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MHD OSCILLATORY FLOW PAST A VERTICAL POROUS PLATE EMBEDDED IN A ROTATING POROUS MEDIUM

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ABSTRACT: *Unsteady hydromagnetic oscillatory flow of a viscous, incompressible, electrically conducting fluid through a porous medium past an infinite vertical porous plate with constant suction has been studied. A transverse uniform magnetic field has been applied in a rotating frame of reference. The porous medium is bounded by a vertical plane surface. The temperature on the vertical surface fluctuates in time about a non-zero constant mean. The analytical expressions for the velocity, temperature, concentration, skin friction, Nusselt number and Sherwood number are presented showing the effects of pertinent parameters.*
KEYWORDS: MHD, porous medium, embedded, temperature and skin-friction

INTRODUCTION

The hydromagnetic convection with heat and mass transfer in a rotating medium has received the attention of many researchers due to its importance in the design of magneto hydrodynamic (MHD) generators and accelerators, geo-physics, the under ground water energy storage systems, nuclear reactors, soil sciences, astrophysics and MHD boundary control of reentry vehicles. In recent years, the problem of free convection has attracted the attention of large number of researchers due to its diverse applications in many branches of science and technology.

On account of their varied importance, such a flow has been studied by Meyer [1]. Attia and Kotb [2] examined the MHD flow between two parallel plates. The unsteady free convective flow through rotating porous medium bounded by an infinite vertical porous plate was considered by Singh [3]. The effect of applied magnetic field on transient free convective flow in a vertical channel was presented by Jha [4]. Singh et al. [5] examined and analyzed the natural convection in a non-rectangular porous cavity. Acharya et al. [6] examined the magnetic field effects on free convection and mass transfer flow through a porous medium with constant suction and constant heat flux. The flow of unsteady visco-elastic electrically conducting fluid between two porous concentric circular cylinders was studied by Dash et al. [7]. Singh et al. [8] analyzed the free convection effects on MHD flow of a rotating viscous liquid in a porous medium past a vertical porous plate. Sengupta and Basak [9] studied the unsteady flow of a visco-elastic Maxwell fluid through porous straight tube under uniform magnetic field. The unsteady MHD free-convection flow past semi-infinite heated porous vertical plate with time-dependent suction and radiative heat transfer was discussed and analysed by Israel-Cookey et al. [10]. Singh et al [11] studied the heat and mass transfer in MHD flow of a viscous fluid past a vertical plate under oscillatory suction velocity, while Das et al. [12] discussed the free convection flow and mass transfer of an elastico-viscous fluid past an infinite vertical porous plate in a rotating porous medium. Panda et al. [13] examined the unsteady free convection MHD flow and mass transfer in a rotating porous medium.

The numerical study of mixed convection heat transfer from thermal sources on a vertical surface had been examined by Ogulu et al [14]. Cheng [15] presented the free convection heat and mass transfer from a horizontal cylinder of elliptic cross section in a micropolar fluid. Thereafter, Panda et al. [16] analyzed the free convection of conducting viscous fluid between two vertical walls filled with porous material. Sharma and Pareek [17] investigated the unsteady flow and heat transfer of an elastico-viscous liquid along an infinite hot vertical porous moving plate with variable free stream and suction. Mahmood and Ali [18] analysed the effect of slip condition on unsteady MHD oscillatory flow of a viscous fluid in a planer channel. Mishra et al. [19] investigated a flow and heat transfer of a MHD visco-elastic fluid in a channel with stretching walls.

In this chapter, an attempt is made to study the hydromagnetic oscillatory flow and heat transfer of a viscous incompressible fluid past a vertical porous plate embedded in a rotating porous medium. The governing equations of motion are solved analytically by using a regular perturbation technique. The behaviour of velocity, temperature, concentration, skin-friction, Nusselt number and Sherwood number for different values of thermo-physical parameters have been computed and the results are presented graphically and discussed qualitatively.

