3)\\
M_{cf32}^{j+1} &= \mu \cdot r_c \cdot R_C^j \cdot sign(\omega_3-\omega_2)\\
M_{bf12}^{j+1} &= \mu \cdot r_b \cdot R_B^j \cdot sign(\omega_1-\omega_2)\\
M_{af}^{j+1} &= \mu \cdot r_a \cdot R_A^j \cdot\\
M_{bf21}^{j+1} &= \mu \cdot r_b \cdot R_B^j \cdot sign(\omega_2-\omega_1)
\end{aligned} \tag{7}$$

Based on the notation from fig. 4, fig. 5 and on the relations (6), (7) can be written equilibrium equations, which will be calculated the reactions from kinematic pairs (elements masses, their their weight forces being known).

$$\begin{cases} R_{EX} = F_{ic}\cdot \cos\gamma_{33} \\ R_{EY} = G_c + G_b - F_{ic}\cdot \sin\gamma_{33} \end{cases} \Rightarrow\ R_E = \sqrt{R_{EX}^2 + R_{EY}^2} \tag{8}$$

Friction from joints do not influence reaction from kinematic coupling E, because the hook has a free motion, and to determine the reaction RE, does not write any equation of moment, which might occur any moment of friction.

$$\begin{cases}
\Sigma F_X(2)=0 \Rightarrow R_{BX}^{j-1} + R_{CX}^{j-1} - F_{i2}\cdot cos\gamma_2 = 0\\
\Sigma F_Y(2)=0 \Rightarrow R_{BY}^{j-1} + R_{CY}^{j-1} - G_2 + F_{i2}\cdot sin\gamma_2 = 0\\
\Sigma F_X(3)=0 \Rightarrow -R_{CX}^{j-1} + R_{DX}^{j-1} - R_{EX} - F_{i3}\cdot cos\gamma_3 - F_{i33}\cdot cos\gamma_{33} = 0\\
\Sigma F_Y(3)=0 \Rightarrow -R_{CY}^{j-1} + R_{DY}^{j-1} - R_{EY} - G_3 - G_{33} + F_{i3}\cdot sin\gamma_3 + F_{i33}\cdot sin\gamma_{33} = 0\\
\Sigma M_C(2)=0 \Rightarrow -R_{BX}\cdot l_2\cdot sin\phi_2 + R_{BY}\cdot l_2\cdot cos\phi_2 - G_2\cdot(l_3-a_3)\cdot cos\phi_2 + M_{i2} + F_{i2}\cdot a_2\cdot sin(\gamma_2+\phi_2)\\
\qquad + M_{Cf32}^j + M_{Bf12}^j = 0\\
\Sigma M_D(3)=0 \Rightarrow -R_{CY}\cdot l_3\cdot cos\phi_3 + R_{CX}\cdot l_3\cdot sin\phi_3 - G_3(l_3-a_3)\cdot cos\phi_3 + F_{i3}(l_3-a_3)\cdot sin(\gamma_3+\phi_3) +\\
\qquad + M_{i3} - G_{33}\cdot a_{33}\cdot cos(\varphi_3+\theta) - F_{i33}\cdot a_{33}\cdot sin(\gamma_{33}-\varphi_3-\theta) - R_{EY}\cdot l_{3'}\cdot cos(\varphi_3+\theta) -\\
\qquad - R_{EY}\cdot l_{3'}\cdot sin(\varphi_3+\theta) + M_{i33} + M_{Cf23}^j - M_{Df}^j + M_{Ef}^j = 0
\end{cases} \tag{9}$$

$$\begin{cases} \Sigma F_X = 0 \Rightarrow -R_{Bx} + F_{i1} \cdot \cos\phi_1 + R_{AX} = 0 \\ \Sigma F_Y = 0 \Rightarrow -R_{BY} + F_{i1} \cdot \sin\phi_1 + R_{AY} - G_1 = 0 \\ \Sigma M_A = 0 \Rightarrow M_e + G_1 \cdot (l_1 - a_1) \cdot \cos\phi_1 + R_{BY} \cdot l_1 \cdot \cos\phi_1 - R_{BX} \cdot (l_1 - a_1) \cdot \sin\phi_1 - M^j_{B21f} + M^j_{Af} = 0 \end{cases} \quad (10)$$

At about j = 1 moments of friction are considered null.

By solving equations system (8, 9, 10) are obtained the X and Y projections of these action forces from kinematic pairs A, B, C and D. The reactions of joints are resultants of these components and are calculated with the following relations:

$$R_A = \sqrt{R^2_{AX} + R^2_{AY}}\ ;\ R_B = \sqrt{R^2_{BX} + R^2_{BY}}\ ;\ R_C = \sqrt{R^2_{CX} + R^2_{CY}}\ ;\ R_D = \sqrt{R^2_{DX} + R^2_{DY}} \quad (11)$$

The calculation relationships (1) – (11) are solved with the help of the program, written in Matlab.

The values of reaction forces are dependent angle on the position of the crank AB, $\varphi 1$; with the above program also can be realized graphical reaction forces variations according to the crank position.

In Fig. 6 is showed a comparison between reaction from kinematic coupling D, calculated with and without friction.

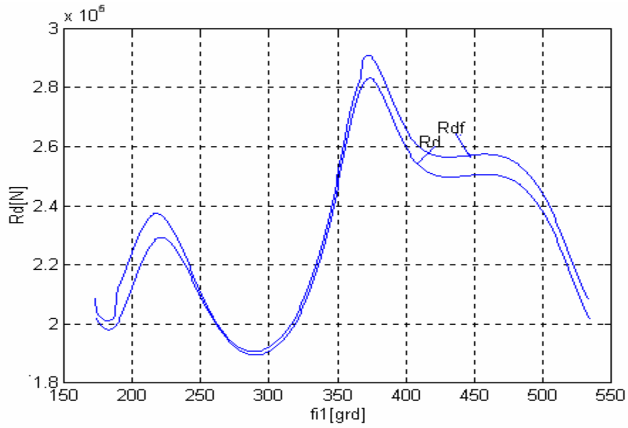


Fig. 6. The variation of the reaction force in the D joint, with and without friction

CONCLUSIONS

After calculations it can make the following conclusions: forces and moments of inertia forces acting on the kinematic elements have relatively low values of the order of 10^4[N] and 10^3[Nm], and the reactions from kinematic pairs have maximum values of the order 10^5[N], and moment the balancing on crank has values of order10^4 [Nm]. By taking into account the friction from joints, the reactions have an increase of approx. 7-8% of calculation without friction.

With the help of these results obtained by these calculations, one can check the bolts in the articulations of the mechanism, respectively the power of the electric motor. Problems that occur consist of the mechanism works under high temperature and friction coefficient can vary with temperature.

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