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[1.] G. C. RANA

THERMAL CONVECTION IN RIVLIN-ERICKSEN ROTATING FLUID PERMEATED WITH SUSPENDED PARTICLES IN THE PRESENCE OF MAGNETIC FIELD AND VARIABLE GRAVITY FIELD IN A POROUS MEDIUM

[1.] DEPARTMENT OF MATHEMATICS, NSCBM GOVT. P. G. COLLEGE, HAMIRPUR-177 005, HIMACHAL PRADESH, INDIA

ABSTRACT: *In this paper, the thermal convection in Rivlin-Ericksen elastico-viscous rotating fluid permeated with suspended particles (fine dust) in the presence of magnetic field and variable gravity field in porous medium is considered. By applying normal mode analysis method, the dispersion relation has been derived and solved numerically. It is observed that the rotation, magnetic field, gravity field, suspended particles and viscoelasticity introduce oscillatory modes. For stationary convection, the rotation has stabilizing effect and suspended particles are found to have destabilizing effect on the system, whereas the medium permeability has stabilizing or destabilizing effect on the system under certain conditions. The magnetic field has destabilizing effect in the absence of rotation, whereas in the presence of rotation, magnetic field has stabilizing or destabilizing effect under certain conditions. The effect of rotation, suspended particles .magnetic field and medium permeability has also been shown graphically.*
KEYWORDS: *Rivlin-Ericksen elastico-viscous fluid, thermal convection, suspended particles, magnetic field, rotation, variable gravity field, porous medium*

INTRODUCTION

A detailed account of the thermal instability of a Newtonian fluid, under varying assumptions of hydrodynamics and hydromagnetics has been given by Chandrasekhar [3]. Chandra [2] observed a contradiction between the theory and experiment for the onset of convection in fluids heated from below. He performed the experiment in an air layer and found that the instability depended on the depth of the layer. A Benard-type cellular convection with the fluid descending at a cell centre was observed when the predicted gradients were imposed for layers deeper than 10 mm. A convection which was different in character from that in deeper layers occurred at much lower gradients than predicted if the layer depth was less than 7 mm, and called this motion, "Columnar instability". He added an aerosol to mark the flow pattern Bhatia and Steiner [1] have studied the thermal instability of a Maxwellian visco-elastic fluid in the presence of magnetic field while the thermal convection in Oldroydian visco-elastic fluid has been considered by Sharma [15].

The medium has been considered to be non-porous in all the above studies. Lapwood [7] has studied the convective flow in a porous medium using linearized stability theory. Scanlon and Segel [14] have considered the effect of suspended particles on the onset of Be'nard convection and found that the critical Rayleigh number was reduced solely because the heat capacity of the pure gas was supplemented by the particles. The suspended particles were thus found to destabilize the layer.

Sharma and Sunil [16] have studied the thermal instability of an Oldroydian viscoelastic fluid with suspended particles in hydromantic in a porous medium. There are many elastico-viscous fluids that cannot be characterized by Maxwell's constitutive relations or Oldroyd's constitutive relations. One such class of fluids is Rivlin-Ericksen elastico-viscous fluid [13]. Srivastava and Singh (1988) have studied the unsteady flow of a dusty elastico-viscous Rivlin-Ericksen fluid through channels of different cross-sections in the presence of time-dependent pressure gradient. Garg et. al. [4] has studied the rectilinear oscillations of a sphere along its diameter in conducting dusty Rivlin-Ericksen fluid in the presence of magnetic field.

Stommel and Fedorov [20] and Linden [8] have remarked that the length scalar characteristic of double diffusive convecting layers in the ocean may be sufficiently large that the Earth's rotation might be important in their formation. Moreover, the rotation of the Earth distorts the boundaries of a hexagonal convection cell in a fluid through a porous medium and the distortion plays an important role in the extraction of energy in the geothermal regions. The problem of thermal instability of a fluid in a porous medium is of importance in geophysics, soil sciences, ground water hydrology and astrophysics. The scientific importance of the field has also increased because hydrothermal circulation is the dominant heat transfer mechanism in the development of young oceanic crust (Lister, [9]).

Thermal instability of a fluid layer under variable gravitational field heated from below or above is investigated analytically by Pradhan and Samal [10]. Although the gravity field of the Earth is varying with height from its surface, we usually neglect this variation for laboratory purposes and treat the field as constant. However, this may not the case for large scale flows in the ocean, the atmosphere or the mantle. It can become imperative to consider gravity as a quantity varying with distance from the centre.

Sharma and Rana [17] have studied Thermal instability of a Walters' (Model B') elastico-viscous in the presence of variable gravity field and rotation in porous medium. Sharma and Rana [18] have also studied the thermosolutal instability of Rivlin-Ericksen rotating fluid in the presence of magnetic field and variable gravity field in porous medium. Kumar and Sharma [6] have studied the effect of suspended particles on thermal convection in viscoelastic fluid in hydromagnetics whereas Rana and Kumar [12] studied thermal instability of Rivlin-Ericksen Elastico-Viscous rotating fluid permitted with suspended particles and variable gravity field in porous medium. Recently, Rana [11] studied thermal instability of compressible Rivlin-Ericksen elastico-viscous rotating fluid permeated with suspended dust particles in porous medium

Keeping in mind the importance in various applications mentioned above, my interest, in the present paper is to study the thermal convection in Rivlin-Ericksen elastico-viscous rotating fluid permitted with suspended particles in the presence of magnetic field and variable field in porous medium.

