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# OSCILLATORY EFFECT ON MAGNETO-HYDRODYNAMIC FLOW AND HEAT TRANSFER IN A ROTATING HORIZONTAL POROUS CHANNEL

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**ABSTRACT:** *This paper analyses the effect of injection/suction on an oscillatory flow of an incompressible electrically conducting viscous fluid in a porous channel. The channel with constant injection/suction and variable temperature rotates about an axis perpendicular to the plates of the channel. A magnetic field of uniform strength is also applied normally to the plates. The upper plate is allowed to oscillate in its own plane whereas the lower plate is kept at rest. The heat transfer is studied under two conditions, the temperature of the upper plate is allowed to oscillate whereas the temperature of the lower plate is kept constant. The originality of the paper is to incorporate the effect of magnetic field when it is fixed relative to the fluid as well as to the moving plate and also to study the heat transfer aspect of the flow in the presence of porous matrix. It is interesting to note that magnetic field fixed relative to the moving plate ($k_1$=1.0) contributes more to the resultant velocity than the magnetic field fixed relative to the fluid ($k_1$=0.0) in case of all the parameters. Another striking result is that frequency of oscillation has a distinct effect when the magnetic field is fixed relative to the fluid. The effect of all the pertinent parameters on phase angle is just opposite to that of resultant velocity owing to the relative positions of the magnetic fields.*
**KEYWORDS:** *porous medium, oscillatory, rotation, suction/injection, heat transfer*

## INTRODUCTION

*It is essential to study the theory of rotating fluids Greenspan (1969) due to its occurrence in various natural phenomena and its applications in various technological situations which are directly governed by the action of Coriolis force. The broad subjects of Oceanography, Meteorology, Atmospheric Science and Limnology all contain some important and essential features of rotating fluids. Several authors like Siegman (1971), Mazumder (1991), Ganapathy (1994), Hayat and Hutter (2004), and Guria et al. (2006) have studied the problem of hydrodynamic flow of a viscous incompressible fluid in a rotating medium. The problem of magneto-hydrodynamic flow of a viscous incompressible electrically conducting fluid in a rotating medium has been studied by many researchers like, Ghosh and Pop (2002), Hayat and Abelman (2007), and Abelman et al. (2009) under different conditions and configurations to analyze various aspects of the problem.*

*Seth et al. (1988) and Singh (2000) considered oscillatory hydromagnetic Couette flow of a viscous incompressible electrically conducting fluid in a rotating system under different conditions. Guria et al. (2009) investigated oscillatory MHD Couette flow of electrically conducting fluid between two parallel plates in a rotating system in the presence of an inclined magnetic field when the upper plate is held at rest and the lower plate oscillates non-torsionally. Das et al. (2009) have studied unsteady hydromagnetic Couette flow of a viscous incompressible electrically conducting fluid in a rotating system when the fluid flow within the channel is induced due to impulsive movement of one of the plates of the channel whereas Singh et al. (1994) considered this problem when one of the plates of the channel is set into uniformly accelerated motion. Seth et al. (1982) analyzed this problem when the lower plate of the channel moves with time dependent velocity U (t) and the upper plate is kept fixed. They considered two particular cases of interest such as, (i) impulsive movement of the plate and (ii) uniformly accelerated movement of the plate. In all these investigations, the channel walls are considered non-porous.*

*However, the study of such fluid flow problems in porous channel have numerous engineering and geophysical applications in the fields of chemical engineering for filtration and purification process, in agriculture engineering to study the underground water resources, in petroleum technology to study the movement of natural gas, oil and water through the channels/reservoirs, mineral and metallurgical industries, designing of cooling systems with the liquid metals, MHD* *generators, MHD pumps, accelerators and flow meters, geothermal reservoirs and underground energy transport etc. In view of these applications, Singh (2004), and Hayat et al. (2007, 2008) considered MHD flow within a parallel plate channel with porous boundaries, under different conditions, in non-rotating/rotating system. Ram and Mishra (1977) applied the equations of motion derived by Ahmadi and Manvi (1971) to study an unsteady MHD flow of conducting fluid through porous medium. Effect of suction and injection on MHD three dimensional Couette flow and heat transfer through a porous medium was studied by Das (2009). Seth et al. (2010) discussed the effects of rotation and magnetic field on unsteady Couette flow in a porous channel. Singh and Mathew (2008) have analyzed injection/suction effect on an oscillatory hydromagnetic flow in a rotating horizontal porous channel. Oscillatory MHD flow in a rotating horizontal porous channel filled with a porous material in presence of suction/injection has also been studied extensively by Pradhan et al. (2011).*

*The objective of the present paper is to study the effect of suction/injection on the flow and heat transfer phenomena of a viscous incompressible electrically conducting fluid in a rotating horizontal porous channel in the presence of a uniform transverse magnetic field when it is fixed relative to the fluid and / or to the moving plate. The plates of the channel are considered porous and flow within the channel is due to the oscillatory motion of the upper plate. The model has important applications in novel magneto hydrodynamic (MHD) energy system, magneto biofluid and designing MHD devises requiring fluid flow control.*

## MATHEMATICAL MODEL

*Consider an oscillatory flow of a viscous incompressible and electrically conducting fluid between two insulating parallel porous plates of infinite length, distance d apart in the presence of a uniform transverse magnetic field $B_0$ applied parallel to z*-axis which is normal to the planes of the plates. Two cases of magnetic fields are considered here, namely (i) magnetic field is fixed relative to the fluid and (ii) magnetic field is fixed relative to the moving plate. The heat transfer aspect of the flow is also studied. A constant injection velocity, $w_0$, is applied at the lower stationary plate and the same constant suction velocity, $w_0$, is applied at the upper plate which is oscillating in its own plane with a velocity $U_0(1+\varepsilon\cos\omega^* t^*)$ about a non-zero uniform mean velocity $U_0$.*