MATHEMATICAL ANALYSIS

An oscillatory flow of a viscous incompressible electrically conducting fluid past an infinite vertical porous plate through a porous medium in a rotating system is considered. The x - axis and y - axis are taken along the sides of the plate and z - axis is normal to the plate and the components of velocity $\vec{q}$ in these directions are taken as u, v and w respectively. The plate is porous, subjected to a suction velocity $w = -w_0$, where w_0 is assumed to be real and positive. The liquid and the plate both are in a state of rigid body rotation with uniform angular velocity Ω about z - axis. The plate is of an infinite extent, therefore all the physical variables depend on z and t only. A uniform magnetic field $B_0 = \mu_e H$, where $H = (0,0,H_0)$ has been applied in the z -direction, that is normal to the flow. The buoyancy force and Hall Effect have been considered to be very small and are neglected. The magnetic Reynolds number is assumed to be small so that the induced magnetic field is negligible in comparison with the applied magnetic field. The free stream velocity is taken as $U = U_0$, where $n << 1$.

Under the above assumptions, the governing equations for the flow under consideration are:

Continuity equation:

$$\frac{\partial w}{\partial z} = 0 \Rightarrow w = -w_0 \tag{1}$$

Momentum equations:

$$\frac{\partial u}{\partial t} - w_0 \frac{\partial u}{\partial z} - 2\Omega v = \frac{\partial U}{\partial t} + \nu \frac{\partial^2 u}{\partial z^2} - \frac{\nu}{K}(u-U) - \frac{\sigma}{\rho}\mu_e^2 H_0^2 (u-U) \tag{2}$$

$$\frac{\partial v}{\partial t} - w_0 \frac{\partial v}{\partial z} + 2\Omega(u-U) = \nu \frac{\partial^2 u}{\partial z^2} - \frac{\nu}{K} v - \frac{\sigma}{\rho}\mu_e^2 H_0^2 v \tag{3}$$

Energy equation:

$$\frac{\partial T}{\partial t} - w_0 \frac{\partial T}{\partial z} = \alpha \frac{\partial^2 T}{\partial z^2} + S(T - T_\infty) \tag{4}$$

Diffusion equation:

$$\frac{\partial C}{\partial t} - w_0 \frac{\partial C}{\partial z} = D \frac{\partial^2 C}{\partial z^2} \tag{5}$$

The boundary conditions for the velocity, temperature and concentration fields are

$$\begin{aligned} &u = 0,\ v = 0,\ T = T_w + \varepsilon(T_w - T_\infty)e^{\text{int}},\ C = C_w + \varepsilon(C_w - C_\infty)e^{\text{int}}, &&\text{at} \quad z = 0 \\ &u \to U_0\left(1 + \varepsilon e^{\text{int}}\right),\ v \to 0,\ T \to T_\infty,\ C \to C_\infty &&\text{as} \quad z \to \infty \end{aligned} \tag{6}$$

where U_0 is the mean free stream velocity, H_0 - the constant magnetic field, K - the permeability of the porous medium, S - the source parameter, α - the thermal diffusivity, D - mass diffusivity, σ - the electrical conductivity of the fluid, T_∞ -the temperature of the fluid far away from the plate, C_∞ - the species concentration in the fluid far away from the plate and other symbols stands for their usual meaning.

In order to write the governing equations and the boundary conditions in dimensionless form, the following non-dimensional quantities are introduced.

$$\begin{aligned} &z^* = \frac{w_0 z}{\nu},\ t^* = \frac{w_0^2 t}{\nu},\ U^* = \frac{U}{U_0},\ n^* = \frac{\nu n}{w_0^2}, T^* = \frac{T - T_\infty}{T_w - T_\infty}, C^* = \frac{C - C_\infty}{C_w - C_\infty} \\ &S^* = \frac{S\nu}{w_0^2},\ q^* = \frac{u}{U_0} + i\frac{v}{U_0} = u + iv,\ K_p = \frac{w_0^2 K}{\nu^2},\ R = \frac{\Omega \nu}{w_0^2}, \\ &M^2 = \frac{\sigma \mu_e^2 H_0^2 \nu}{\rho w_0^2},\ \Pr = \frac{\nu}{\alpha},\ Sc = \frac{\nu}{D},\ \alpha = \frac{\kappa}{\rho C_p} \end{aligned} \tag{7}$$

where Pr is the Prandtl number, Gr - the Grashof number, Gc - the modified Grashof number, M - the Magnetic parameter, Sc - the Schmidt number, S - the source parameter α - the thermal diffusivity, R – the rotation parameter, K_p - the permeability parameter.

