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INFLUENCE OF GEOMETRIC PARAMETERS ON THE MINIMUM THICKNESS OF THE HARDENED LAYER OF THE TOOTH FLANK

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ABSTRACT: *This article describes the effect of the geometrical parameters of the standard involute and non-standard convex-concave (C-C) gearing on the thickness of the hardened layer. The thickness of hardened layer is important from the aspect of wear on gearing. In case of involute gearing is the thickness of the hardened layer defined by various authors, what is on the other hand determinated also by the standard STN 01 4686-5. In case of C-C gearing there are not available any standards, and therefore it is possible to determine the thickness of the hardened layer only by the means of modern simulation methods.*
KEYWORDS: *Non-standard gearing, convex-concave (C-C) gearing, involute gearing, geometrical paremeters of gearing, hardened layer thickness, load carrying capacity of gearings.*

INTRODUCTION

The simplest gearing is created with one pair of meshing gears, where the tooth faces are creating a kinematics couple. This kinematics couple serves to the transition of rotational movement and mechanical energy. The basic criterion of a gearing is to acquire a continual tooth mesh what is defined in the fundamental law of gearing Fig. 1: the continual mesh of two profiles occurs, when the mutual normal line in the point of contact proceeds through the pole of relative motion C (pitch point) in every moment [1]. Point C then divides the axis spacing in ratio, which equals to the gear ratio [2]:

$$i=\frac{\omega_1}{\omega_2}=\frac{z_2}{z_1}=\frac{r_{w2}}{r_{w1}}=\frac{O_2C}{O_1C}=const. \qquad (1)$$

where: $\omega_{1,2}$ - angular speed (velocity), $z_{1,2}$ - number of teeth, $r_{w1,2}$ -rolling radius

The elemental displacement of both profiles in the direction of the normal line has to fulfill the criteria (1), it means that the normal components of the peripheral velocities must be equal:

$$v_{n1}=v_{n2}=v_n \qquad (2)$$

$v_{n1,2}$ - normal components of peripheral velocity.

The profiles meeting the requirements (1) and (2) are called conjugated profiles and the C-C gearing must also fulfill these requirements. Except of the constant gear ratio (whit exception, when it does not acts on the gearing with variable gear ratio) the gearing in general has to meet also other requirements [3]:

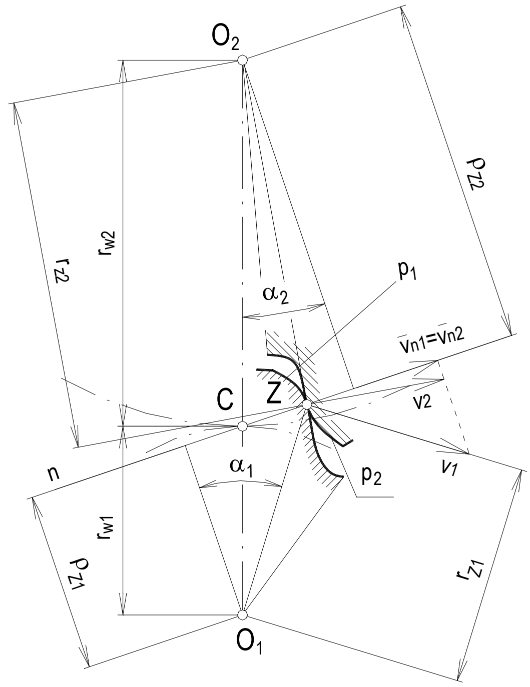

Figure 1. Criterion of correct gear mesh

a) *Functional requirement: teeth not undercut, determined width of tooth addendum (s_a=0,2 m_n or s_a=0,4m_n), absence of interference, minimum meshing duration (a_ε>1,2)*
b) *Operation requirements: similar values of bending strength and gearing slides, high stiffness in contact pressure transmission, wear resistance.*

c) Economical requirements: simple manufacturing, simple control and assembly, easy operation and maintenance.

d) Continuous and silent operation at the required period of service.