FORMULATION OF THE PROBLEM

*Consider an infinite horizontal layer of an electrically conducting Rivlin-Ericksen elastico-viscous fluid of depth d in a porous medium bounded by the planes z = 0 and z = d in an isotropic and homogeneous medium of porosity $\in$ and permeability k_1, which is acted upon by a uniform rotation $\Omega(0, 0, \Omega)$, uniform vertical magnetic field H(0,0,H) and variable gravity **g**(0, 0, -g), g = λg_0, g_0(>0) is the value of g at z = 0 and λ can positive or negative as gravity increases or decreases upward from its value g_0.*

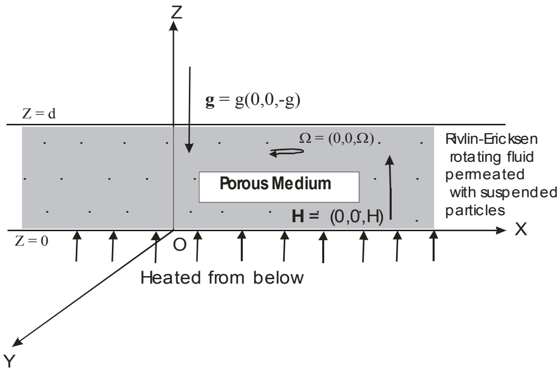

Figure 1. Schematic sketch of physical situation

This layer is heated from below such that a steady adverse temperature gradient $\beta = \left(\left|\frac{dT}{dz}\right|\right)$ is maintained. The character of equilibrium of this initial static state is determined by supposing that the system is slightly disturbed and then following its further evolution. The hydromagnetic equations in porous medium [Chandrasekhar (1981), Joseph (1976), Rivlin and Ericksen (1955), relevant to the problem are

$$\frac{1}{\in}\left[\frac{\partial \vec{q}}{\partial t}+\frac{1}{\in}(\vec{q}.\nabla)\vec{q}\right] = -\frac{1}{\rho_0}\nabla p+\vec{g}\left(1+\frac{\delta\rho}{\rho_0}\right)-\frac{1}{k_1}\left(\upsilon+\upsilon'\frac{\partial}{\partial t}\right)\vec{q}+\frac{2}{\in}\left(\vec{q}\times\vec{\Omega}\right)+\frac{K'N}{\rho_0\in}\left(\vec{q}_d-\vec{q}\right)+\frac{\mu_e}{4\pi\rho_0}\left(\nabla\times\vec{H}\right)\times\vec{H}, \quad (1)$$

$$\nabla.\vec{q}=0 \quad (2)$$

$$E\frac{\partial T}{\partial t}+(q.\nabla)T+\frac{nNc_{pt}}{\rho_o C_f}\left[\in\frac{\partial}{\partial t}+q_d.\nabla\right]T=\kappa\nabla^2 T\ , \quad (3)$$

$$\nabla.\vec{H}=0 \quad (4)$$

$$\in\frac{\partial\vec{H}}{\partial t}=\nabla\times\left(\vec{q}\times\vec{H}\right)+\in\eta\nabla^2\vec{H}\ . \quad (5)$$

where $E=\in+(1-\in)\left(\frac{\rho_s C_s}{\rho_o C_f}\right)$ ρ_s, C_s, ρ_o, C_f *denote the density and heat capacity of solid (porous) matrix and fluid respectively. The equation of state is*

$$\rho=\rho_o\left[1-\alpha\left(T-T_o\right)\right], \quad (6)$$

where the suffix zero refers to values at the reference level z = 0. Here $\rho,\upsilon,\upsilon',p,\in,T,\mu_e,\alpha,q(0,0,0)$ *and H(0,0,H) stand for density, kinematic viscosity, kinematic viscoelasticity, pressure, medium porosity, temperature, magnetic permeability, thermal coefficient of expansion, velocity of the fluid and magnetic field. Here* $q_d(\overline{x},t)$ *and* $N(\overline{x},t)$ *denote the velocity and number density of the particles respectively,* $K'=6\pi\rho\upsilon\eta$, *where* η *is particle radius, is the Stokes drag coefficient,* $q_d=(l,r,s)$ *and* $\overline{x}=(x,y,z)$.

If mN is the mass of particles per unit volume, then the equations of motion and continuity for the particles are

$$mN\left[\frac{\partial q_d}{\partial t}+\frac{1}{\in}(q_d.\nabla)q_d\right]=K'N(q-q_d), \tag{7}$$

$$\in\frac{\partial N}{\partial t}+\nabla.(Nq_d)=0. \tag{8}$$

The presence of particles adds an extra force term proportional to the velocity difference between particles and fluid and appears in the equation of motion (1). Since the force exerted by the fluid on the particles is equal and opposite to that exerted by the particles on the fluid, there must be an extra force term, equal in magnitude but opposite in sign, in the equations of motion for the particles (7). The buoyancy force on the particles is neglected. Interparticles reactions are not considered either since we assume that the distance between the particles are quite large compared with their diameters. These assumptions have been used in writing the equations of motion (7) for the particles.