*The fluid as well as plates of the channel are in a state of rigid body rotation with uniform angular velocity $\Omega^*$ about z*- axis. Choose the origin on the lower plate lying in x*- y* plane and x*-axis parallel to the direction of motion of the upper plate. Since plates of the channel are infinite along x* and y* directions and are electrically non-conducting, all physical variables, except pressure, will be functions of z* and t*. Since magnetic Reynolds number is very small for metallic liquids so the induced magnetic field may be neglected in comparison with the applied one. This is the well known low magnetic Reynolds number approximation (Cramer and Pai 1973). Initially, (t*<0), both the fluid and plates are assumed to be at rest. When t*>0, the upper plate starts moving with a velocity proportional to U*(t*) in a co-ordinate system rotating with the fluid. The equations of continuity, motion and energy in vector form are:*

$$\nabla . q = 0 \tag{1}$$

$$\frac{\partial q}{\partial t^*} + (q.\nabla)q + 2\Omega n \times q = -\frac{1}{\rho}\nabla p + \upsilon\nabla^2 q + \frac{1}{\rho} J \times B - \frac{\mu}{\rho k^*} q \tag{2}$$

$$\nabla \times B = \mu_e J \text{ (Ampere's Law)} \tag{3}$$

$$\nabla \times E = \frac{\partial B}{\partial t^*} \text{ (Faraday's Law)} \tag{4}$$

$$\nabla . B = 0 \text{ (Maxwell's Law)} \tag{5}$$

$$\nabla . J = 0 \text{ (Gauss's Law)} \tag{6}$$

$$J = \sigma(E + q \times B) \text{ (Ohm's Law for a moving conductor without incorporating Hall current)} \tag{7}$$

$$\rho C_p \frac{DT^*}{Dt^*} = \nabla \bullet (K\nabla T) \tag{8}$$

*where q, B, E, J are, the velocity, the magnetic field, the electric field and the current density vector respectively where as n is the unit vector in the z* direction, σ is fluid electrical conductivity, $\mu_e$ is the magnetic permeability of the fluid, and t* denotes time.*

*The physical model of the problem is illustrated in figure 1 below, where q=( u*, v*, w*) is the velocity vector in the x*, y*, z* directions respectively.*

*$B=(0, 0, B_0)$, $E=(E_x, E_y, E_z)$, $J=(J_x, J_y, 0)$ where $B_0$ is a constant. It is assumed that no*

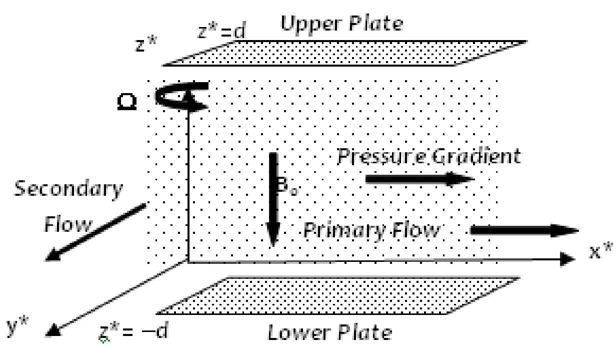


*Figure 1. Physical Model and Coordinate System*

*applied and polarization voltage exists (i.e., E=0). This then corresponds to the case when no energy is added to or extracted from the fluid by the electric field. Now, the equation for the conservation of electric charge, $\nabla.J = 0$, leads to $J_{z^*}$ = constant. As in the case of vertical velocity, we immediately see that $J_{z^*} = 0$. Eq. (7) thus yields*

$$J_{x^*} = \sigma B_0^2 v, \; J_{y^*} = -\sigma B_0^2 u^* \tag{9}$$

*In view of the above considerations, Eq. (2) and (8) can be rewritten in the component form as*

$$\frac{\partial u^*}{\partial t^*} + w^*\frac{\partial u^*}{\partial z^*} - 2\Omega^* v^* = -\frac{1}{\rho}\frac{\partial p^*}{\partial x^*} + \upsilon\frac{\partial^2 u^*}{\partial z^{*2}} - \frac{\sigma B_0^2}{\rho}u^* - \frac{\mu}{\rho k^*}u^* \tag{10}$$

$$\frac{\partial v^*}{\partial t^*} + w^*\frac{\partial v^*}{\partial z^*} + 2\Omega^* u^* = -\frac{1}{\rho}\frac{\partial p^*}{\partial y^*} + \upsilon\frac{\partial^2 v^*}{\partial z^{*2}} - \frac{\sigma B_0^2}{\rho}v^* - \frac{\mu}{\rho k^*}v^* \tag{11}$$

$$\frac{\partial T^*}{\partial t^*} + w^*\frac{\partial T^*}{\partial z^*} = \frac{K}{\rho C_P}\frac{\partial^2 T^*}{\partial z^{*2}} \tag{12}$$

*The initial and boundary conditions for the present problem are*

$$u^* = v^* = 0, \quad 0 \le z^* \le d \quad \text{and} \quad t^* \le 0$$

$$u^* = v^* = 0, \quad w^* = w_0, T^* = T_0 \quad \textit{at } z^* = 0,$$

$$u^* = U^*(t^*) = U_0(1 + \varepsilon\cos\omega^* t^*), \quad v^* = 0, w^* = w_0 \quad \text{at } z^* = d$$

$$T^* = T_d \quad \text{at } z^* = d \quad \textit{(Constant plate temperature)} \tag{13}$$

$$T^* = T_d + \varepsilon(T_0 - T_d)\cos\omega^* t^* \quad \text{at } z^* = d \quad \textit{(Oscillatory plate temperature) at } t^* > 0$$

*We note that Eq. (10) is valid when the magnetic field is fixed with respect to the fluid. On the other hand, when the magnetic field is fixed relative to the moving plates, Eq. (10) is replaced by the following equation given by (Raptis and Singh 1986):*

$$\frac{\partial u^*}{\partial t^*} + w^*\frac{\partial u^*}{\partial z^*} - 2\Omega^* v^* = -\frac{1}{\rho}\frac{\partial p^*}{\partial x^*} + \upsilon\frac{\partial^2 u^*}{\partial z^{*2}} - \frac{\sigma B_0^2}{\rho}(u^* - U^*) - \frac{\mu}{\rho k^*}u^* \tag{14}$$

*Now, Eq (10) and (14) can be rewritten as the single equation*

$$\frac{\partial u^*}{\partial t^*} + w^*\frac{\partial u^*}{\partial z^*} = \frac{-1}{\rho}\frac{\partial p^*}{\partial x^*} + \upsilon\frac{\partial^2 u^*}{\partial z^{*2}} + 2\Omega^* v^* - \frac{\sigma B_0^2}{\rho}(u^* - k_1 U^*) - \frac{\upsilon}{k^*}u^* \tag{15}$$

*where* $k_1 = \begin{cases} 0, & \text{if } B_0 \text{ is fixed relative to the fluid} \\ 1, & \text{if } B_0 \text{ is fixed relative to the moving plate} \end{cases}$

