RADIATION AND MASS TRANSFER EFFECTS ON MHD FREE CONVECTION FLOW THROUGH POROUS MEDIUM PAST AN EXPONENTIALLY ACCELERATED VERTICAL PLATE WITH VARIABLE TEMPERATURE

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ABSTRACT: The purpose of this paper is to study the effect of mass transfer and thermal radiation on MHD free convection flow past an exponentially accelerated vertical plate in a porous medium with variable temperature and concentration. The fluid considered is gray, absorbing emitting radiation but not a non-scattering medium. The dimensionless governing equations under the Boussinesq approximation are solved by the Laplace transform method. The effects of various physical parameters on velocity, temperature and concentration are studied. The results are shown graphically and the numerical values of Skin friction are presented in tabular form. The analysis reveals that the Lorentz force opposes the motion of the fluid more effectively in absence of porous matrix. Further it is interesting to note that flow of fluid with higher thermal diffusivity in the presence of porous matrix prevents the back flow.

KEYWORDS: Radiation, magnetic field, exponential, accelerated vertical plate, heat transfer, chemical reaction

INTRODUCTION

The radiative heat and mass transfer flow of an electrically conducting fluid has wide applications in geophysics, geothermal, engineering and solar physics. It plays an important role in manufacturing industries for the design of nuclear power plants, gas turbines, steel rolling and various propulsion device for space vehicles, missiles, combustion and furnace design, energy utilization, aircrafts, satellites, remote sensing for astronomy and space exploration are examples of such engineering areas.

Radiation effects on mixed convection along an isothermal vertical plate were studied by Hossain and Takhar [4]. Radiation effects on MHD free convection flow of a gas past semi-infinite vertical plate were analyzed by Takhar et al [13]. They derived the solutions by expanding the stream function and temperature into a series and then the numerical procedure used to solve the differential equation. Seddeek [11] have analyzed thermal radiation and buoyancy effects on MHD free convective heat generating flow over an accelerating permeable surface with temperature dependent velocity. Abo-Eldahab and Ghonaim [1] studied the radiation effect on heat transfer of a micro polar fluid through a porous medium. Raptis and Perdikis [10] studied the effects of thermal radiation and free convection flow past a moving vertical plate. The governing equations were solved analytically. Das [3] have analyzed radiation effects on flow past an impulsively started infinite isothermal vertical plate. The governing equations were solved by the Laplace transform technique. Mathucumaraswamy and Janakiraman [5] studied MHD and radiation effects on moving isothermal vertical plate with variable mass diffusion. Radiation effects on MHD free convection flow over a vertical plate with heat and mass flux was studied by Sivaiah et al [12].

Radiation and mass transfer effects on two dimensional flows past an impulsively started vertical plate has been studied by Prasad et al [7]. Bhaskar et al [2] studied radiation and mass transfer effects on MHD free convection flow past an impulsively started isothermal vertical plate with dissipation. Prasad et al [8] have studied the radiation and mass transfer effects on an unsteady MHD convection flow past a semi-infinite vertical permeable moving plate embedded in a porous medium with viscous dissipation. Mathucumaraswamy and Vijayalakshmi [6] studied MHD and chemical reaction on flow past an impulsively started semi-infinite vertical plate with thermal radiation. They solved the governing equations by finite difference scheme. Radiation effects on MHD flow past an impulsively started exponentially accelerated vertical plate with variable temperature in the presence of heat generation was studied by Kumar and Verma [14]. Rajesh and Verma [9] have solved radiation and mass transfer effects on MHD free convection flow past an exponentially accelerated plate with variable temperature. The literature survey so far no work has been reported to study the flow of exponentially accelerated plate in the presence of porous matrix and chemically reactive species.
In the present analysis the effect of radiation and mass transfer on MHD free convection flow of a viscous incompressible electrically conducting fluid past an exponentially accelerated vertical plate embedded in a porous medium with variable temperature has been studied. The dimensionless governing equations are solved by using the Laplace transform technique.

**Mathematical Analysis**

Consider the unsteady free convection flow of a viscous incompressible electrically conducting fluid past an infinite vertical plate with variable temperature embedded in a uniform porous medium. Here the x'-axis is taken along the plate in the vertically upward direction and the y'-axis is taken normal to the plate in the fluid. A magnetic field of uniform strength $B_0$ is applied normal to the plate.