The initial state of the system is taken to be quiescent layer (no settling) with a uniform particle distribution number. The initial state is

$$q=(0,0,0), q_d=(0,0,0),\ T=-\beta z+T_o, \rho=\rho_o(1+\alpha\beta z), N=N_o, \text{a constant.} \tag{9}$$

THE PERTURBATION EQUATIONS

Let $q(u,v,w)$, $q_d(l,r,s)$, ϑ, δp and $\delta\rho$ denote, respectively, the perturbations in fluid velocity $q(0,0,0)$, the perturbation in particle velocity $q_d(0,0,0)$, temperature T, pressure p and density ρ.

The change in density $\delta\rho$ caused by perturbation ϑ temperature is given by

$$\delta\rho=-\alpha\rho_o\theta. \tag{10}$$

The linearized perturbation equations governing the motion of fluids are

$$\frac{1}{\in}\frac{\partial q}{\partial t}=-\frac{1}{\rho_o}\nabla\delta p-\vec{g}\alpha\vartheta-\frac{1}{k_1}\left(\upsilon+\upsilon'\frac{\partial}{\partial t}\right)\vec{q}+\frac{2}{\in}\left(\vec{q}\times\vec{\Omega}\right)+\frac{K'N}{\rho_o\in}\left(\vec{q}_d-\vec{q}\right)+\frac{\mu_e}{4\pi\rho_o}\left(\nabla\times\vec{h}\right)\times\vec{H}, \tag{11}$$

$$\nabla.\vec{q}=0, \tag{12}$$

$$\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\vec{q}_d=\vec{q}, \tag{13}$$

$$(E+b\in)\frac{\partial\theta}{\partial t}=\beta(w+bs)+\kappa\nabla^2\theta, \tag{14}$$

$$\nabla.\vec{h}=0 \tag{15}$$

$$\in\frac{\partial H}{\partial t}=\left(\vec{H}.\nabla\right)\vec{q}+\in\eta\nabla^2\theta, \tag{16}$$

where $b=\dfrac{mNC_{pt}}{\rho_o c_f}$, and w, s, are the vertical fluid and particles velocity.

In the Cartesian form, equations (11)-(16) can be expressed as

$$\begin{aligned}\frac{1}{\in}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\frac{\partial u}{\partial t}=&-\frac{1}{\rho_o}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\frac{\partial}{\partial x}(\delta p)-\frac{1}{k_1}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\left(\upsilon+\upsilon'\frac{\partial}{\partial t}\right)u\\&-\frac{mN}{\in\rho_o}\frac{\partial u}{\partial t}+\frac{\mu_e H}{4\pi\rho_o}\left(\frac{\partial h_x}{\partial z}-\frac{\partial h_z}{\partial x}\right)+\frac{2}{\in}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\Omega v,\end{aligned} \tag{17}$$

$$\begin{aligned}\frac{1}{\in}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\frac{\partial v}{\partial t}=&-\frac{1}{\rho_o}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\frac{\partial}{\partial y}(\delta p)-\frac{1}{k_1}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\left(\upsilon+\upsilon'\frac{\partial}{\partial t}\right)v\\&-\frac{mN}{\in\rho_o}\frac{\partial v}{\partial t}+\frac{\mu_e H}{4\pi\rho_o}\left(\frac{\partial h_y}{\partial z}-\frac{\partial h_z}{\partial y}\right)+\frac{2}{\in}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\Omega u,\end{aligned} \tag{18}$$

$$\begin{aligned}\frac{1}{\in}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\frac{\partial w}{\partial t}=&-\frac{1}{\rho_o}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\frac{\partial}{\partial z}(\delta p)-\frac{1}{k_1}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\left(\upsilon+\upsilon'\frac{\partial}{\partial t}\right)w\\&-\frac{mN}{\in\rho_o}\frac{\partial u}{\partial t}+\frac{\mu_e H}{4\pi\rho_o}\left(\frac{\partial h_y}{\partial x}-\frac{\partial h_x}{\partial y}\right)+g\alpha\theta,\end{aligned} \tag{19}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0, \tag{20}$$

$$(E+b\in)\frac{\partial\theta}{\partial t}=\beta(w+bs)+\kappa\nabla^2\theta, \tag{21}$$

$$\in\frac{\partial h_x}{\partial t}=H\frac{\partial u}{\partial z}+\in\eta\nabla^2 h_x, \tag{22}$$

$$\in\frac{\partial h_y}{\partial t}=H\frac{\partial u}{\partial z}+\in\eta\nabla^2 h_y, \tag{23}$$

$$\in\frac{\partial h_z}{\partial t}=H\frac{\partial u}{\partial z}+\in\eta\nabla^2 h_z, \tag{24}$$

$$\frac{\partial h_x}{\partial x}+\frac{\partial h_y}{\partial y}+\frac{\partial h_z}{\partial z}=0. \tag{25}$$