In view of the Equation (7), and dropping stars (*), Equations (2)-(5) reduce to

$$\frac{\partial q}{\partial t} - \frac{\partial q}{\partial z} + \left(M^2 + 1/K_p + 2iR\right)(q - U) = \frac{\partial U}{\partial t} + \frac{\partial^2 q}{\partial z^2} \tag{8}$$

$$\frac{\partial T}{\partial t}-\frac{\partial T}{\partial z}=\frac{1}{\Pr}\frac{\partial^2 T}{\partial z^2}+ST \tag{9}$$

$$\frac{\partial C}{\partial t}-\frac{\partial C}{\partial z}=\frac{1}{Sc}\frac{\partial^2 C}{\partial z^2} \tag{10}$$

The corresponding boundary conditions are:

$$\begin{aligned} &q=0, \quad T=1+\varepsilon e^{\text{int}}, \quad C=1+\varepsilon e^{\text{int}} && \text{at} \quad z=0 \\ &q=1+\varepsilon e^{\text{int}}, \quad T=0, \quad C=0 && \text{as} \quad z\to\infty \end{aligned} \tag{11}$$

SOLUTION OF THE PROBLEM

The equations (8) - (10) are nonlinear partial differential equations, whose exact solutions are not possible. Hence, in the neighborhood of the plate, the velocity, temperature and concentration of the fluid are assumed to be

$$q(z,t)=(1-q_0)+\varepsilon(1-q_1)e^{\text{int}} \tag{12}$$

$$T(z,t)=T_0(z)+\varepsilon T_1(z)e^{\text{int}} \tag{13}$$

$$C(z,t)=C_0(z)+\varepsilon C_1(z)e^{\text{int}} \tag{14}$$

In view of these equations, the dimensionless governing equations (8) – (10) reduce to the following system of equations.

Zeroth order equations

$$q_0''+q_0'-\left(M^2+\frac{1}{K_p}+2iR\right)q_0=0 \tag{15}$$

$$T_0''+T_0'+\left(S\Pr+1\right)T_0=0 \tag{16}$$

$$C_0''+ScC_0'=0 \tag{17}$$

The corresponding boundary conditions are

$$\begin{aligned} &q_0=1, \; T_0=1, \; C_0=1 && \text{at} \quad z=0 \\ &q_0\to 0, T_0\to 0, C_0\to 0 && \text{as} \quad z\to\infty \end{aligned} \tag{18}$$

First order equations

$$q_1''+q_1'-\left(M^2+\frac{1}{K_p}+2iR+in\right)q_1=0 \tag{19}$$

$$T_1''+\Pr T_1'+\Pr\left(S-in\right)T_1=0 \tag{20}$$

$$C_1''+ScC_1'+ScC_1=0 \tag{21}$$

The corresponding boundary conditions are

$$\begin{aligned} &q_1=1, \; T_1=1, \; C_1=1 && \text{at} \quad z=0 \\ &q_1\to 0, \; T_1\to 0, \; C_1\to 0 && \text{as} \quad z\to\infty \end{aligned} \tag{22}$$

Solving the Equations (15) – (17) subject to (18), and the Equations (19) – (21) subject to (22), and using Equations (12) - (14), we get the expressions for the velocity, temperature and concentration as

$$q(z,t)=\left(1-e^{-\lambda_1 z}\right)+\varepsilon\left(1-e^{-\lambda_2 z}\right)e^{\text{int}}=u+iv \tag{23}$$

$$T(z,t)=e^{-\lambda_3 z}+\varepsilon e^{-\lambda_4 z+\text{int}} \tag{24}$$

$$C(z,t)=e^{-Sc z}+\varepsilon e^{-\lambda_5 z+\text{int}} \tag{25}$$

The skin-friction, Nusselt number and Sherwood number are important physical parameters for this type of boundary layer flow.