MATERIAL AND METHODOLOGY

It is evident from the fundamental law of gearing, that the main structural element of the involute gearing is the tooth face profile, characterized by the known basic geometrical parameters [1,2,3,4].

C-C gearing – the path of contact is created from two arcs with inflection point C (Fig.2): $z_{1,2}$, m_n, α_C, r_{kh}, r_{kd} and the tool parameters h_a^*, h_p^*, r_f^*.

Geometrical parameters and the shape of the path of contact clearly determines the type of the gear. The meshing conditions of the spur gears are defined on the face plane, where the general point of contact X is translating through the path of contact defined with point AE, while for the moment are two teeth in mesh (points AB, DE) – Fig. 3,4.

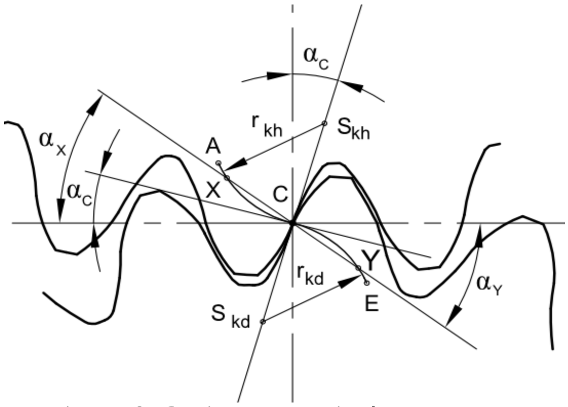

Figure 2. Basic geometrical parameters of the C-C gearing [4]

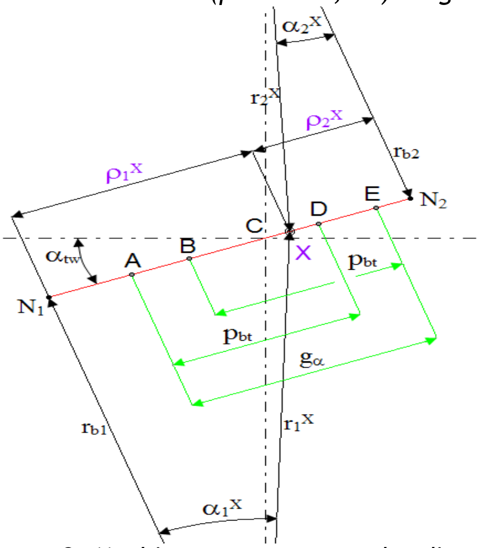

Figure 3. Meshing parameters and radius of curvature of involute gearing

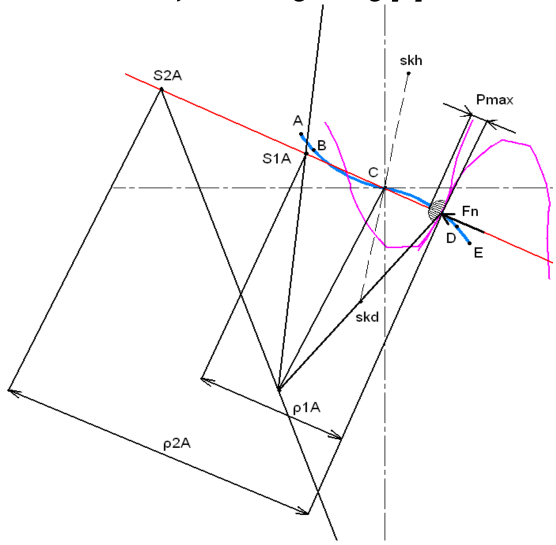

Figure 4. Meshing parameters and radius of curvature for C-C gearing [5]