*$\upsilon$ is the kinematic viscosity, t is time, $\rho$ is the density and p* is the modified pressure, T is the temperature, K is the thermal conductivity, $C_P$ is the specific heat at constant pressure and k* is the permeability of the medium. From Eq. (1), it is clear that, $w^* = w_0$ (constant). Substituting $w^* = w_0$ and the modified pressure gradients under the usual boundary layer approximation i.e. from Eqs. (10) and (11) are:*

$$\frac{\partial U^*}{\partial t^*} + \frac{\sigma B_0^2}{\rho}U^* + \frac{\upsilon}{k^*}U^* = \frac{-1}{\rho}\frac{\partial p^*}{\partial x^*}; \quad 2\Omega^* U^* = \frac{-1}{\rho}\frac{\partial p^*}{\partial y^*}$$

*Substituting the above pressure gradients in Eqs. (11) and (15), we get*

$$\frac{\partial u^*}{\partial t^*} + w_0\frac{\partial u^*}{\partial z^*} = \frac{\partial U^*}{\partial t^*} + \upsilon\frac{\partial^2 u^*}{\partial z^{*2}} + 2\Omega^* v^* - \frac{\sigma B_0^2}{\rho}[u^* - (1 + k_1)U^*] - \frac{\upsilon}{k^*}(u^* - U^*) \tag{16}$$

$$\frac{\partial v^*}{\partial t^*} + w_0\frac{\partial v^*}{\partial z^*} = \upsilon\frac{\partial^2 v^*}{\partial z^{*2}} - 2\Omega^*(u^* - U^*) - \frac{\sigma B_0^2}{\rho}v^* - \frac{\upsilon}{k^*}v^* \tag{17}$$

*The following non-dimensional quantities are introduced $\eta = z^*/d$, $t = \omega^* t^*$, $u = u^*/U_0$, $v = v^*/U_0$, $\Omega = \Omega^* d^2/\upsilon$ (rotation parameter), $\omega = \omega^* d^2/\upsilon$ (frequency parameter), $S = w_0 d/\upsilon$ (injection/ suction parameter) and $M = B_0 d\sqrt{\frac{\sigma}{\mu}}$ (Hartmann number), $K_p = k^*/d^2$ (permeability parameter) and, $T = \frac{T^* - T_d}{T_0 - T_d}$ (temperature of the fluid) in the Eqs. (12), (16) and (17) we get*

$$\omega\frac{\partial u}{\partial t} + S\frac{\partial u}{\partial \eta} = \frac{\partial^2 u}{\partial \eta^2} + \omega\frac{\partial U}{\partial t} + 2\Omega v - M^2[u - (1 + k_1)U] - \frac{1}{K_p}(u - U) \tag{18}$$

$$\omega\frac{\partial v}{\partial t}+S\frac{\partial v}{\partial \eta}=\frac{\partial^2 v}{\partial \eta^2}-2\Omega(u-U)-\left(M^2+\frac{1}{K_p}\right)v \qquad (19)$$

$$\frac{\partial T}{\partial t}+\frac{S}{\omega}\frac{\partial T}{\partial \eta}=\frac{1}{P_r.\omega}\frac{\partial^2 T}{\partial \eta^2} \qquad (20)$$

*The corresponding transformed boundary conditions are*

$$\begin{aligned}
&u = v = 0, \quad \text{at } 0 \le \eta \le 1,\ t \le 0\\
&u = v = 0,\ T = 1, \quad \text{at } \eta = 0 \text{ at } t > 0\\
&u = U(t) = 1+\varepsilon\cos t,\ v = 0, \text{ at } \eta = 1\\
&T = 0 \text{ at } \eta = 1 \text{ (Constant plate temperature)}\\
&T = \varepsilon\cos t \text{ at } \eta = 1 \text{ (Oscillatory plate temperature)}
\end{aligned} \qquad (21)$$

## SOLUTION OF THE PROBLEM

*Let us combine Eqs. (18) and (19) into a single equation, by introducing a complex function q = u+iv, and we get*

$$\omega\frac{\partial q}{\partial t}+S\frac{\partial q}{\partial \eta}=\frac{\partial^2 q}{\partial \eta^2}+\omega\frac{\partial U}{\partial t}-\left(2i\Omega+M^2+\frac{1}{K_p}\right)q+\left(2i\Omega+M^2(1+k_1)+\frac{1}{K_p}\right)U \qquad (22)$$

*and the boundary conditions (21) for q can be written*

$$\left.\begin{aligned}
&q = 0 \text{ at } 0 \le \eta \le 1,\ t \le 0\\
&q = 0 \text{ at } \eta = 0,\ t > 0\\
&q = U(t) = 1+\frac{\varepsilon}{2}\left(e^{it}+e^{-it}\right) \text{at } \eta = 1,\ t > 0
\end{aligned}\right\} \qquad (23)$$

*Following Lighthill (1954) we have considered the solutions as given below*

$$q(\eta,t)=q_0(\eta)+\frac{\varepsilon}{2}\left\{q_1(\eta)e^{it}+q_2(\eta)e^{-it}\right\} \qquad (24)$$

$$\text{and } T = T_0(\eta)+T_1(\eta)\varepsilon\cos t \qquad (25)$$

*Substituting equation (24) and (25) into equations (20) and (22) and comparing the harmonic and non-harmonic terms, we get*

$$q_0''-Sq_0'-\left(2i\Omega+M^2+\frac{1}{K_p}\right)q_0=-\left(2i\Omega+(1+k_1)M^2+\frac{1}{K_p}\right) \qquad (26)$$

$$q_1''-Sq_1'-\left(i(2\Omega+\omega)+M^2+\frac{1}{K_p}\right)q_1=-\left(i(2\Omega+\omega)+(1+k_1)M^2+\frac{1}{K_p}\right) \qquad (27)$$

$$q_2''-Sq_2'-\left(i(2\Omega-\omega)+M^2+\frac{1}{K_p}\right)q_2=-\left(i(2\Omega-\omega)+(1+k_1)M^2+\frac{1}{K_p}\right) \qquad (28)$$

$$T_0''-S\Pr T_0'=0 \qquad (29)$$

$$T_1''-S\Pr T_1'+\left(\Pr\omega\tan t\right)T_1=0 \qquad (30)$$

*The corresponding transformed boundary conditions including the two different cases of temperature become*

$$\begin{aligned}
&q_0 = q_1 = q_2 = 0, \text{ at } 0 \le \eta \le 1,\ t \le 0.\\
&q_0 = q_1 = q_2 = 0,\ T_0 = 1,\ T_1 = 0, \text{ at } \eta = 0\\
&q_0 = q_1 = q_2 = 1, \quad \text{at } \eta = 1\\
&T_0 = 0,\ T_1 = 0 \text{ at } \eta = 1, \text{ Constant plate temperature}\\
&T_0 = 0,\ T_1 = 1 \text{ at } \eta = 1 \text{ Oscillatory plate temperature, at } t > 0
\end{aligned} \qquad (31)$$

