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VERIFICATION OF AN OPTIMAL COMPOSITE SANDWICH STRUCTURE BY FINITE ELEMENT ANALYSIS

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ABSTRACT: *This study shows the optimization method for a new complex structural model [laminated carbon fiber reinforced plastic (CFRP) deck plates with polystyrene foam (EPS) inner layer]. The structure is designed for minimal weight taking into consideration 6 design constraints. After optimization the structure is examined using finite element analysis, which includes the determination of deflections and stress distributions and calculation of eigenfrequencies.*
KEYWORDS: *laminated carbon fiber reinforced plastic (CFRP), polystyrene foam (EPS), optimization*

INTRODUCTION

Fibre composites have many advantageous properties, which can not be available by other materials, e.g. high strength, low density, good thermal insulation, corrosion resistance, high bending stiffness, good vibration damping and aesthetic appearance. These advantageous characteristics can be due to special material structure.

Composites are used in many industries (space-, military-, automotive-, construction-, machine- and chemical industry) due to the above mentioned advantageous properties. Application of composite materials instead of traditional metals provides significant weigh saving due to the low density of composite materials. Weight reduction is a very important property which is utilized in many fields of industry, e.g. automotive-, air and space- and construction industries.

Design procedure, material- and shape optimization of composite structures are much more complex compared to traditional metal structures due to orthotropic or anisotropic nature of composites [1, 3, 4, 5, 6]. Unfortunately simple practical design procedures and calculation methods are not available due to this complexity.

A NEW SANDWICH STRUCTURE

There is a basic requirement in case of many structures that these should be light weight, cost effective and good mechanical and heat insulation properties. A new sandwich plate model under consideration provides an alternative for the above mentioned requirements. A new model is depicted in Figure 1. The CFRP (Carbon Fibre Reinforced Plastic) plates are constructed from laminated layers. All of the fibers of a layer and laminate are arranged in the longitudinal direction. The fibre volume fraction of a layer is 61% and the matrix volume fraction is 39%.

The inner layer is an EPS (high strengths polystyrene insulator foam). The inner layer is glued to the upper and lower deck plates. This kind of structures can be applied in many applications (e.g. space-, military-, automotive-, construction-, machine- and chemical industry).

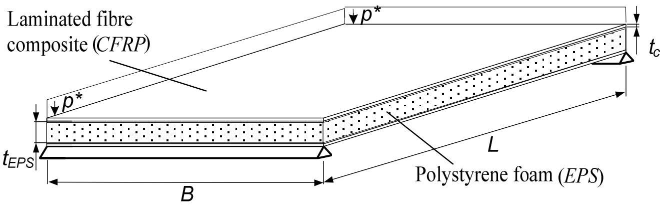

Figure 1: Sandwich structure

The aim of the calculation is the optimal design of a sandwich structure (Figure 1). The structure is simply supported, and a uniformly distributed loading of $3.5\cdot10^{-3}$ N/mm^2 (p^*) acts on the total surface of the structure (in the calculations the p = 7 N/mm line pressure was taking into consideration). The dimensions of the structure are: L = 2250 mm, B = 2000 mm.

The material parameters of a pre-impregnated CFRP layer are given as follows: the thickness of a layer t^* = 0.2 mm ($t_c = n\cdot t^*$), the longitudinal Young's modulus $E_x = E_c$ =120 GPa and the transverse modulus E_y = 9 GPa. The shear modulus are $G_{xy} = G_{yz} = G_{xz}$ = 4.4 GPa. The specific mass of the CFRP plate ρ_c = 180 g/m^2, and Poisson's ratios ν_{xy} = 0.25 and ν_{yx} = 0.019.

Table 1 includes the parameters of different EPS layers applied during the optimisation. The density is denoted by

Table 1: Parameters of different EPS layers

	EPS 30	EPS 70	EPS 100	EPS 150	EPS 200
$\rho_{EPS}\left[\frac{kg}{mm^3}\right]$	$10\cdot10^{-9}$	$15\cdot10^{-9}$	$20\cdot10^{-9}$	$25\cdot10^{-9}$	$30\cdot10^{-9}$
$K_{EPS}\left[\frac{W}{mK}\right]$	0.048	0.04	0.036	0.034	0.033

ρ_{EPS} and K_{EPS} is the thermal conductivity of the different EPS layers (EPS 30, EPS 70, EPS 100, EPS 150 and EPS 200). E_{EPS} = 3.5 MPa is the Young's modulus.