For defining the reduced radius of curvature S_{red} in case of involute gearing (mesh of two convex surfaces) the following equation applies:

$$\frac{1}{\rho_{red}}=\frac{1}{\rho_1}+\frac{1}{\rho_2}=\frac{\rho_1+\rho_2}{\rho_1 \cdot \rho_2} \tag{3}$$

where: $\rho_{1,2}$ - radius of curvature

In case of C-C gearing and also the inner involute gearing (mesh of concave – and convex + surface) the following applies:

$$\frac{1}{\rho_{red}}=\frac{1}{\rho_1}-\frac{1}{\rho_2}=\frac{\rho_2-\rho_1}{\rho_1 \cdot \rho_2} \tag{4}$$

The magnitude of Hertz´s pressure can be determined by [5]:

$$\sigma_H = 0,418 \, . \sqrt{\frac{F_N}{b} . \frac{2.E_1.E_2}{E_1+E_2} . \left(\frac{1}{\rho_1} \pm \frac{1}{\rho_2}\right)} \tag{5}$$

where: ρ_H - reduced radius of curvature by Hertz, F_N - normal force, b - face width, E_1, E_2, μ_1, μ_2 - are elastic constants of materials of each cylinder.

The tangentional component of the peripheral velocity plays a significant role, where with the right correction of the gearing it is possible to influence and reduce the magnitude of friction which comes whit a temperature reduction in the contact area of the gearing. The calculation of the tangentional components of the peripherical velocity in case of C-C gearing is more complicated [6] (the location and orientation of the components of the peripherical velocity is changing) because of the shape of the path of contact. Distribution of the normal and tangentional components of the velocity in the involute gearing is evident from Fig 5, in case of C-C gearing (Fig. 6) these components cannot be clearly determined due to the curved shape of the path of contact.

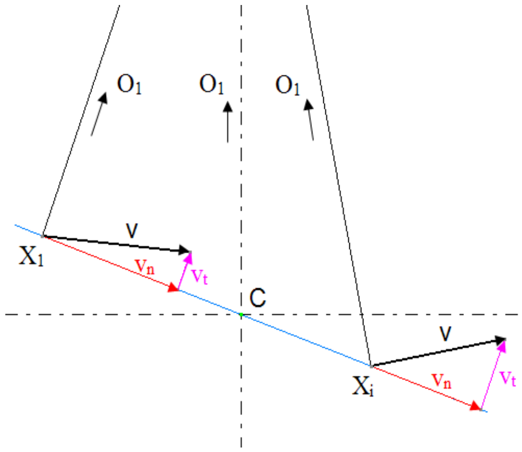

Figure 5. Distribution of velocity components in case of involute gearing [6]

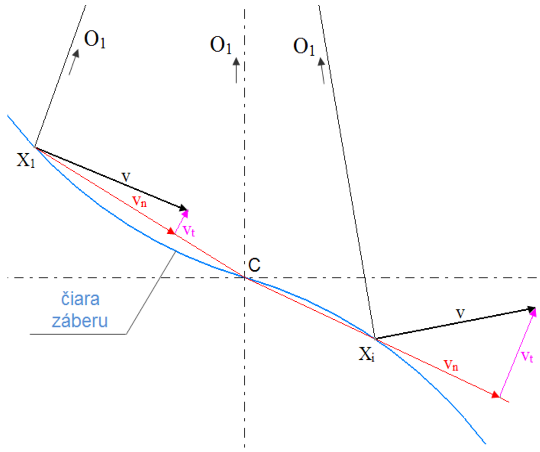

Figure 6. Distribution of velocity components in case of C –C gearing [6]

The determination of the gearing slide could be done with the following expressions:

$$\vartheta_1 = \frac{v_{t1} - v_{t2}}{v_{t1}}\ , \quad \vartheta_2 = \frac{v_{t2} - v_{t1}}{v_{t2}} \tag{6}$$

where: $v_{t1,2}$ - tangential component of peripheral velocity

From Figure 7 it is evident, that the values of gearing slide in case of C-C gearing are lower than in case of involute gearing [6].