*The solutions of Eqs.(26) to (30) under the boundary conditions (31) are given by*

$$q_0(\eta)=\frac{(e^{\lambda_1\eta}-e^{\lambda_2\eta})+B_1[(e^{(\lambda_2+\lambda_1\eta)}-e^{(\lambda_1+\lambda_2\eta)})+(e^{\lambda_2\eta}-e^{\lambda_1\eta})]}{(e^{\lambda_1}-e^{\lambda_2})}+B_1 \qquad (32)$$

$$q_1(\eta)=\frac{(e^{\lambda_3\eta}-e^{\lambda_4\eta})+B_2[(e^{(\lambda_4+\lambda_3\eta)}-e^{(\lambda_3+\lambda_4\eta)})+(e^{\lambda_4\eta}-e^{\lambda_3\eta})]}{(e^{\lambda_3}-e^{\lambda_4})}+B_2 \quad (33)$$

$$q_2(\eta)=\frac{(e^{\lambda_5\eta}-e^{\lambda_6\eta})+B_3[(e^{(\lambda_6+\lambda_5\eta)}-e^{(\lambda_5+\lambda_6\eta)})+(e^{\lambda_6\eta}-e^{\lambda_5\eta})]}{(e^{\lambda_5}-e^{\lambda_6})}+B_3 \quad (34)$$

$$T_0(\eta)=\frac{e^{SPr}-e^{SPr\eta}}{e^{SPr}-1},\ T_1(\eta)=0 \quad (35)$$

$$T_0(\eta)=\frac{e^{S\Pr}-e^{S\Pr\eta}}{e^{S\Pr}-1},\ T_1(\eta)=\frac{e^{\frac{(S\Pr+\alpha)\eta}{2}}-e^{\frac{(S\Pr-\alpha)\eta}{2}}}{e^{\frac{(S\Pr+\alpha)}{2}}-e^{\frac{(S\Pr-\alpha)}{2}}} \quad (36)$$

$$T=\frac{e^{S\Pr}-e^{S\Pr\eta}}{e^{S\Pr}-1},\ T=\frac{e^{S\Pr}-e^{S\Pr\eta}}{e^{S\Pr}-1}+\varepsilon\cos t.\frac{e^{\frac{(S\Pr+\alpha)\eta}{2}}-e^{\frac{(S\Pr-\alpha)\eta}{2}}}{e^{\frac{(S\Pr+\alpha)}{2}}-e^{\frac{(S\Pr-\alpha)}{2}}} \quad (37)$$

where $\alpha=\sqrt{S^2Pr^2-4Pr\omega tant}$, $B_1=1+\dfrac{k_1M^2}{2i\Omega+M^2+\frac{1}{K_p}}$, $B_2=1+\dfrac{k_1M^2}{i(2\Omega+\omega)+M^2+\frac{1}{K_p}}$, $B_3=1+\dfrac{k_1M^2}{i(2\Omega-\omega)+M^2+\frac{1}{K_p}}$

$$\lambda_1=\frac{\left(S+\sqrt{S^2+4(2i\Omega+M^2+\frac{1}{K_p})}\right)}{2},\ \lambda_2=\frac{\left(S-\sqrt{S^2+4(2i\Omega+M^2+\frac{1}{K_p})}\right)}{2}\ \lambda_3=\frac{\left(S+\sqrt{S^2+4\{i(2\Omega+\omega)+M^2+\frac{1}{K_p}\}}\right)}{2},$$

$$\lambda_4=\frac{\left(S-\sqrt{S^2+4\{i(2\Omega+\omega)+M^2+\frac{1}{K_p}\}}\right)}{2}\ \lambda_5=\frac{\left(S+\sqrt{S^2+4\{i(2\Omega-\omega)+M^2+\frac{1}{K_p}\}}\right)}{2},\ \lambda_6=\frac{\left(S-\sqrt{S^2+4\{i(2\Omega-\omega)+M^2+\frac{1}{K_p}\}}\right)}{2}$$

Now, for the resultant velocities and the shear stresses of the steady and unsteady flow, we write

$$q_0(\eta) = u_0(\eta) + iv_0(\eta) \quad (38)$$

and $q_1(\eta)eit+q_2(\eta)e\text{-}it = u_1(\eta) + iv_1(\eta)$ (39)

The resultant velocity or amplitude and phase difference of the steady and unsteady flow are given by

$$R_0=\sqrt{u_0^2+v_0^2},\quad \theta_0=\tan^{-1}(v_0/u_0) \quad (40)$$

and $R_1=\sqrt{u_1^2+v_1^2},\quad \theta_1=\tan^{-1}(v_1/u_1)$ (41)

where $u_o$, $u_1$ are the primary and $v_0$, $v_1$ are the secondary velocities of the steady and unsteady flow respectively.

Shear Stress at the Stationary Plate for the Steady flow:

$$\left(\frac{\partial q_0}{\partial\eta}\right)_{\eta=0}=\tau_{ox}+i\tau_{oy}=\frac{(\lambda_1-\lambda_2)(1-B_1)+B_1(\lambda_1e^{\lambda_2}-\lambda_2e^{\lambda_1})}{(e^{\lambda_1}-e^{\lambda_2})}$$

The amplitude and phase difference are given by

$$\tau_{0r}=\sqrt{\tau_{0x}^2+\tau_{0y}^2},\quad \theta_{0r}=\tan^{-1}\left(\frac{\tau_{0y}}{\tau_{0x}}\right) \quad (42)$$

Shear Stress at the Stationary Plate for the Unsteady flow:

$$\left(\frac{\partial u_1}{\partial\eta}\right)_{\eta=0}+i\left(\frac{\partial v_1}{\partial\eta}\right)_{\eta=0}=\left(\frac{\partial q_1}{\partial\eta}\right)_{\eta=0}e^{it}+\left(\frac{\partial q_2}{\partial\eta}\right)_{\eta=0}e^{-it}=\tau_{1x}+i\tau_{1y}$$

$$=\left[\frac{(\lambda_3-\lambda_4)(1-B_2)+B_2(\lambda_3e^{\lambda_4}-\lambda_4e^{\lambda_3})}{(e^{\lambda_3}-e^{\lambda_4})}\right]e^{it}+\left[\frac{(\lambda_5-\lambda_6)(1-B_3)+B_3(\lambda_5e^{\lambda_6}-\lambda_6e^{\lambda_5})}{(e^{\lambda_5}-e^{\lambda_6})}\right]e^{-it} \quad (43)$$