OBJECTIVE FUNCTION AND DESIGN CONSTRAINTS

The aim of the study is the optimal design of a new sandwich structure. The structure was designed for minimal weight. Design constraints on maximum deflection of the total structure, stress in the composite plates, stress in the polystyrene foam, eigenfrequency of the structure, thermal insulation of the structure and size constraints for design variables (number of layers of deck plates - n; thickness of polystyrene inner layer - t_{EPS}) are considered in the calculation. The systematic searching method was used in the single objective optimization which examines all of possible variations. The calculations were completed by the application of MathCAD software.

Mass function

The total mass of the structure is the sum of the CFRP and EPS components:

$$m = 2\,\rho_c\,[B\,L\,(n\,t^*)] + \rho_{EPS}\,[B\,L\,t_{EPS}] \tag{1}$$

where t^* is the thickness of a laminate, $\rho_c = 180\cdot 10^{-9}$ kg/mm^3 is the density of the laminate, ρ_{EPS} is the density of one of the applied insulator inner layer (see in Table 1).

DESIGN CONSTRAINTS

Deflection of the total structure

$$w_{\max} = \frac{5p\,L^4}{384(E_c I_c + E_{EPS} I_{EPS})} \le \frac{L}{200} \tag{2}$$

where: I_c, I_{EPS} are moment of inertia of the CFRP plate and EPS layer,
E_c, E_{EPS} are reduced modulus of elasticity of the CFRP lamina and Young's modulus of EPS component.

Stress in the composite plates

The moment acting on the total structure is distributed on the components of the structure. X_cM is the part of total moment which is acting on composite plate.

$$\frac{X_c M}{I_c}\cdot\frac{h_{EPS} + nt}{2} \le \sigma_{Call} \tag{3}$$

where: $M = \frac{pL^2}{8}$; $\sigma_{Call} = \frac{\sigma_T}{\gamma_c}$ allowable stress; X_cM moment acting on composite plate; σ_T tensile strength of composite lamina; γ_c safety factor (=2).

Stress in the polystyrene foam

$$\frac{X_{EPS} M}{I_{EPS}}\cdot\frac{t_{EPS}}{2} \le \sigma_{EPSall} \tag{4}$$

where: $X_{EPS} = \frac{E_{EPS} I_{EPS}}{E_{EPS} I_{EPS} + E_c I_c}$; $\sigma_{EPSall} = \frac{\sigma_{EPS\max}}{\gamma_{EPS}}$ allowable stress; $X_{EPS}M$ moment acting on EPS layer; σ_{EPSmax} compression strength of EPS; γ_{EPS} safety factor (=2).

Eigenfrequency of the total structure

$$f_1 = \frac{\pi}{2L^2}\sqrt{\frac{10^3(E_c I_c + E_{EPS} I_{EPS})}{m}} \ge f_0 \tag{5}$$

m: weight/unit length of the structure [kg/mm], f_0: limitation for eigenfrequency (50 Hz).

Thermal insulation of the structure

There is an assumption that the temperature out of the structure is (T_{out}) -20°C (253.15 K), inner the structure is (T_{in}) +15°C (288.15 K). The global thermal conductivity of composite laminate is, K_{CFRP} = 0.86 W/mK and the thermal conductivity of insulator layer is provided by the manufacturer (see in Table 1).

Figure 2 shows the section of the structure and parameters of structural components. There is a constraint for the maximal heat loss of the structure (Q_{all}), the value of it is 30 W (on a unit surface F=1 m^2).