The induced magnetic field is neglected as the magnetic Reynolds number of the flow is taken to be very small. It is assumed that at time $t' \leq 0$, both the plate and the fluid are at the same temperature $T'_w$ and concentration $C'_w$. At time $t' > 0$, the plate is exponentially accelerated with a velocity $u = u_0 \exp(a't')$ in its own plane.

The temperature of the plate and the concentration level are also raised or lowered to, $T'_w + (T'_w - T'_w)A_{t'}$, $C'_w + (C'_w - C'_w)A_{t'}$. The fluid considered here is a gray, absorbing/emitting radiation but a non-scattering medium. The viscous dissipation is assumed to be negligible. Under these assumptions the governing boundary layer equations, usual Boussinesq's approximations are

$$
\frac{\partial u'}{\partial t'} = \rho \sigma C_p \frac{\partial T'}{\partial t'} + \frac{\alpha}{\rho} \frac{\partial^2 u'}{\partial y'^2} - \frac{v}{K_p} u'
$$

(1)

$$
\rho C_p \frac{\partial T'}{\partial t'} = \frac{\partial^2 T'}{\partial y'^2} - \frac{\partial q}{\partial y'}
$$

(2)

$$
\frac{\partial C'}{\partial t'} = \frac{\partial^2 C'}{\partial y'^2} - K'_c \left(C' - C'_w\right)
$$

(3)

The initial and boundary conditions are

$$
\left\{ \begin{array}{ll}
  u' = 0, T' = T'_w, C' = C'_w & \text{for all } y', t' \leq 0 \\
  u' = u_0 \exp(a't'), T' = T'_w + (T'_w - T'_w)A_{t'}, C' = C'_w + (C'_w - C'_w)A_{t'} & \text{at } y' = 0 \text{ for } t' \geq 0 \\
  u' \rightarrow 0, T' \rightarrow T'_w, C' \rightarrow C'_w & \text{as } y' \rightarrow \infty
\end{array} \right.
$$

(4)

where $A = \frac{u_0^2}{\nu}$.

The local radiant for the case of an optically thin gray gas is expressed by

$$
\frac{\partial q}{\partial y'} = -4a^* \sigma \left(T'_w^4 - T'^4\right)
$$

(5)

It is assumed that the temperature difference within the flow are sufficiently small such that $T'^4$ may be expressed as a linear function of the temperature. This is accomplished by expanding $T'^4$ in a Taylor series about $T'_w$ and neglecting the higher order terms, thus

$$
T'^4 \approx 4T'_w^3T' - 3T'_w^4
$$

(6)

Using equation (6) in (5)

$$
\frac{\partial q}{\partial y'} = -16a^* \sigma T'_w^3 \left(T'_w - T'\right)
$$

(7)

Now equation (2) reduces to

$$
\rho C_p \frac{\partial T'}{\partial t'} = \frac{\partial^2 T'}{\partial y'^2} + 16a^* \sigma T'_w^3 \left(T'_w - T'\right)
$$

(8)

On introducing the non-dimensional quantities and parameters

$$
U = \frac{u'}{u_0}, \quad t = \frac{t' u_0^2}{\nu}, \quad Y = \frac{y' u_0}{\nu}, \quad T = \frac{T' - T'_w}{T'_w - T'_w}, \quad G_c = \frac{g \beta \nu (C'_w - C'_w)}{u_0^3}, \quad \Gamma_c = \frac{C' - C'_w}{C'_w - C'_w}, \quad
$$

$$
G_c = \frac{g \beta \nu (C'_w - C'_w)}{u_0^3}, \quad P_r = \frac{\mu C_p}{K}, \quad S_c = \frac{\nu}{D}, \quad M = \frac{\sigma B_0^2 \nu}{\rho u_0^3}, \quad R = \frac{16a^* \nu^2 \sigma T'_w^3}{K u_0^3}, \quad a = \frac{a' \nu}{u_0^2}, \quad K_p = \frac{K_p u_0^2}{\nu^2}, \quad K_c = \frac{K_c \nu}{u_0^2}
$$

(9)
in equations (1), (8) and (3) leads to
\[
\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial y^2} - MU - \frac{U}{K_p} + G_T + G_C C
\]  
(10)
\[
\frac{\partial T}{\partial t} = \frac{1}{P_r} \frac{\partial^2 T}{\partial y^2} - \frac{R_T}{P_r}
\]  
(11)
\[
\frac{\partial C}{\partial t} = \frac{1}{S_c} \frac{\partial^2 C}{\partial y^2} - K_C C
\]  
(12)