The amplitude and phase difference are given by

$$\tau_{1r}=\sqrt{\tau_{1x}^2+\tau_{1y}^2},\quad \theta_{1r}=\tan^{-1}\left(\frac{\tau_{1y}}{\tau_{1x}}\right)$$

## RESULTS AND DISCUSSION

*a) Steady case*

*Figure 2 presents the effect of porosity parameter $K_p$ and magnetic parameter M on resultant velocity $R_0$. It is observed that the resultant velocity $R_0$ is not affected by the porosity parameter $K_p$ but an increase in magnetic parameter increases it. Further, it is interesting to note that magnetic field fixed relative to the moving plate ($k_1$=1.0) contributes more to the resultant velocity than the magnetic field fixed relative to the fluid ($k_1$=0.0).*

*Figure 3 exhibits the effect of rotational parameter Ω on resultant velocity $R_0$. Curve VI depicts the effect without rotation (Ω=0.0) and without porous medium ($k_p$=100). In this case, the resultant velocity assumes the lowest value in both the cases $k_1$=0.0 and $k_1$=1.0. Again comparing the curve VI ($K_p$=100) with I ($K_p$=1), it is clearly seen that presence of porous matrix without rotation increases the resultant velocity. Further, it is to note that resultant velocity increases with the increase in the value of rotational parameter. On careful observation, it is revealed that on increasing the rotation of frame of reference, the difference between the resultant velocity in respect of $k_1$=0.0 and $k_1$=1.0 decreases and ultimately it coincides with curve IV. Thus, it may be concluded that with higher value of rotation the relative position of magnetic field with respect to moving plate and the fluid has no effect.*

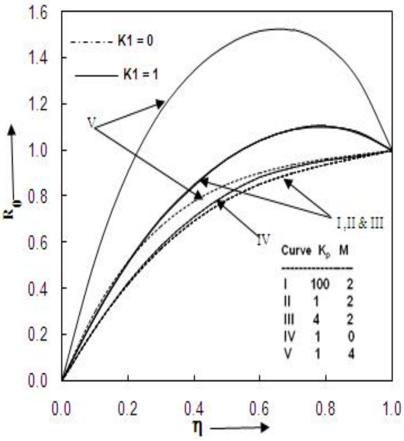


*Figure 2. Resultant velocity $R_0$ for various values of $K_p$ and M (Ω=5, S=2)*

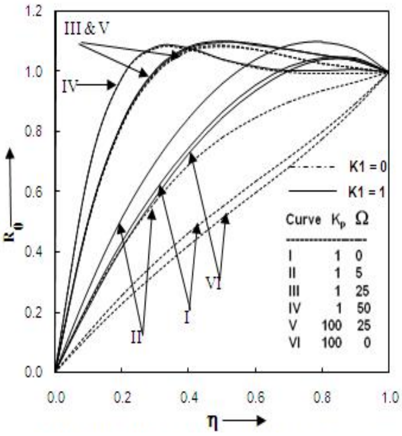


*Figure 3. Resultant velocity $R_0$ for various values of $K_p$ and Ω (M=2, S=2)*

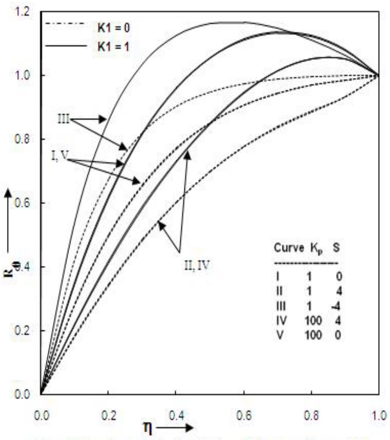


*Figure 4. Resultant velocity $R_0$ for various values of $K_p$ and S (Ω=5, M=2)*

*The effect of suction /injection parameter S on resultant velocity $R_0$ is shown through curves II and III in Figure 4. It is observed that in case of suction (S<0) the resultant velocity $R_0$ increases significantly (curve III) whereas for injection (S>0) it decreases (curve II). Therefore, injection is found to be counterproductive for enhancing the resultant velocity. It is important to note that the resultant velocity $R_0$, for magnetic field fixed relative to the moving plate ($k_1$=1.0) is always greater than its counterpart that is the magnetic field fixed relative to the fluid ($k_1$=0.0) in all the cases. Also, another striking result is that in the absence or presence of suction /injection, porous media has no significant effect on the resultant velocity $R_0$ for both $k_1$=0.0 and $k_1$=1.0.*

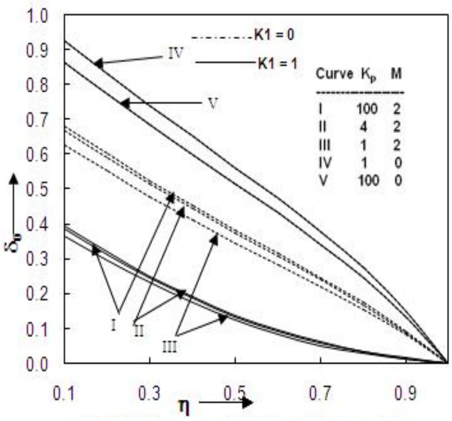


*Figure 5. Phase angle $\delta_0$ for various values of $K_p$ and M (Ω=5, S=2)*

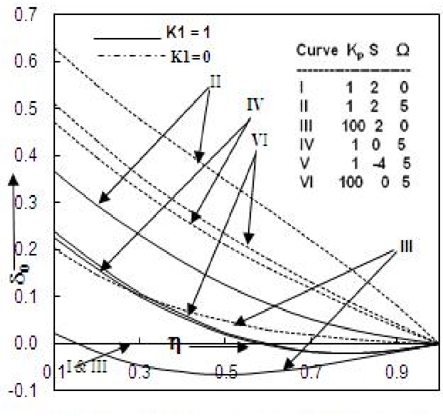


*Figure 6. Phase angle $\delta_0$ for various values of $K_p$, S and Ω (M=2)*

*Figure 5 exhibits the variation of phase angle $\delta_0$ for different values of the parameters Kp and M. It is noted that phase angle decreases steadily span wise for different values of porosity parameter. Relative position of magnetic field with respect to the fluid ($k_1$=0.0) has a greater phase angle in all the cases than its counterpart ($k_1$=1.0) which is of opposite effect on resultant velocity. Further, it is to note that as the magnetic field increases, phase angle also increases significantly.*