Based on the aforementioned facts it is possible to compare the advantages and disadvantages of the involute and C-C gearings.

Involute gearing - the benefits are due to the characteristics of the involute profile:

- *simple profile and easy manufacturing,*
- *possibilities of tooth profile correction,*
- *correct mesh and constant gear ratio, even after the change of center distance,*
- *good reliability, lifetime and mechanical efficiency.*

The drawbacks of the involute gearing:

- *possibility of undercutting the tooth dedendum for the low number of teeth,*
- *risk of teeth tapering for the high angle of mesh,*
- *development of higher surface pressure (two convex surfaces engaging on each other- external gearing)*
- *development of higher gearing slide and higher losses by friction,*
- *noise and vibration,*
- *the rigid constraint not allows vibration damping and dynamical loading,*
- *requirements on the accuracy*

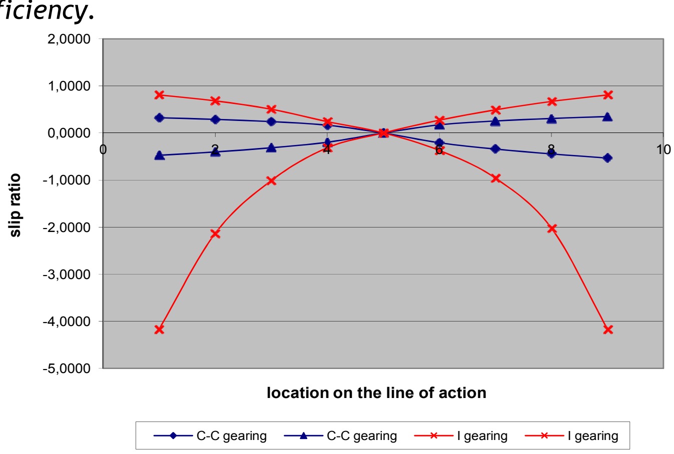

Figure 7. Course of the gearing slide ratio values

C-C gearing - the benefits comes from the advantage of mesh of convex tooth faces with concave tooth faces [7]:

- *lower contact pressures (high load carrying capacity in contact),*
- *better gearing slide ratio what can affect the lower wear, noisiness, and losses by friction with final, effect on the longer life time and durability*

C-C gearing disadvantages:

- *more complicated shape of tooth face,*
- *high requirements on finishing operation,*
- *reaching the accurate distance between axes.*

EXPERIMENTAL METHODS

In force transmissions the widely used base material is steel where at the maximum load rating (high carrying capacity) it has to preserve its stiffness with ductile core and increased hardness of tooth faces with the fallowing characteristics [7].

- *gear teeth resistant against brittle fracture at impact loading,*
- *high stiffness and hardness of tooth faces in contact,*
- *good resistance of tooth faces against wear and seizing,*
- *increasing of fatigue strength of surface layer in tensile.*

The geometry itself with the properly selected material does not ensure increased hardness of tooth faces, therefore it is necessary to modify the material by the means of thermochemical design: cementation, nitriding, carbonitriding, boriding, or surface hardening (Table 1).

Table 1. Datas of thermochemical surface engineering techniques [8]

Technology / Properties	Cemented	Carbonitriding	Nitriding	Nitrocemented	Borided	Surface hardened
Difunded elements	C	C+N	N	N+C, N+C+O	B	-
Temerature [°C]	850-950	600-630	500-550	820-860	800-1000	30-70 °C over Ac3, 30-70 °C over Ac1
Thickness of layer [mm]	0,15-0,2.mn (max. 2 mm)	0,05	0,2-0,6	0,4-0,8 (ammonia) 0,05-0,2 (bath)		2,5-6 or 1-2
Layer hardness [HV],[HRC]	60-62 HRC	1000 HV	60-65 HRC, (1000-1200 HV)	56-60 HRC, (700-800 HV)	1500-2000 HV	45-55 HRC

Hard layers established with the aforementioned technologies on the surface are of high resistance against abrasion, while the depth of the layers and the resulting hardness are not equal (Figure 8).