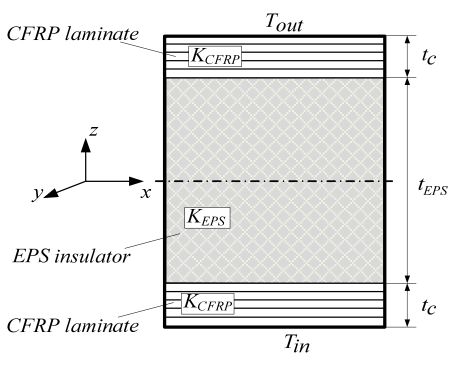

Figure 2: Section of the structure

Based on the Fourier principle [7, 8] the next equation (6) inequality can be written for the sandwich structure, assumed that the structural components are homogenous:

$$Q_{act} = F\,\frac{T_{in} - T_{out}}{\frac{t_c}{K_{CFRP}} + \frac{t_{EPS}}{K_{EPS}} + \frac{t_c}{K_{CFRP}}} \le Q_{all} \quad \text{[W]} \tag{6}$$

Size constraints for design variables

$$8 \le n \le 12 \qquad [mm] \qquad (7)$$
$$30 \le t_{EPS} \le 80$$

These represent physical limitations on the design variables [mm], taking economical and manufacturing aspects into consideration.

NUMERICAL RESULTS OF SINGLE OBJECTIVE OPTIMIZATION

Mass saving can be a prime design aim of sandwich structures because the composite materials are very expensive. Table 2 shows the result of mass optimization of the analyzed structure based on the mass objective function (eq. 1) and design constraints (eqs. 2-7). The examination includes variation of different numbers of deck layers (8, 10 and 12 pieces) and different types of EPS materials.

The obtained total mass for case of different structure alternatives are as follows (Table 2)

Table 2: Result of mass optimization

Mass optimization: m [kg]						
Number of layers in CFRP deck plates: n [pieces]	Thickness of inner EPS foam layer: t_{EPS} [mm]	Type of applied EPS polystyrene inner foam layer				
		EPS 30	EPS 70	EPS 100	EPS 150	EPS 200
In case of 8 CFRP layers in a deck plate	30	-	-	-	-	-
	40	-	-	-	-	-
	50	-	16.335	17.46	18.585	19.71
	60	15.66	17.01	18.36	19.71	21.06
	80	16.56	18.36	20.16	21.96	23.76
In case of 10 CFRP layers in a deck plate	30	-	-	-	-	-
	40	-	-	-	-	21.6
	50	-	19.575	20.7	21.825	22.95
	60	18.9	20.25	21.6	22.95	24.3
	80	19.8	21.6	23.4	25.2	27.0
In case of 12 CFRP layers in a deck plate	30	-	-	-	-	-
	40	-	-	-	23.94	24.84
	50	-	22.815	23.94	25.065	26.19
	60	22.14	23.49	24.84	26.19	27.54
	80	23.04	24.84	26.64	28.44	30.24

There are some cells of the table where mass data are not found, because in these cases one of/more design constraints are not fulfilled. These critical constraints were the maximum deflection and the thermal insulation of the total structure.

It can be summarised that the total mass of the structure is increasing by the increasing of the number of layers of deck plates. The optimal sandwich structure is a laminated deck plates with 8 CFRP layers with EPS 30 typed inner layer which thickness is 60 mm.

FINITE ELEMENT ANALYSIS OF THE ORTIMAL STRUCTURE

In this section, we present finite element analysis of a multilayered plate subjected to a uniformly distributed load of intensity $p^* = 3.5 \cdot 10^{-3}$ N/mm^2. The pressure load is applied to the top cover sheet as shown in Figure 1. The rectangular plate is simply supported on two opposite edges and free on the remaining two edges (see Figure 3).

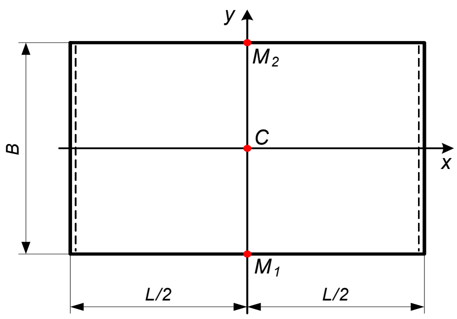

Figure 3: Three different points on the plate

Model description

We employ the conventional shell model which uses two principal assumptions: The normal of original mid-surface remain straight during the deformation and the stress in the direction normal to the mid-surface is zero. Our finite element analysis is based on the Mindlin-Reissner shell theory, which takes into account the transverse shear deformation. In addition, small displacements and strains are assumed.