*Figure 6 depicts the variation of phase angle $\delta_0$ for different values of the parameters S and Ω. One most interesting result is that phase angle assumes negative value for higher value of rotation parameter (Ω≥ 25) for the layers (η>0.4), little away from the lower plate. Also, the negative value of phase angle appears in case of suction (s=-4.0) and without suction that is for impermeable wall. Further, it is seen that the role of porosity is to decrease the phase angle in both the cases $k_1$=0.0 and $k_1$=1.0.*

*b) Unsteady case:*

*Figure 7(a),(b)&(c) depicts the resultant velocity in case of the unsteady motion. In comparison with the steady case, it is seen that the effects of all the parameters Kp, S, Ω and M remain same as that of steady flow except the magnitude which is nearly twice than that of the steady case.*

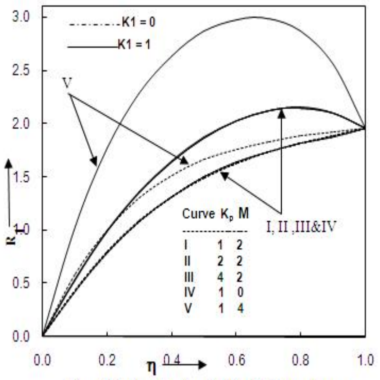


*Figure 7(a). Resultant velocity $R_1$ for various values of $K_p$ and M ($\Omega$=5, S=2, $\omega$=5)*

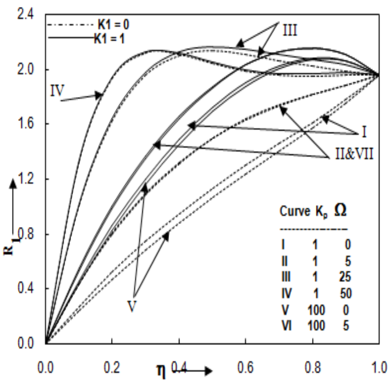


*Figure 7(b). Resultant velocity $R_1$ for various values of $K_p$ and $\Omega$ (M=2, S=2, $\omega$=5)*

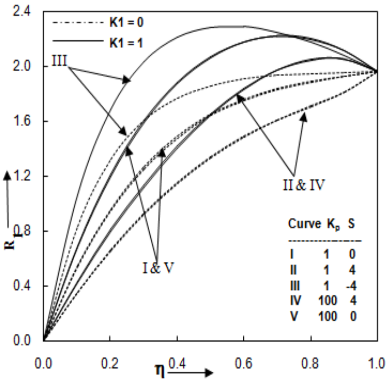


*Figure 7(c). Resultant velocity $R_1$ for various values of $K_p$ and S ($\Omega$=5, M=2, $\omega$=5)*

*Figure 8 shows the effect of frequency of oscillation ω on resultant velocity $R_1$. It is observed that the frequency parameter ω has noticeable effect when magnetic field is fixed relative to the fluid ($k_1$=0.0) but in case of its counterpart, that is ($k_1$=1.0) there is no significant effect. Therefore, the above result suggests that while studying the unsteady motion, the magnetic field is to be considered with respect to the fluid so that the flow parameters have distinct role to play in modifying the resultant velocity. Moreover, it is seen that the resultant velocity $R_1$ decreases with an increasing value of ω for both the cases that is $k_1$=0.0 and $k_1$=1.0.*

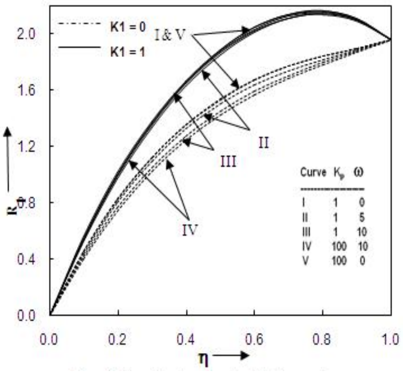


*Figure 8. Resultant velocity $R_1$ for various values of $K_p$ and $\omega$ ($\Omega$=5, S=2, M=2)*

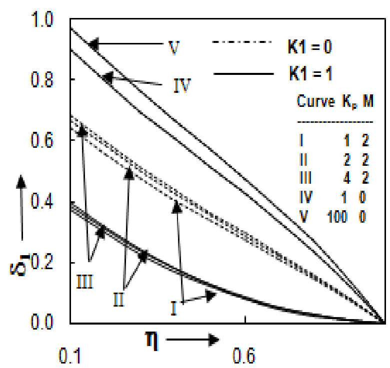


*Figure 9(a). Phase angle $\delta_1$ for various values of $K_p$ and M ($\Omega$=5, S=2, $\omega$=5)*

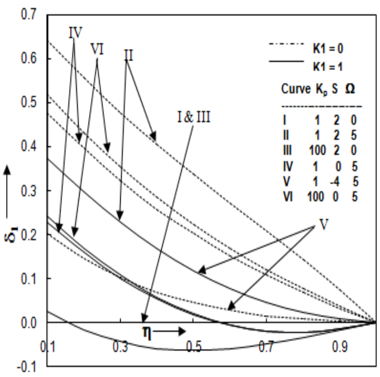


*Figure 9(b). Phase angle $\delta_1$ for various values of $K_p$, S and $\Omega$ (M=2, $\omega$=5)*

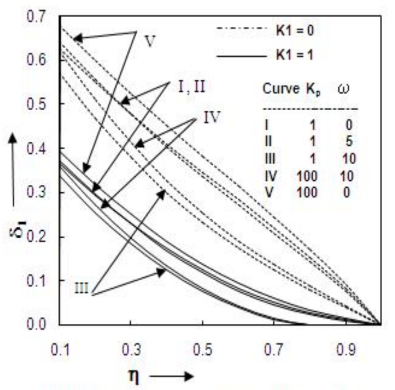


*Figure 10. Phase angle $\delta_1$ for various values of $K_p$ and $\omega$ ($\Omega$=5, S=2, M=2)*