The nitride and carbonitride layers are the hardest and fatigue resistant, but they have small depth which not allow high loading with surface pressures (the core would deform and the nitrided layer would damage), therefore they are appropriate for gearings with intensive abrasive loadings and lower surface pressures. For higher surface pressures it is necessary to use alloyed steels with higher toughness and core strength.

Cemented and nitrocemented layers has lower hardness (~ 800HV) but are essentially thicker and therefore has good wear resistance and if the core strength is high enough they tolerate higher surface pressures.

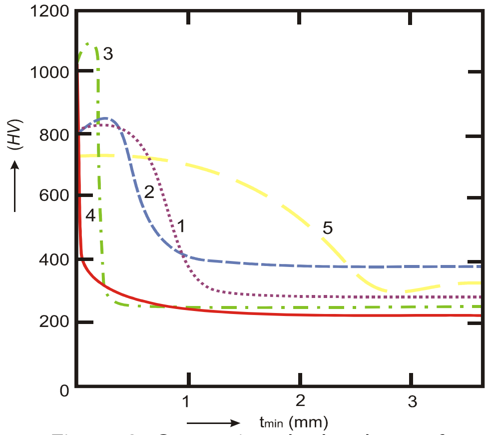

Figure 8. Comparing the hardness of various layers [9]: 1 - cemented, 2 - nitrocemented, 3 - nitriding, 4 - carbonitriding, 5 - surface hardened

Surface hardened layers have lower hardness (max. 750 HV) but they are the thickest and therefore they tolerate high surface pressures.

Other possibilities to increase the carrying capacity are the creation of coatings by depositing thin layers on the base material. Methods of coating deposition are the following [10], [8]:

***1) CVD Method (Chemical Vapor Deposition)** - The principle of CVD is in heating up the coated substrate in vacuum to high temperature (900-1050°C) and with response of chemical compounds, supplied to the surface of the material in vapor state, the solid state is formed.*

- *Benefits: High temperature stability of the created layers, possibility to develop complicated layers not only nitrides of metals, high adhesion and wear resistance, uniform thickness of the layers on the surfaces with complex shape.*
- *Disadvantages: maintaining the base substrate at high temperature to reach steady chemical structure and high power demands long operation cycle (8-10 h) due to long heating and cooling, ecological problems with disposal of exhaust gases produced during deposition, tensile strengths in the layer (different coefficient of thermal expansion).*

***2) Coating with PACD method** (Plasma assisted chemical vapor deposition) - Presence of plasma allows lowering the temperature of layer forming at 470-530°C. No dimensional changes occur during the coating process. These coatings have extremely low coefficient of friction (below 0,1). The equipment for PACVD coatings enables except of coating deposition also surface nitriding and cleaning the parts by the means of ion etching. Coatings created in this way comply with high requirements of quality, mainly abrasion resistance, life and hardness.*

***3) PVD Method** (Physical vapour deposition) - The principle of PVD is based on transformation of deposited material to vapour phase (ion sputtering) in vacuum and depositing on the substrate at low temperatures (150-500°C). The coating thickness is ranging from 1-to 5 μm.*

- *Benefits: the most friendly method of coating deposition (no toxic materials are used), high wear resistance of the layers, low coefficient of friction, possibility to form wide range of various combinations of the layers, small thickness and easy reproduction of the layers, possibility to form accurate layer thickness, possibility to control internal tensions in the coating, high speed of coating deposition with good adhesion.*
- *Drawbacks: difficulties with deposition on polymers, high costs on purchasing and operation.*

***4) Thermal spraying** - The material is deposited as a powder or wire and it is brought to the equipment where it is melted and forwarded to the substrate. With impacting the surface the smelted particles are spread and the drops are coupled among each other, where the coating is formed during the cooling process. The thickness is ranging from 0,2-2 mm.*

- □ *Benefits:* good wear resistance, excellent tribological characteristics, oxidation resistance, corrosion, electro insulation and electro-conductive coatings.