Operating equation (17) and (18) by $\frac{\partial}{\partial x}$ *and* $\frac{\partial}{\partial y}$ *respectively, adding and using equation (22)-(25), we get*

$$\frac{1}{\in}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\frac{\partial}{\partial t}\left(\frac{\partial w}{\partial z}\right)=-\frac{1}{\rho_0}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\left(\nabla^2-\frac{\partial^2}{\partial z^2}\right)\delta p-\frac{1}{k_1}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\left(\upsilon+\upsilon'\frac{\partial}{\partial t}\right)\left(\frac{\partial w}{\partial z}\right)$$
$$-\frac{mN}{\in\rho_0}\frac{\partial}{\partial t}\left(\frac{\partial w}{\partial z}\right)+\frac{\mu_e H}{4\pi\rho_0}\nabla^2 h_z-\frac{2}{\in}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\Omega\zeta, \tag{26}$$

where $\zeta=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}$ *is the z-component of vorticity.*

Operating equation (19) and (26) by $\left(\nabla^2-\frac{\partial^2}{\partial z^2}\right)$ *and* $\frac{\partial}{\partial z}$ *respectively and adding to eliminate* δp *between equations (19) and (26), we get*

$$\frac{1}{\in}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\frac{\partial}{\partial t}\left(\nabla^2 w\right)=-\frac{1}{k_1}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\left(\upsilon+\upsilon'\frac{\partial}{\partial t}\right)\nabla^2 w+g\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\alpha\theta$$
$$-\frac{mN}{\in\rho_0}\frac{\partial}{\partial t}\left(\nabla^2 w\right)+\frac{\mu_e H}{4\pi\rho_0}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\frac{\partial}{\partial t}\left(\nabla^2 h_z\right)-\frac{2}{\in}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\Omega\frac{\partial\zeta}{\partial z},, \tag{27}$$

where $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial}{\partial y^2}+\frac{\partial}{\partial z^2}$.

Operating equation (17) and (18) by $-\frac{\partial}{\partial y}$ *and* $\frac{\partial}{\partial x}$ *respectively and adding, we get*

$$\frac{1}{\in}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\frac{\partial\zeta}{\partial t}=-\frac{1}{k_1}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\left(\upsilon+\upsilon'\frac{\partial}{\partial t}\right)\zeta-\frac{mN}{\in\rho_0}\frac{\partial\zeta}{\partial t}+\frac{\mu_e H}{4\pi\rho_0}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\frac{\partial\xi}{\partial t}+\frac{2}{\in}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\Omega\frac{\partial w}{\partial z}, \tag{28}$$

where $\xi=\frac{\partial h_y}{\partial x}-\frac{\partial h_x}{\partial y}$ *is the z-component of current density.*

Operating equations (22) and (23) by $-\frac{\partial}{\partial y}$ *and* $\frac{\partial}{\partial x}$ *respectively and adding, we get*

$$\in\frac{\partial\xi}{\partial t}=H\frac{\partial\xi}{\partial t}+\in\eta\nabla^2\xi. \tag{29}$$

DISPERSION RELATION AND DISCUSSION

Analyzing the disturbances into normal modes, we assume that the perturbation quantities have x, y and t dependence of the form

$$[w,s,\theta,\zeta,h_z,\xi]=[W(z),S(z),\Theta(z),Z(z),K(z)]\exp\left(ik_x+ik_y+nt\right), \tag{30}$$

Where k_x *and* k_y *are the wave numbers in the x and y directions,* $k=\left(k_x{}^2+k_y{}^2\right)^{1/2}$ *is the resultant wave number and n is the frequency of the harmonic disturbance, which is, in general, a complex constant. Using expression (30) in (27)-(29), (24) and (21) become*

$$\frac{n}{\in}\left(\frac{d^2}{dz^2}-k^2\right)W=-gk^2\alpha\Theta-\frac{1}{k_1}\left(\upsilon+\upsilon' n\right)\left(\frac{d^2}{dz^2}-k^2\right)W-\frac{mNn}{\in\rho_0\left(\frac{m}{K'}n+1\right)}\left(\frac{d^2}{dz^2}-k^2\right)W$$
$$-\frac{2\Omega}{\in}\frac{dZ}{dz}+\frac{\mu_e H}{4\pi\rho_0}\frac{d}{dz}\left(\frac{d^2}{dz^2}-k^2\right)K, \tag{31}$$

$$\frac{n}{\in}Z=-\frac{1}{k_1}\left(\upsilon+\upsilon' n\right)W-\frac{mNn}{\in\rho_0\left(\frac{m}{K'}n+1\right)}Z-\frac{2\Omega}{\in}\frac{dW}{dz}+\frac{\mu_e H}{4\pi\rho_0}\frac{d}{dz}DX, \tag{32}$$

$$\in nX=H\frac{dZ}{dz}+\in\eta\left(\frac{d^2}{dz^2}-k^2\right)X, \tag{33}$$

$$\in nK=H\frac{dW}{dz}+\in\eta\left(\frac{d^2}{dz^2}-k^2\right)K, \tag{34}$$

$$(E+b\in)n\Theta=\beta(W+bS)+\kappa\left(\frac{d^2}{dz^2}-k^2\right)\Theta, \tag{35}$$