*Figure 9(a) and (b) presents the variation of phase angle $\delta_1$ of unsteady flow for different values of $K_{p,}$ Ω, Sand M. It is observed that the effects of all the parameters on phase angle remain same as that of steady flow except the magnitude which is nearly twice than the steady case*

*Figure 10 exhibits the variation of phase angle $\delta_1$ for different values of frequency parameter ω. It is seen that phase angle assumes higher values when the magnetic field is fixed relative to the fluid for all the values of the parameters. It is also to note that permeability of the medium is responsible for decreasing the phase angle in both, presence (ω≠0) or absence (ω=0) of oscillation as well as in both the relative positions of magnetic field.*

*Figure 11 shows the temperature distribution for various values of Prandtl number Pr, frequency of oscillation ω and* *suction/injection parameter S when the plates are maintained at constant temperature. It is seen that higher Prandtl number fluid in the presence of injection contributes to increase the thickness of the thermal boundary layer whereas suction decreases it. Again, it is seen that frequency of oscillation ω has no effect.*

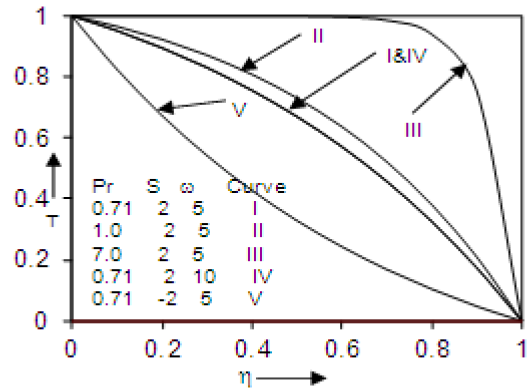


*Figure 11. Constant plate temperature distribution for various values of Pr, S &ω*

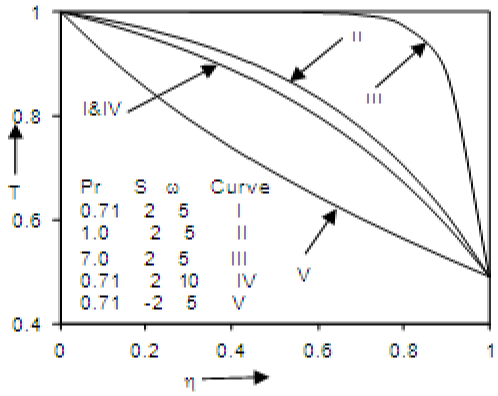


*Figure 12. Oscillatory plate temperature distribution for various values of Pr, S & ω*

*From Figure 12 it is revealed that the role of Pr and S remains same for both constant plate temperature and oscillatory plate temperature except ω where, an increase in ω leads to increase the temperature at all the points of the flow domain.*

*Table 1 presents the numerical values of amplitude of oscillation ($\tau_{or}$) and phase difference ($\theta_{or}$) of shear stress at the stationary plate (η=0) for the steady flow. It is observed that an increase in rotation parameter Ω, magnetic parameter M and suction parameter S leads to increase the amplitude of shear stress but presence of injection and porous matrix decreases it. Further, one striking result is that absence of injection/suction (S=0), without rotation and porous matrix, amplitude of shear stress decreases where as in the absence of magnetic field it increases. Thus application of magnetic field associated with the presence of injection/suction, porous matrix and rotation are responsible for reduction of the amplitude of shear stress. Thus, it may be concluded that the reduction of amplitude of shear stress leads to flow stability which is desirable. Moreover, an increase in the value of rotation and injection parameters leads to increase in phase angle whereas the reverse effect is observed in case of suction (S<0) and magnetic parameter M. The above discussion is true for both the cases k1=0.0 and k1=1.0.*

*Table 1: Values $\tau_{or}$ of and $\theta_{or}$ for different values of $k_1$, S, $\Omega$, M and $k_p$*

| $k_1$ | $k_p$ | $\Omega$ | M | S | $\tau_{or}$ | $\theta_{or}$ |
|---|---|---|---|---|---|---|
| 0 | 100 | 5 | 2 | 2 | 2.5295 | 0.7657 |
| 1 | 100 | 5 | 2 | 2 | 3.0589 | 0.4765 |
| 0 | 1 | 5 | 2 | 2 | 2.5757 | 0.7092 |
| 1 | 1 | 5 | 2 | 2 | 3.1412 | 0.4461 |
| 0 | 1 | 5 | 2 | 0 | 3.3227 | 0.5560 |
| 1 | 1 | 5 | 2 | 0 | 4.1176 | 0.3043 |
| 0 | 100 | 5 | 2 | 0 | 3.2579 | 0.5992 |
| 1 | 100 | 5 | 2 | 0 | 4.0220 | 0.3209 |
| 0 | 1 | 5 | 2 | -4 | 5.5462 | 0.2715 |
| 1 | 1 | 5 | 2 | -4 | 7.0860 | 0.0808 |
| 0 | 1 | 5 | 0 | 2 | 2.4839 | 0.9597 |
| 1 | 1 | 5 | 0 | 2 | 2.4839 | 0.9597 |
| 0 | 100 | 5 | 0 | 2 | 2.5087 | 1.0269 |
| 1 | 100 | 5 | 0 | 2 | 2.5087 | 1.0269 |
| 0 | 1 | 0 | 2 | 2 | 1.4863 | 0 |
| 1 | 1 | 0 | 2 | 2 | 2.5499 | 0 |
| 0 | 1 | 25 | 2 | 2 | 6.3826 | 0.8298 |
| 1 | 1 | 25 | 2 | 2 | 6.4509 | 0.7514 |
| 0 | 100 | 0 | 2 | 2 | 1.2898 | 0 |
| 1 | 100 | 0 | 2 | 2 | 2.3992 | 0 |