From the mentioned methods of deposition of thin coatings the PVD method seems to be the most suitable to deposit thin coating on gears, which belongs to the most advanced methods, operates with low temperatures (max. 500°C) and allows to create also several hundred layers, so called multilayers and nanolayers with thickness up to few μm and last but not least it is economically reasonable [15].

Before deposition it is necessary to remove impurities from the surface (due to good adhesion). The most important requirements in coating deposition are, that the layers must have good mechanical stability (no cracking) and must have good adhesion to the substrate (no delamination). From the aspect of increasing the carrying capacity of gear wheels it is necessary to improve the following mechanical characteristics: high surface hardness, high stiffness, resistance against corrosion and high temperature oxidation, abrasion resistance, long lifetime [11]. Moreover the coating has to resist to temperatures around 400°C, low coefficient of fiction, maximum surface roughmes Ra=0,6 μm etc. [10].

RESULTS AND DISCUSSION

From the aspect of minimum hardened layer thickness determination on the geared transmissions are dangerous mainly high shear stresses, developing from the contact pressures in the surface layers, which can cause pitting and plastic deformation in the area below the coating and to fracture of the layer [12]. Involute gearing:

1) *The magnitude of minimum depth of hardened layer is specified by standard STN 01 4686-5 in the control of fatigue in contact*
2) *a) for cemented and nitrocemented gear wheels the following applies:*

$$h_{t\min} = \frac{J_{HV}}{J_{HV} - 120} . 4{,}16.10^{-3} . d_1 . \frac{u}{u+1} \tag{7}$$

where: h_{tmin} - min thicknes of the hardened layer, J_{HV} - tooth core hardness, d_1 - circular pitch, u - transference number.

b) for nitrided gear wheels applies:

$$h_{t\min} = \frac{J_{HV}}{J_{HV} - 150} . 2{,}38.10^{-3} . d_1 . \frac{u}{u+1} \tag{8}$$

2) *According to GLAUBITZ [13] has to be the minimum thickness of the hardened layer t_E greater than the depth of maximum shear stress:*

$$t_E = 2.(z)_{\tau\max} \tag{9}$$

where: $(z)_{\tau max}$ - max shear stress depth

3) *LINHART [14] in his work determines the requirement of minimum hardened layer in the way, that on the interface of the layer and the core must not exceed the yield stress:*

$$0{,}5\,\sigma_{Kt} = p_H \left\{ 1 - \frac{\frac{t_{\min}}{a}}{\left[1 + \left(\frac{t_{\min}}{a}\right)^2\right]^{0{,}5}} \right\} \frac{t_{\min}}{a} \tag{10}$$

where: σ_{Kt} - yield stregth, p_H - max. tensile stress according to Hertza, t_{min} - min thicknes of hardened layer, a - half width of contact area.

<u>C-C gearing:</u> In case of C-C gearing it is possible to determine the minimum thickness of hardened layer on the base of numerical simulations [11]. Maximum von MISESS stresses and maximum shear stresses were evaluated, while the minimum thickness of the hardened layer was determined from the maximum shear stress. Simulation were run on 108 models, which varied in angle of path of contact in point C (a_c), which acquired values: ac=6°-23°; further they varied in radius of curvature of path of contact (r_k), which acquired values: r_k=13,17,22 mm. Other geometrical parameters of C-C gearing are shown in Table 2.

From the results of maximum shear stresses were specified equations for the determination of minimum thickness of hardened layer in point B and D (Fig. 4), which where defined by a trend line.