The initial and boundary conditions in non-dimensional form are
\[
U = 0, T = 0, C = 0 \text{ for all } y, t \leq 0
\]  
\[
U = \exp(at), T = t, C = t \text{ at } y = 0
\]  
(13)
\[
U \rightarrow 0, T \rightarrow 0, C \rightarrow 0 \text{ as } y \rightarrow \infty \text{ as } t \rightarrow 0
\]

Using Laplace transform technique in the equations (10), (11) and (12) and then using the initial and boundary conditions (13) the solutions are as follows
\[
T = \frac{1}{2} \left[ \left( t + \frac{yP_r}{2 \sqrt{R}} \right) e^{-\sqrt{R} \tau} \text{erfc} \left( \frac{y}{2 \sqrt{t}} \right) + \left( t - \frac{yP_r}{2 \sqrt{R}} \right) e^{-\sqrt{R} \tau} \text{erfc} \left( \frac{y}{2 \sqrt{t}} \right) \right]
\]  
(14)
\[
C = \frac{1}{2} \left[ \left( t + \frac{y}{2 \sqrt{K_c}} \right) e^{-\sqrt{K_c} \tau} \text{erfc} \left( \frac{y}{2 \sqrt{t}} \right) + \left( t - \frac{y}{2 \sqrt{K_c}} \right) e^{-\sqrt{K_c} \tau} \text{erfc} \left( \frac{y}{2 \sqrt{t}} \right) \right]
\]  
(15)
\[
U = \frac{e^{-at}}{2} \left[ e^{\sqrt{\lambda} \tau} \text{erfc} \left( \frac{y}{2 \sqrt{t}} + \sqrt{\lambda t} \right) + e^{-\sqrt{\lambda} \tau} \text{erfc} \left( \frac{y}{2 \sqrt{t}} - \sqrt{\lambda t} \right) \right] + \frac{\alpha}{2} \left[ e^{bt} \text{erfc} \left( \frac{y + \sqrt{\lambda t}}{2 \sqrt{t}} \right) + e^{-bt} \text{erfc} \left( \frac{y - \sqrt{\lambda t}}{2 \sqrt{t}} \right) \right]
\]  
(16)
\[
\text{where: } \lambda = M + \frac{1}{K_p}, \quad \alpha = \frac{G_t}{1 - P_r}, \quad \alpha_1 = \frac{G_c}{1 - S_c}, \quad b = \frac{R - \lambda}{1 - P_r}, \quad d = \frac{S_c K_c - \lambda}{1 - S_c}.
\]

**SKIN FRICTION**

We now study the expression for skin-friction (τ) which measure shear stress at the plate is presented in the following form:
\[
\tau = \left( \frac{\partial U}{\partial y} \right)_{y=0} = e^a \left[ \sqrt{\lambda + a} \text{erfc} (\lambda + a \sqrt{t}) + \frac{1}{\sqrt{\pi t}} e^{-\lambda a \sqrt{t}} \right]
\]
\[ + \alpha \left[ \frac{e^{bt}}{b^2} \left( \sqrt{\lambda + b} \right) \left( \text{erfc} \left( \sqrt{\lambda + b} \right) \right) - \frac{1}{\sqrt{\pi t}} \right] e^{-t \lambda} - \frac{1}{\sqrt{\pi t}} + \frac{1}{b} \left( t + \frac{1}{b} \right) \times \]
\[ \left( \text{erfc} \left( \sqrt{\frac{R}{P_r} + b} \right) \right) \left( \text{erfc} \left( \sqrt{\frac{R}{P_r} + b} \right) \right) - \frac{1}{2b \sqrt{\lambda}} \left( \text{erfc} \left( \sqrt{\frac{R}{P_r} + b} \right) \right) + \left( \frac{1}{b} \left( t + \frac{1}{b} \right) \times \right) \]
\[ \left( \sqrt{\lambda \left( 1 - \text{erfc} \left( \sqrt{\lambda t} \right) \right)} - e^{-\lambda t} \right) + \frac{1}{2b \sqrt{\lambda}} \left( \text{erfc} \left( \sqrt{\frac{R}{P_r} + b} \right) \right) - \frac{1}{b} \left( t + \frac{1}{b} \right) \times \]
\[ \left( \sqrt{\lambda \left( 1 - \text{erfc} \left( \sqrt{\lambda t} \right) \right)} - e^{-\lambda t} \right) + \frac{1}{2b \sqrt{\lambda}} \left( \text{erfc} \left( \sqrt{\frac{R}{P_r} + b} \right) \right) - \frac{1}{b} \left( t + \frac{1}{b} \right) \times \]
\[ \left( \frac{1}{d} \left( t + \frac{1}{d} \right) \right) \left( \sqrt{S_c \left( 1 - \text{erfc} \left( \sqrt{K_c t} \right) \right)} + \frac{S_c}{\sqrt{\pi t}} \right) \]
\[ \left( \frac{1}{d} \left( t + \frac{1}{d} \right) \right) \left( \sqrt{S_c \left( 1 - \text{erfc} \left( \sqrt{K_c t} \right) \right)} + \frac{S_c}{\sqrt{\pi t}} \right) \]