Equations (31) – (35) in non-dimensional form, become

$$\left[\frac{\sigma}{\in}\left(1+\frac{M}{1+\tau_1\sigma}\right)+\frac{1+F\sigma}{P_l}\right]\left(D^2-a^2\right)W+\frac{g\alpha a^2d^2\Theta}{\upsilon}+\frac{2\Omega d^3}{\in\upsilon}DZ-\frac{\mu_e Hd}{4\pi\rho_0\upsilon}\left(D^2-a^2\right)DK=0, \tag{36}$$

$$\left[\frac{\sigma}{\in}\left(1+\frac{M}{1+\tau_1\sigma}\right)+\frac{1+F\sigma}{P_l}\right]Z=\left(\frac{2\Omega d}{\in\upsilon}\right)DW-\frac{\mu_e Hd}{4\pi\rho_0\upsilon}DX, \tag{37}$$

$$\left(D^2-a^2-p_1\sigma\right)X=-\left(\frac{Hd}{\in\eta}\right)DZ, \tag{38}$$

$$\left(D^2-a^2-p_2\sigma\right)K=-\left(\frac{Hd}{\in\eta}\right)DW, \tag{39}$$

$$\left(D^2-a^2-Ep_1\sigma\right)\Theta=-\left(\frac{\beta d}{\kappa}\right)\left(\frac{B+\tau_1\sigma}{1+\tau_1\sigma}\right)W, \tag{40}$$

where we have put $a=kd$, $\sigma=\frac{nd^2}{\upsilon}$, $\tau=\frac{m}{K'}$, $\tau_1=\frac{\tau\upsilon}{d^2}$, $F=\frac{\upsilon'}{d^2}$, $E_1=E+b\in$, B = b+1, $P_l=\frac{k_1}{d^2}$, is the dimensionless medium permeability, $p_1=\frac{\upsilon}{\kappa}$, is the thermal Prandtl number, $p_1=\frac{\upsilon}{\eta}$, is the magnetic Prandtl number and $D^*=d\frac{d}{dz}$ and the superscript * is suppressed. Applying the operator $\left(D^2-a^2-p_2\sigma\right)$ to the equation (37) to eliminate X between equations (37) and (38), we get

$$\left\{\left[\frac{\sigma}{\in}\left(1+\frac{M}{1+\tau_1\sigma}\right)+\frac{1+F\sigma}{P_l}\right]\left(D^2-a^2-p_2\sigma\right)+\frac{Q}{\in}D^2\right\}W=\frac{2\Omega d^2}{\upsilon}\left(D^2-a^2-p_2\sigma\right)DW\cdot \tag{41}$$

Eliminating K, Θ and Z between equations (36) – (41), we obtain

$$\begin{aligned}&\left\{\left[\frac{\sigma}{\in}\left(1+\frac{M}{1+\tau_1\sigma}\right)+\frac{1+F\sigma}{P_l}\right]\left(D^2-a^2\right)\left(D^2-a^2-p_2\sigma\right)\left(D^2-a^2-E_1p_1\sigma\right)\right\}W\\&+Ra^2\lambda\left(\frac{B+\tau_1\sigma}{1+\tau_1\sigma}\right)\left(D^2-a^2-p_2\sigma\right)W+\frac{Q}{\in}\left(D^2-a^2\right)\left(D^2-a^2-E_1p_1\sigma\right)W\\&=\frac{\frac{T_A}{\in^2}\left(D^2-a^2-p_2\sigma\right)^2\left(D^2-a^2-E_1p_1\sigma\right)}{\left[\frac{\sigma}{\in}\left(1+\frac{M}{1+\tau_1\sigma}\right)+\frac{1+F\sigma}{P_l}\right]\left(D^2-a^2-p^2\sigma\right)+\frac{Q}{\in}D^2}D^2W=0,\end{aligned} \tag{42}$$

where: $R=\frac{g_0\alpha\beta d^4}{\upsilon\kappa}$, is the thermal Rayleigh number, $Q=\frac{\mu_e H^2d^2}{4\pi\rho_0\upsilon\eta}$, is the Chandrasekhar number and $T_A=\left(\frac{2\Omega d^2}{\upsilon}\right)^2$, is the Taylor number.