Than for the point B applies:

$$t_{\min} = 0{,}3012.\alpha_C^{0{,}0056} \tag{11}$$

and for the point D applies:

$$t_{\min} = 0{,}2913.\alpha_C^{0{,}0282} \tag{12}$$

Table 2. Gearing parameters

a_C	m	z_1 / z_2	D_1 / D_2	D_{a1} / D_{a2}	D_{f1} / D_{f2}	$a=a_w$	$j=j_w$	r_k [mm]		
								13	17	22
[°]	[mm]	[-]	[mm]	[mm]	[mm]	[mm]	[mm]	ε_a [-]	ε_a [-]	ε_a [-]
23	4	28 / 42	112 / 168	120 / 176	102,8 / 158,8	140	0,142	1,167	1,219	1,266
22	4	28 / 42	112 / 168	120 / 176	102,8 / 158,8	140	0,142	1,176	1,23	1,281
21	4	28 / 42	112 / 168	120 / 176	102,8 / 158,8	140	0,142	1,185	1,242	1,296
20	4	28 / 42	112 / 168	120 / 176	102,8 / 158,8	140	0,142	1,194	1,254	1,311
19	4	28 / 42	112 / 168	120 / 176	102,8 / 158,8	140	0,142	1,204	1,268	1,328
18	4	28 / 42	112 / 168	120 / 176	102,8 / 158,8	140	0,142	1,216	1,283	1,346
17	4	28 / 42	112 / 168	120 / 176	102,8 / 158,8	140	0,142	1,228	1,299	1,367
16	4	28 / 42	112 / 168	120 / 176	102,8 / 158,8	140	0,142	1,24	1,315	1,387
15	4	28 / 42	112 / 168	120 / 176	102,8 / 158,8	140	0,142	1,253	1,332	1,408
14	4	28 / 42	112 / 168	120 / 176	102,8 / 158,8	140	0,142	1,267	1,351	1,431
13	4	28 / 42	112 / 168	120 / 176	102,8 / 158,8	140	0,142	1,282	1,37	1,456
12	4	28 / 42	112 / 168	120 / 176	102,8 / 158,8	140	0,142	1,297	1,39	1,481
11	4	28 / 42	112 / 168	120 / 176	102,8 / 158,8	140	0,142	1,314	1,411	1,507
10	4	28 / 42	112 / 168	120 / 176	102,8 / 158,8	140	0,142	1,331	1,434	1,535
9	4	28 / 42	112 / 168	120 / 176	102,8 / 158,8	140	0,142	1,349	1,457	1,564
8	4	28 / 42	112 / 168	120 / 176	102,8 / 158,8	140	0,142	1,367	1,481	1,594
7	4	28 / 42	112 / 168	120 / 176	102,8 / 158,8	140	0,142	1,387	1,507	1,626
6	4	28 / 42	112 / 168	120 / 176	102,8 / 158,8	140	0,142	1,407	1,533	1,659

a_C - angle of point C, m - modulus, z - number of teeth, D - diameter of the pitch circle, D_a - addendum diameter, D_b - dedendum diameter , a_w - axial distance, j_w - backlash

CONCLUSIONS

The presented article deals with the effect of geometrical parameters of C-C gearing on the determination of the minimum thickness of hardened layer. This was specified by numerical simulation by the means of FEM analysis, where for the point B applies the expression (11) and for the point D expression (12) [11]. In the mentioned expression the minimum thickness of the hardened layer depends on the angle of the path of contact in point C - $a_{c,}$ while the final value has to be round to hundredth. Starting point for the design of hardened layer then could be the most unfavorable situation from the expressions (11) and (12) to prevent its failure and damage of the base material.

Subject for further examination is to continue in research on increasing the load carrying capacity of C-C gearing by the means of thin hard layers (multi layers, nano layers) while in [10] is evident the effect of TiN thin hard layer. Further research is carried on with application of mono-, multi- and nano layer on the model of strength analysis.

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