\[ (17) \]

**RESULTS AND DISCUSSIONS**

The following discussions are carried to bring out the effects of various parameters on flow, heat and mass transfer phenomena. The present study brings the result of Rajesh and Verma [9] as a particular case. When \( K_p \) is very large but the case of without chemical reaction associated with mass transfer cannot be discussed as a particular case. Moreover, the case in the absence of thermal buoyancy \( (G_r = 0) \) as well as buoyancy effect due to concentration difference \( (G_r = 0) \) can be studied as a particular case. All the profiles exhibit the asymptotic behavior in the flow domain due to exponential motion of the plate. This is due to the induced flow generated by the exponentially accelerated plate. Some of the interesting results are as follows.

Backflow occurs due to high value of porosity parameter (i.e. absence of porous medium) and low value of prandtl number in the case of cooling of the plate, while the plate is at higher concentration (figure-1 curve-IV and figure-7 curve-IV). The above result agrees well with the result obtained by Rajesh and Verma [9]. Thus we may conclude that the backflow can be prevented due to the presence of porous media as well as fluid with higher thermal diffusivity. One more interesting point is to note about the higher value of time span. In this case back flow occurs, while heating of the plate.

From figures 1 & 2 it is observed that an increase in \( M \) leads to decrease the velocity for both cooling and heating of the plate. This is due to the increasing Lorentz force which reduces the velocity with high value of porous parameter \( K_p \). The velocity decreases for cooling of the plate and reverse effect is observed on heating of the plate.

The following results are figured out with the help of figures 3 & 4 when acceleration parameter increases the velocity increases for both cooling and heating of the plate. Due to an increase in radiation parameter the velocity decreases in case of cooled plate. The reverse effect is observed in case of heating of the plate. This is obviously consistent with physical conditions because the loss of energy due to radiation cannot be compensated incase of cooling of the plate.
Figure 3: Velocity profile $G_r=10, G_c=5, M=3, K_p=0.5, S_c=0.6, K_c=0.2, t=0.2, P_r=0.71$

Figure 4: Velocity profile $G_r=-10, G_c=5, M=3, K_p=0.5, S_c=0.6, K_c=0.2, t=0.2, P_r=0.71$

Figure 5: Velocity profile $G_r=10, G_c=5, M=3, K_p=0.5, a=0.5, R=4, t=0.2, P_r=0.71$

Figure 6: Velocity profile $G_r=-10, G_c=5, M=3, K_p=0.5, a=0.5, R=4, t=0.2, P_r=0.71$

Figure 7: Velocity profile $G_r=10, G_c=5, M=3, K_p=0.5, a=0.5, R=4, S_c=0.60, K_c=0.2$

Figure 8: Velocity profile $G_r=-10, G_c=5, M=3, K_p=0.5, a=0.5, R=4, S_c=0.60, K_c=0.2$

Figure 9: Velocity profile $t=0.2, P_r=0.71, M=3, K_p=0.5, a=0.5, R=4, S_c=0.60, K_c=0.2$

Figure 10: Velocity profile $t=0.2, P_r=0.71, M=3, K_p=0.5, a=0.5, R=4, S_c=0.60, K_c=0.2$

Figure 11: Temperature profile

Figure 12: Concentration profile
Figures 5 & 6 show the variation of velocity for different values of $S_c$ and $K_c$. An increase in $S_c$ and $K_c$ resulted in the decrease of velocity for cooled plate and the reverse effect is observed in a heated plate. For heavier species with destructive reaction caused a reduction in the velocity in case of a cooled plate. This is due to an increasing resistive force for heavier species associated with a destructive reaction. The heat energy lost could not be compensated due to destructive reaction as well as cooling of the plate.