Here we assume that the temperature at the boundaries is kept fixed, the fluid layer is confined between two boundaries and adjoining medium is electrically non conducting. The boundary conditions appropriate to the problem are [Chandrasekhar, (1981); Veronis, (1965)]

$$W=D^2W=DZ=\Theta=0 \quad \text{at } z = 0 \text{ and } 1 \tag{43}$$

and the components of h are continuous. Since the components of the magnetic field are continuous and the tangential components are zero outside the fluid, we have

$$DK = 0, \tag{44}$$

on the boundaries. Using the boundary conditions (4.14) and (4.15), we can show that all the even order derivatives of W must vanish for z = 0 and z = 1 and hence, the proper solution of equation (4.13) characterizing the lowest mode is

$$W = W_0 \sin \pi z; \quad W_0 \text{ is a constant.} \tag{45}$$

Substituting equation (45 in (42), we obtain the dispersion relation

$$\begin{aligned} R_1 x\lambda = &\left[\frac{i\sigma_1}{\in}\left(1+\frac{M}{1+\tau_1\pi^2 i\sigma_1}\right)+\frac{1+F\pi^2 i\sigma_1}{P}\right](1+x)(1+x+E_1p_1i\sigma_1)\left(\frac{1+\tau_1\pi^2 i\sigma_1}{B+\tau_1\pi^2 i\sigma_1}\right) \\ &+\frac{Q}{\in}(1+x)(1+x+E_1p_1i\sigma_1)\left(\frac{1+\tau_1\pi^2 i\sigma_1}{B+\tau_1\pi^2 i\sigma_1}\right)+\frac{\frac{T_A}{\in^2}(1+x+E_1p_1i\sigma_1)}{\frac{i\sigma_1}{\in}\left(1+\frac{M}{1+\tau_1\pi^2 i\sigma_1}\right)+\frac{1+F\pi^2 i\sigma_1}{P}}\left(\frac{1+\tau_1\pi^2 i\sigma}{B+\tau_1\pi^2 i\sigma_1}\right), \end{aligned} \tag{46}$$

where $R_1=\frac{R}{\pi^4}, T_{A_1}=\frac{T_A}{\pi^4}, x=\frac{a^2}{\pi^2}, i\sigma_1=\frac{\sigma}{\pi^4}, P=\pi^2 P_l, Q_1=\frac{Q}{\pi^4}$.

Equation (46) is required dispersion relation accounting for the effect of suspended particles, magnetic field, medium permeability, variable gravity field, rotation on thermal convection in Rivlin-Ericksen elastico-viscous fluid in porous medium.

STABILITY OF THE SYSTEM AND OSCILLATORY MODES

Here we examine the possibility of oscillatory modes, if any, in Rivlin-Ericksen elastico-viscous fluid due to the presence of suspended particles, rotation, magnetic field, viscoelasticity and variable gravity field. Multiply equation (36) by W^* the complex conjugate of W, integrating over the range of z and making use of equations (37)-(70) with the help of boundary conditions (43) and (44), we obtain

$$\begin{aligned} &\left[\frac{\sigma}{\in}\left(1+\frac{M}{1+\tau_1\sigma}\right)+\frac{1+F\sigma}{P_l}\right]I_1 - \frac{\mu_e \in \eta}{4\pi\rho_0}\left(\frac{1+\tau_1\sigma^*}{B+\tau_1\sigma}\right)(I_2+p_2\sigma^* I_3) - \frac{\alpha a^2\lambda g_0\kappa}{\upsilon\beta}\left(\frac{1+\tau_1\sigma^*}{B+\tau_1\sigma}\right)(I_4+E_1p_1\sigma^* I_5) \\ &+d^2\left[\frac{\sigma^*}{\in}\left(1+\frac{M}{1+\tau_1\sigma}\right)+\frac{1+F\sigma^*}{P_l}\right]I_6+\frac{\mu_e\in\eta d^2}{4\pi\rho_0}\left(\frac{1+\tau_1\sigma^*}{B+\tau_1\sigma}\right)(I_7+p_2\sigma^* I_8)=0, \end{aligned} \tag{47}$$

where

$$I_1=\int_0^1\left(|DW|^2+a^2|W|^2\right)dz, I_2=\int_0^1\left(\left|D^2K\right|^2+a^2K^2+2a^2|DK|^2\right)dz, I_3=\int_0^1\left(|DK|^2+a^2|K|^2\right)dz, I_4=\int_0^1\left(|D\Theta|^2+a^2|\Theta|^2\right)dz,$$

$$I_4=\int_0^1\left(|D\Theta|^2+a^2|\Theta|^2\right)dz,\ I_5=\int_0^1\left(|\Theta|^2\right)dz, I_6=\int_0^1\left(|Z|^2\right)dz,\ I_7=\int_0^1\left(|DX|^2+a^2|X|^2\right)dz,\ I_5=\int_0^1\left(|X|^2\right)dz$$