Knowing the velocity field, the skin-friction at the plate can be obtained, which in non-dimensional form is given by

$$\tau=\left.\frac{\partial q}{\partial z}\right|_{z=0}=\alpha_1+\varepsilon\left(\alpha_2\cos nt-\beta_2\sin nt\right)+i\left[\beta_1+\varepsilon\left(\beta_2\cos nt+\alpha_2\sin nt\right)\right]$$

The skin- friction can also be expressed as $\tau = \tau_p + i\tau_s$, *where* τ_p, τ_s *are the primary and secondary skin-frictions.*

For $nt = \pi/2$, *the transient primary and secondary skin-frictions are*

$$\tau_p = \left.\frac{\partial u}{\partial z}\right|_{z=0} = \alpha_1 - \varepsilon\beta_2 \; ; \quad \tau_s = \left.\frac{\partial v}{\partial z}\right|_{z=0} = \beta_1 + \varepsilon\alpha_2$$

Knowing the temperature field, the rate of heat transfer coefficient can be obtained, which in non-dimensional form, in terms of Nusselt number, is given by

$$Nu = -\left.\frac{\partial T}{\partial z}\right|_{z=0} = \left(Nu\right)_p + i\left(Nu\right)_s$$

where $\left(Nu\right)_p = \lambda_3 + \varepsilon\left(\alpha_3 \cos nt - \beta_3 \sin nt\right)$ *and* $\left(Nu\right)_s = \varepsilon\left(\alpha_3 \sin nt - \beta_3 \cos nt\right)$

For $nt = \pi/2$, *we have* $\left(Nu\right)_p = \lambda_3 - \varepsilon\beta_3$ *and* $\left(Nu\right)_s = \varepsilon\alpha_3$

Knowing the concentration field, the rate of mass transfer coefficient can be obtained, which in non-dimensional form, in terms of Sherwood number, is given by

$$Sh = -\left.\frac{\partial C}{\partial z}\right|_{z=0} = \left(Sh\right)_p + i\left(Sh\right)_s$$

where $\left(Sh\right)_p = Sc + \varepsilon\lambda_5 \cos nt$ and $\left(Sh\right)_s = \varepsilon\lambda_5 \sin nt$

For $nt = \dfrac{\pi}{2}$, we have $\left(Sh\right)_p = Sc$ and $\left(Sh\right)_s = \varepsilon\lambda_5$

Here the constants are not give because sake of brevity.

RESULTS AND DISCUSSION

In order to get a physical insight in to the problem the effects of various governing parameters on the physical quantities are computed and represented in Figures 1-13 and discussed in detail.

The effect of magnetic field intensity is illustrated in Fig.1. While all other participating parameters in the flow field remain constant and as the magnetic field intensity increases, a significant change in the primary velocity within the boundary layer region is observed. It is seen that as the magnetic field intensity increases, the primary velocity within the boundary layer enhances. However as we move far away from the plate, it is noticed that the magnetic intensity does not alter the nature of the velocity field.

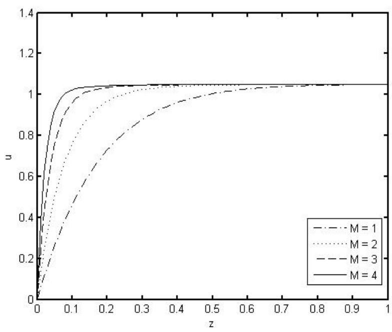

Fig.1. Effect of magnetic parameter (M) on the primary velocity profiles (R=1.0, Kp=10, t=π/4, n=0.01, ε=0.050)

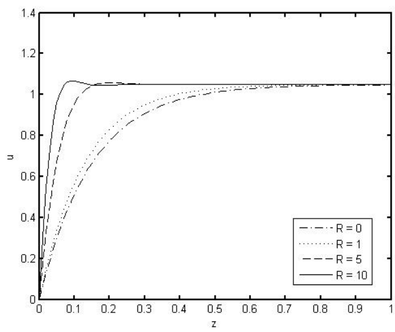

Fig.2. Effects of rotation parameter (R) on the primary velocity profiles (M=1, Kp=10, n=0.01, t=π/4, ε=0.050)

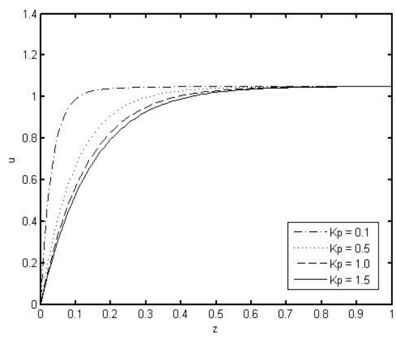

Fig.3. Influence of permeability parameter (Kp) on the primary velocity profiles (M=1, n=0.01, R=1, t=π/4, ε=0.050)