Figures 7 & 8 exhibit the effect of $t$ and $P_r$ on velocity. The increase in $t$ and $P_r$ increases the velocity for the cooled plate and decreases for a heated plate.

Figures 9 & 10 show the effects of buoyancy parameters $G_r$ and $G_c$. An increase in $G_r$ and $G_c$ increases the velocity for cooled plate and the reverse effect is observed in case of the heated plate. This is obviously due to increasing convective current.

Figure 11 shows the temperature variation due to different value of $P_r$, $t$ and $R$. An increase in $P_r$, $t$, and $R$ decrease the temperature but with larger time span temperature increases. This decrease of temperature may be attributed to the loss of heat energy due to radiation as well as low diffusivity.

Figure 12 exhibits the concentration variation in the flow domain. This result is analogous to temperature distribution. In case of mass transfer, $S_c$ and $K_c$ play the similar role as $P_r$ and $R$ in temperature distribution.

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<thead>
<tr>
<th>Table 1 (Skin friction for cooling of the plate)</th>
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<td>$M$</td>
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<th>Table 2 (Skin friction for heating of the plate)</th>
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Skin friction is a measure of shearing stress experienced at the solid surface. It is seen that skin friction increases with an increase in $M$ and $a$ for both heating and cooling of the plate. This suggests that greater Lorentz force with increasing acceleration of the plate gives rise to increasing skin friction. It is also observed that the skin friction increases with an increase in $R, S_c, K_c$, and $P_r$ but decreases with an increase in $G_r, G_c$, and $t$ for a cooled plate but the reverse effect is observed in case of a heated plate. The porosity parameter reduces the skin friction for both heating and cooling of the plate. This suggests that presence of porous matrix reduces the frictional drag at the plate.

**CONCLUSIONS**

1. Flow of fluid with higher thermal diffusivity in the presence of porous matrix prevents the back flow.
2. Presence of heated plate flow exposed to higher time span causes back flow.
3. Lorentz force opposes the motion of the fluid more effectively in the absence of porous matrix.
4. Presence of heavier diffusing species with destructive reaction reduces the velocity in a cooled plate. This is obvious due to absorption of heat energy both for destructive reaction and cooling of the plate.

5. The decrease of temperature may be attributed to the loss of heat energy due to radiation as well as low diffusion.

6. An increase in Lorentz force with the presence of porous matrix reduces the skin friction for both heating and cooling of the plate.

**APPENDICES**

- $a^*$ - Absorption coefficient
- $a'$ - Dimensional acceleration parameter
- $a$ - Dimensionless accelerating parameter
- $t'$ - Dimensional time
- $t$ - Dimensionless time
- $C_p$ - Specific heat at constant pressure
- $C'$ - Dimensional concentration in the fluid
- $C'_{\infty}$ - Concentration of the fluid near the plate
- $C'_{\infty}'$ - Concentration of the fluid far away from the plate
- $C$ - Dimensionless fluid concentration
- $T'$ - Dimensional temperature of the fluid
- $T'_{\infty}$ - Constant temperature of the plate
- $T'_{\infty}'$ - Temperature of the fluid far away from the plate
- $U$ - Dimensionless velocity
- $U$ - Dimensionless velocity
- $M$ - Magnetic parameter
- $D$ - Chemical molecular diffusivity
- $g$ - Acceleration due to gravity
- $G_{\gamma}$ - Thermal Grashof number
- $G_{\gamma}$ - Mass Grashof number
- $K$ - Thermal conductivity of the fluid
- $K_{p}$ - Permeability of porous medium
- $K_{p}$ - Porosity parameter
- $K'$ - Reaction rate constant
- $K_{r}$ - Chemical reaction parameter
- $P_{r}$ - Prandtl number
- $q_{r}$ - Radiative heat flux in the $y$ - direction
- $S_{c}$ - Schmidt number
- $R$ - Radiation parameter
- $x'$, $y'$ - co-ordinate axes along and perpendicular to the plate
- $y$ - Dimensionless co-ordinate axis normal to the plate.
- $\mu$ - Coefficient of viscosity
- $\nu$ - Kinematic coefficient of viscosity
- $\rho$ - Fluid density
- $\sigma$ - Electric conductivity
- $\tau$ - Skin friction
- $\alpha$ - Thermal diffusivity
- $\theta$ - Volumetric coefficient of thermal expansion
- $b'$ - Volumetric coefficient of expansion with concentration
- erf - Complementary error function.

**REFERENCES**