The integral part I_1-I_8 are all positive definite. Putting $\sigma = i\sigma_i$ in equation (5.1), where σ_i is real and equating the imaginary parts, we obtain

$$\left\{\begin{aligned} &\left[\frac{1}{\in}\left(1+\frac{M}{1+\tau_1 i\sigma_i}\right)+\frac{F}{P_l}\right]\left(I_1-d^2I_4\right) \\ &-\frac{\mu_e\in\eta}{4\pi\rho_0}\left(\frac{1+\tau_1\sigma^*}{B+\tau_1\sigma}\right)\left(\left(\frac{\tau_1(B-1)}{B^2+\tau_1^2\sigma_i^2}\right)I_2+\left(\frac{B+\tau_1^2\sigma_i^2}{B^2+\tau_1^2\sigma_i^2}\right)p_2I_3\right) \\ &+\frac{\alpha a^2\lambda g_0\kappa}{\upsilon\beta}\left[\left(\frac{\tau_1(B-1)}{B^2+\tau_1^2\sigma_i^2}\right)I_4+\left(\frac{B+\tau_1^2\sigma_i^2}{B^2+\tau_1^2\sigma_i^2}\right)E_1p_1I_5\right]d^2 \\ &+\frac{\mu_e\in\eta d^2}{4\pi\rho_0}\left(\frac{\tau_1(B-1)}{B^2+\tau_1^2 i\sigma_i^2}\right)I_6+\left(\frac{B+\tau_1^2\sigma_i^2}{B^2+\tau_1^2\sigma_i^2}\right) \end{aligned}\right\}=0, \tag{48}$$

Equation (48) implies that $\sigma_i = 0$ or $\sigma_i \neq 0$ which mean that modes may be non oscillatory or oscillatory. The oscillatory modes introduced due to presence of rotation, magnetic field, suspended particles, viscoelasticity and variable gravity field.

THE STATIONARY CONVECTION AND DISCUSSION

For stationary convection putting $\sigma = 0_i$ in equation (46) reduces it to

$$R_1=\frac{1+x}{\lambda xB}\left[\frac{1+x}{P}+\frac{Q_1}{\in}+\frac{T_{A_1}(1+x)P}{\{\in(1+x)+Q_1P\}\in}\right], \tag{49}$$

which expresses the modified Rayleigh number R_1 as a function of the dimensionless wave number x and the parameters T_{A_1}, B, P, Q_1 and Rivlin-Ericksen elastico-viscous fluid behave like an ordinary Newtonian fluid since elastico-viscous parameter F vanishes with σ.

To study the effects of suspended particles, rotation and medium permeability, we examine the behavior of $\frac{dR_1}{dB}, \frac{dR_1}{dT_{A_1}}, \frac{dR_1}{dQ_1}$ and $\frac{dR_1}{dP}$ analytically. Equation yields

$$\frac{dR_1}{dB}=\frac{1+x}{\lambda x B^2}\left[\frac{1+x}{P}+\frac{Q_1}{\in}+\frac{T_{A_1}(1+x)P}{\{\in(1+x)+Q_1P\}\in}\right], \tag{50}$$

which is negative implying thereby that the effect of suspended particles is to destabilize the system when the gravity increases upward from its value g_0 (i.e., $\lambda\rangle 0$). Also in fig. 2, R_1 decreases with the increase in suspended particles parameter B. Thus suspended particles have destabilizing effect, which clearly verifies the result numerically. This destabilizing effect is an agreement of the earlier work of Scanlon and Segel [14] and Rana [11]. From equation (49), we get

$$\frac{dR_1}{dT_{A_1}}=\frac{1+x}{\lambda x B}\left[\frac{(1+x)P}{\{\in(1+x)+Q_1P\}\in}\right], \tag{51}$$

which shows that rotation has stabilizing effect on the system when gravity increases upwards from its value g_0 (i.e., $\lambda\rangle 0$). which is an agreement with the result derived by Sharma and Rana [17] and Rana [11]. Also in fig. 3, R1 increases with the increase in T_A. Thus rotation has stabilizing effect, which clearly verifies the result numerically.

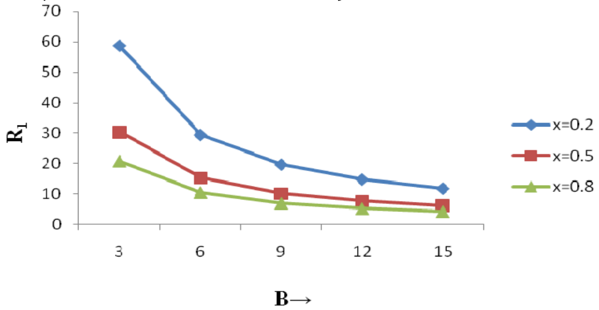

Fig.2. Variation of Rayleigh number R1 with suspended particles for $\lambda=2, T_{A_1}=5, Q_1=10, \in=0.2, P=0.2$, for fixed wave numbers x = 0.2, x = 0.5 and x = 0.8.

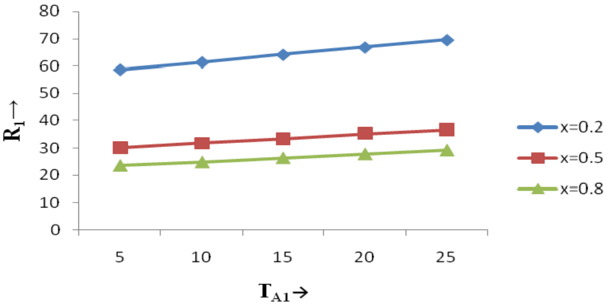

Fig.3. Variation of Rayleigh number R1 with rotation T_A for B = 3, $\lambda=2, Q_1=10, \in=0.2, P=0.2$, $Q_1=10$, for fixed wave numbers x = 0.2, x = 0.5 and x = 0.8.