The influence of rotation parameter on the velocity profiles is illustrated in Fig.2. It is observed that as the rotation parameter increases, a significant rise in the primary velocity within the boundary layer region is noticed. However, as we move far away from the plate, it is noticed that the influence of rotation parameter is not that significant as was noticed in the boundary layer region. Fig.3 shows the influence of permeability of the bounding surface on the velocity profiles. It is observed that the influence of the permeability of the boundary is more prominent and influences the velocity within the boundary layer region while the effect is not felt as we move away from the bounding surface. It is seen that as the porosity increases, a significant drop in the velocity is noticed. Such a drop can be attributed due to the fact that the fluid is trapped inside the pores of the boundary and even percolation through the boundary. However, the phenomenon is not same as we move far away from the boundary.

The behavior of secondary flow with respect to the applied magnetic field intensity is shown in Fig.4. While all other participating parameters are held constant, it is seen that as the magnetic field *intensity is increased, a significant drop in the secondary velocity is noticed. Further, as we move away from the boundary, for a constant magnetic parameter the velocity decreases. But in the boundary layer region only an increase is noticed. Fig.5 illustrates the effect of rotation parameter on the secondary velocity. It is observed that the secondary velocity increases within the boundary layer region as the rotation parameter increases, where as away from the boundary this trend gets reversed. The influence of the permeability of the bounding surface on the secondary velocity is shown in Fig.6. It is seen that as the permeability parameter increases the secondary velocity increases.*

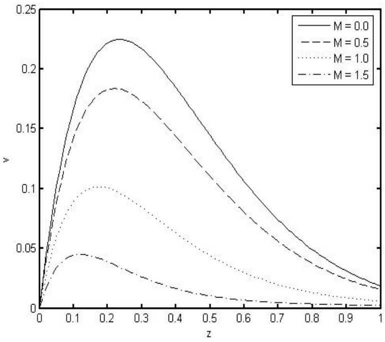

Fig.4. Influence of magnetic parameter (M) on the secondary velocity profiles (R=1.0, Kp=10, t=π/4, n=0.05, ε=0.050)

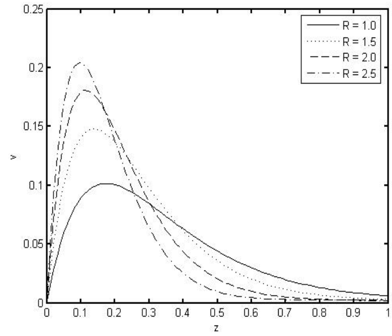

Fig.5. Effect of rotation parameter (R) on the secondary velocity profiles (M=1.0, Kp=10, t=π/4, n=0.05, ε=0.050)

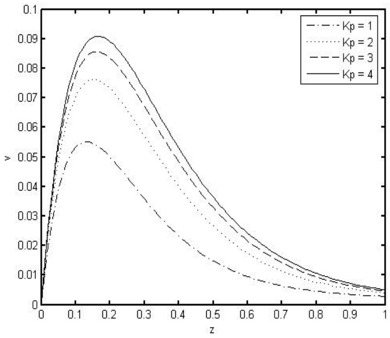

Fig.6. Influence of permeability parameter (Kp) on the secondary velocity profiles (M=1, n=0.05, R=1, t=π/4, ε=0.050)

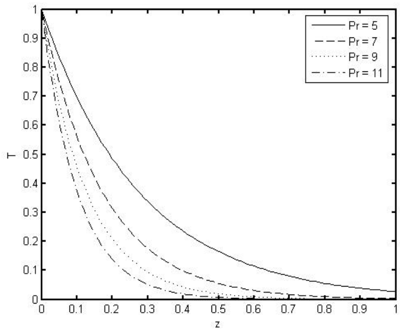

Fig.7. Effects of Prandtl number (Pr) on the temperature profiles (S=1, n=0.05, ε=0.050)

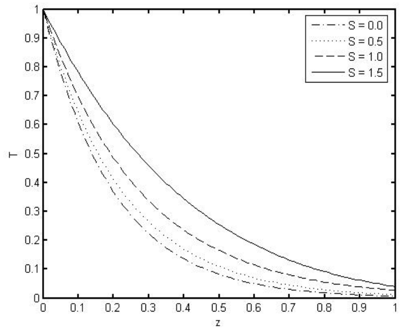

Fig.8. Effects of source parameter (S) on the temperature profiles (P=5, n=0.05, ε=0.050)