To investigate the stress distribution and deflection, we set up a finite element model (three-layer thin shell) in Adina finite element analysis software [9]. An 8-node shell element is employed in present analyses. More detailed descriptions of finite element procedures can be found in Bathe's book [10].

Validation of the optimal plate

First, we study a case in which the material model is isotropic in all three layers. For simplicity, we declare three points on the plate to compare the approximate analytical results with the numerical results. Let the point C be the central point of the plate and the mid-points M_1 and M_2 take on the free edges.

Under the previous boundary conditions, the plate behaves like the saddle shape hyperbolic paraboloid and the point C will be its saddle point. Of course, our finite element solution shows a good agreement with approximate analytical results, which are based on deflection of beam as seen in equation (2).

Table 3: Comparison of numerical results in isotropic case

	Designed constrains	Finite element result	
		at the point C	at the point M
Deflection of the structure	3.205	3.039	3.368
Stress in the composite plates	22.472	22.40	23.79
Stress in the polystyrene foam	$5.4 \cdot 10^{-5}$	$2.64 \cdot 10^{-6}$	$2.8 \cdot 10^{-6}$

In further discussion we assume that the material model of CFRP laminates is orthotropic and the mechanical behaviour of EPS insulator remains isotropic. The corresponding material properties are given in Section 2. If we repeat the finite element analysis with the orthotropic material model, we get the following results.

As appeared from the numerical simulation the maximum value of deformation is 4.501 mm and it occurs at points M_1 and M_2 (see Figure 3 and 4) and the deflection is 1.124 mm at the point C. The maximum effective stress is 28.13 MPa, but it decreases quickly in the vicinity of points M (see Figure 5). The stress value is about 19 MPa at the point C on the cover sheet. Observe that these values are close enough to results, which are derived from the equations (2, 3, 4). Thus in this particularly case the finite element results are shown an acceptable agreement with the approximate analytical results. The first three eigenfrequencies from the finite element analysis are the follows:

f_1 = 193.5 Hz; f_2 = 203.5 Hz; f_3 = 537.0 Hz

Notice that from equation (5) the first eigenfrequency of the whole structure is 172.71 Hz.

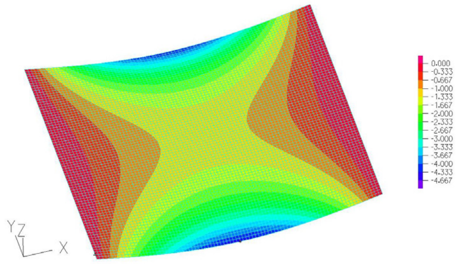

Figure 4: Deformation of the mid-surface in the z axis of the global coordinate system

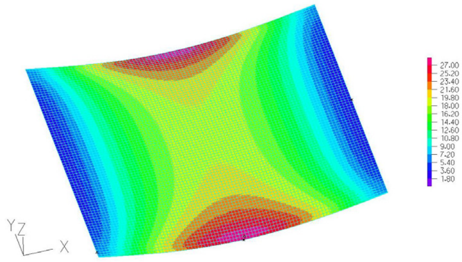

Figure 5: Effective stress distribution on the cover sheets

Consequently, the approximate analytical calculation is used well in the optimization of multilayered plates.

CONCLUSIONS

There is a basic requirement in case of many structures that these should be light weight, cost effective and good mechanical and heat insulation properties. A new sandwich plate model under consideration provides an alternative for the above mentioned requirements.

Optimization method of a new complex structural model was elaborated. The structure was built from laminated carbon fiber reinforced plastic (CFRP) deck plates and polystyrene foam (EPS) inner layer. The structure was designed for minimal weight taking into consideration 6 design constraints.

In an optimum design procedure the type and thickness of polystyrene foam inner layer as well as number of layers of laminated plates are determined, which fulfil the design constraints and minimize the mass of the total structure.

In summary, the deflections and effective stresses of multilayered plates are determined by FEM. When we compare the finite element analysis with the approximate analytical solutions, we find an acceptable difference. Under this condition the approximate analytical formulae for the deflection and stress calculations in Section 3 are adequate to use in optimization.

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