From equation (49), we get

$$\frac{dR_1}{dQ_1}=\frac{1+x}{\lambda x B}\left[\frac{1}{\in}-\frac{T_{A_1}(1+x)P^2}{\{\in(1+x)+Q_1P\}^2\in}\right], \tag{52}$$

which implies that magnetic field stabilizes the system, if

$$\{\in(1+x)+Q_1P\}^2\rangle T_{A_1}(1+x)P^2$$

and destabilizes the system, if

$$\{\in(1+x)+Q_1P\}^2\langle T_{A_1}(1+x)P^2$$

when gravity increases upwards from its value g_0 (i.e., $\lambda\rangle 0$), which is an agreement with the result derived by Sharma and Rana [18], Bhatia and Stiener [1] and Sharma and Sunil [16]. Also in fig. 4, R_1 increases/decreases with the increase in magnetic field parameter Q_1, Hence, magnetic field has stabilizing/destabilizing effects, which clearly verify the result numerically.

In the absence of rotation, magnetic field has destabilizing effect on the system, when gravity increases upwards from its value g_0 (i.e., $\lambda\rangle 0$).

It is evident from equation (49) that

$$\frac{dR_1}{dP}=\frac{1+x}{\lambda x B}\left[\frac{1}{P^2}-\frac{T_{A_1}(1+x)}{\{\in(1+x)+Q_1P\}^2\in}\right], \tag{53}$$

From equation (53), we observe that medium permeability has destabilizing effect when $\{\in(1+x)+Q_1P\}^2\langle T_{A_1}(1+x)P^2$ and medium permeability has a stabilizing effect when $\{\in(1+x)+Q_1P\}^2\rangle T_{A_1}(1+x)P^2$, when gravity increases upwards from its value g_0 (i.e., $\lambda\rangle 0$).

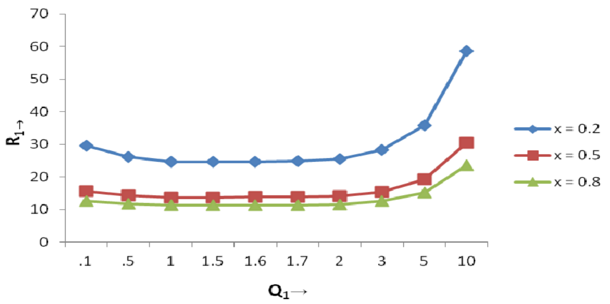

Fig.4. Variation of Rayleigh number R1 with magnetic field Q_1 for $\lambda = 2, T_{A_1} = 5, Q_1 = 10, \in = 0.2, P = 0.2$ B = 3, for fixed wave numbers x = 0.2, x = 0.5 and x = 0.8.

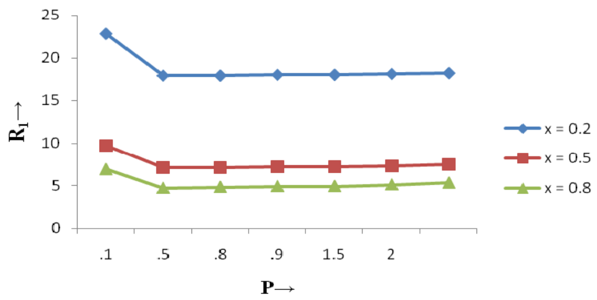

Fig.5. Variation of Rayleigh number R1 with medium permeability P for B = 3, $\lambda = 2, T_{A_1} = 5, Q_1 = 10, \in = 0.2,$ for fixed wave numbers x = 0.2, x = 0.5 and x = 0.8.

Also in fig. 5, R1 increase/decrease with the increase in medium permeability parameter P. Hence, medium permeability has destabilizing/stabilizing effects, which clearly verify the result numerically. This destabilizing effect is an agreement of the earlier work of Scanlon and Segel [14], Sharma and Rana [17], Sharma [15] and Rana [11].

NOMENCLATURE & GREEK SYMBOLS

q - Velocity of fluid
qd - Velocity of suspended particles
p - Pressure
g - Gravitational acceleration vector
g - Gravitational acceleration
k_1 - Medium permeability
T - Temperature
t - Time coordinate
cf - Heat capacity of fluid
Cpt - Heat capacity of particles
mN - Mass of particle per unit volume
k - Wave number of disturbance
kx, ky - Wave numbers in x and y directions
p1 - Thermal Prandtl number
Pl - Dimensionless medium permeability
ε - Medium porosity
ρ - Fluid density
μ - Fluid viscosity
μ' - Fluid viscoelasticity
υ - Kinematic viscosity
ν' - Kinematic viscoelasticity
η - Particle radius
k - Thermal diffusitivity
α - Thermal coefficient of expansion
β - Adverse temperature gradient
ϑ - Perturbation in temperature
n - Growth rate of the disturbance
δ - Perturbation in respective physical quantity
ζ - Z-component of vorticity
Ω - Rotation vector having components (0, 0, Ω)
μe - Magnetic permeability

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