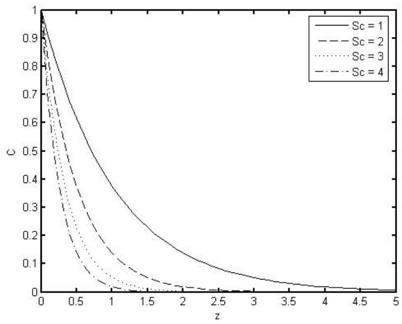

Fig.9. Effects of Schmidt number (Sc) on the concentration profiles (n=1, t=π/2, ε=0.050)

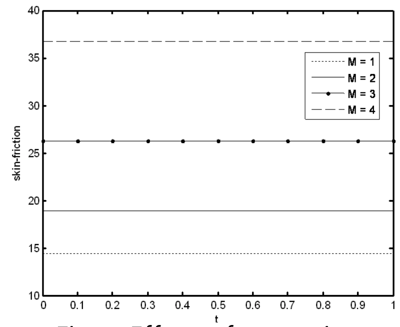

Fig.10. Effects of magnetic parameter (M) on the skin-friction (n=0.01, R=1, Kp=10, ε=0.050)

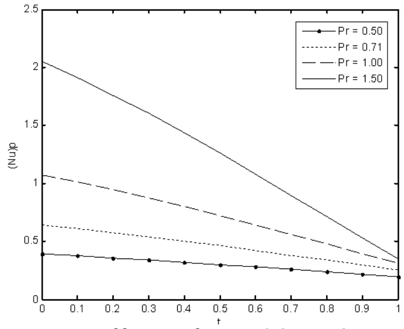

Fig.11. Effects of Prandtl number (Pr) on the primary Nusselt number

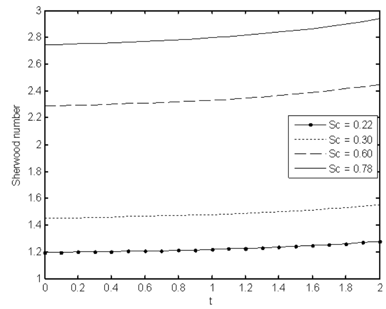

Fig. 12. Effects of Prandtl number (Pr) on the secondary Nusselt number

The influence of Prandtl number on the temperature field is shown in Fig.7. As the Prandtl number increases, a significant drop in the temperature is observed. However, as we move away from the boundary such an influence by the Prandtl number does not show any contribution on the temperature field.

Fig.8 shows the influence of the source parameter on the temperature field. In general it is seen that the effect of the source parameter is of exponential model. For constant values of the parameters that appear in the field equations and as the source parameter is increased, it is seen that the temperature increases. Far away from the bounding surface, not much of significant effect by the source parameter is seen.

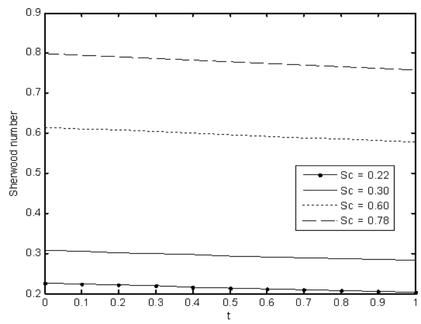

Fig.13. Effects of Schmidt number (Sc) on the Sherwood number

The influence of the Schmidt number on the concentration profiles in shown in Fig.9. It is noticed that as the Schmidt number increases, the temperature drops out significantly. As was noticed in the earlier cases, as we move away from the boundary, the contribution by the Schmidt number is not that significant.

The effect of magnetic field intensity is illustrated in Fig. 10. As the magnetic field intensity increases the skin-friction increases. Fig.11 illustrates the effect of Prandtl number (Pr) on the primary Nusselt number of the fluid under consideration. As Prandtl number increases the primary Nusselt number increases.

Fig.12 illustrates the effect of Prandtl number (Pr) on the secondary Nusselt number of the fluid under consideration. As Prandtl number increases the secondary Nusselt number increases over certain period of time and then the trend gets reversed. Fig.13 illustrates the effect of Schmidt number (Sc) on the Sherwood number of the fluid under consideration. As Schmidt number increases the Sherwood number increases.

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