*Table 2: Values of $\tau_{1r}$ and $\theta_{1r}$ for various values of $K_1$, S, $\Omega$, M, $K_p$ and ω*

| $k_1$ | $k_p$ | $\Omega$ | M | S | $\tau_{or}$ | $\theta_{or}$ | $k_1$ |
|---|---|---|---|---|---|---|---|
| 0 | 100 | 2 | 5 | 2 | 5 | 4.75869 | 0.79521 |
| 1 | 100 | 2 | 5 | 2 | 5 | 5.81319 | 0.49052 |
| 0 | 1 | 2 | 5 | 2 | 5 | 4.85796 | 0.73482 |
| 1 | 1 | 2 | 5 | 2 | 5 | 5.98130 | 0.45878 |
| 0 | 1 | 0 | 5 | 2 | 5 | 6.28722 | 0.57122 |
| 1 | 1 | 0 | 5 | 2 | 5 | 7.90197 | 0.31183 |
| 0 | 100 | 0 | 5 | 2 | 5 | 6.14392 | 0.61705 |
| 1 | 100 | 0 | 5 | 2 | 5 | 7.71021 | 0.32894 |
| 0 | 1 | -4 | 5 | 2 | 5 | 10.67874 | 0.27532 |
| 1 | 1 | -4 | 5 | 2 | 5 | 13.77252 | 0.08581 |
| 0 | 1 | 2 | 5 | 0 | 5 | 4.64057 | 1.00919 |
| 1 | 1 | 2 | 5 | 0 | 5 | 4.64057 | 1.00919 |
| 0 | 100 | 2 | 5 | 0 | 5 | 4.68438 | 1.08659 |

| $k_1$ | $k_p$ | $\Omega$ | M | S | $\tau_{or}$ | $\theta_{or}$ | $k_1$ |
|---|---|---|---|---|---|---|---|
| 1 | 100 | 2 | 5 | 0 | 5 | 4.68438 | 1.08659 |
| 0 | 1 | 2 | 0 | 2 | 5 | 2.82477 | 0.00000 |
| 1 | 1 | 2 | 0 | 2 | 5 | 4.87487 | 0.00000 |
| 0 | 1 | 2 | 25 | 2 | 5 | 12.49222 | 0.84087 |
| 1 | 1 | 2 | 25 | 2 | 5 | 12.60830 | 0.76168 |
| 0 | 100 | 2 | 0 | 2 | 5 | 2.46425 | 0.00000 |
| 1 | 100 | 2 | 0 | 2 | 5 | 4.59097 | 0.00000 |
| 0 | 1 | 2 | 5 | 2 | 10 | 4.57494 | 0.69044 |
| 1 | 1 | 2 | 5 | 2 | 10 | 5.83291 | 0.43583 |
| 0 | 1 | 2 | 5 | 2 | 0 | 5.04864 | 0.70923 |
| 1 | 1 | 2 | 5 | 2 | 0 | 6.15720 | 0.44612 |
| 0 | 100 | 2 | 5 | 2 | 0 | 4.95806 | 0.76571 |
| 1 | 100 | 2 | 5 | 2 | 0 | 5.99593 | 0.47647 |

*Table 2 presents the numerical values of amplitude ($\tau_{1r}$) and phase difference ($\theta_{1r}$) of the shear stress at the stationary plate (η=0) in case of the unsteady flow. It is quite remarkable to note that* *the rotational parameter Ω, magnetic parameter M, suction and porosity of the medium have same effect as that of steady case. Only ω, the frequency of oscillation decreases the amplitude but increases it for some values of ω (ω<5). Further, it is noted that in absence of porosity and frequency of oscillation, amplitude of shear stress decreases but phase angle increases slightly. Hence, presence of porous matrix and oscillatory motion are unfavorable for reduction of shearing stress.*

***Nusselt Number (Nu)***

*The rate of heat transfer between the fluid and the plate is analyzed through non-dimensional Nusselt number and the calculations of this number for constant plate temperature and oscillatory plate temperature are given in Table 3 and Table 4 respectively.*

*Table 3: Nusselt number for constant plate temperature*

| Pr | s | $N_u$ |
|---|---|---|
| 0.71 | 2 | 0.4526 |
| 1 | 2 | 0.3130 |
| 7 | 2 | 1.1641e-005 |
| 0.71 | 4 | 0.1762 |
| 0.71 | -2 | 1.8726 |
| 0.71 | -4 | 3.0162 |

$$Nu = -\left(\frac{\partial T}{\partial \eta}\right)_{\eta=0} = \frac{\mathrm{SPr}}{e^{\mathrm{SPr}} - 1}$$

$$Nu = -\left(\frac{\partial T}{\partial \eta}\right)_{\eta=0} = \frac{\mathrm{SPr}}{e^{\mathrm{SPr}} - 1} + \frac{2\varepsilon \cos t\sqrt{S^2\,Pr^2 - 4\,Pr\,\omega \tan t}}{e^{\left(\frac{S\,Pr + \sqrt{S^2\,Pr^2 - 4\,Pr\,\omega\tan t}}{2}\right)} - e^{\left(\frac{S\,Pr - \sqrt{S^2\,Pr^2 - 4\,Pr\,\omega\tan t}}{2}\right)}}$$

*It is evident that Nusselt numbers (Nu) for both air (Pr=0.71) and water (Pr=7.0) decrease with an increase in the value of Prandtl number Pr as well as injection parameter (S>0) but in case of suction (S<0), it increases. Moreover, it is seen that Nusselt number decreases with an increase in frequency of oscillation ω. Therefore, it may be concluded that the increasing frequency of oscillation in a high Prandtl number fluid in the presence of injection do not favour the high rate of heat transfer.*

*Table 4: Nusselt number for oscillatory plate temperature*

| Pr | s | ω | Nuo |
|---|---|---|---|
| 0.71 | 2 | 0 | 0.2308 |
| 0.71 | 2 | 5 | 0.2028 |
| 1.0 | 2 | 5 | 0.1324 |
| 7.0 | 2 | 5 | 2.7092e-006 |
| 0.71 | 4 | 5 | 0.0800 |
| 0.71 | -2 | 5 | 0.8392 |
| 0.71 | -4 | 5 | 1.3688 |
| 0.71 | 2 | 10 | 0.1696 |

## CONCLUSIONS

*The theoretical study of oscillatory MHD flow and heat transfer through a porous medium in a rotating system reveals the following facts.*

*a) Steady case:*

- *Permeability of the medium has no noticeable effect on the resultant velocity.*
- *Enhancement of resultant velocity is well marked in case of the relative position of magnetic field fixed with respect to the moving plate whereas reverse effect is observed in case of phase angle for all the cases.*
- *Without rotation and without porous medium are found to be counter-productive on the resultant velocity in both the positions of magnetic field whereas presence of porous medium without rotation favours the enhancement of resultant velocity.*
- *Absence of rotation and porous medium is found to be counterproductive on the resultant velocity in both the positions of magnetic field whereas presence of porous medium without rotation favours the enhancement of resultant velocity.*
- *No tangible effect is marked due to relative positions of the magnetic fields with respect to fluid and moving plate under the influence of greater Coriolis force.*
- *Presence of injection reduces the resultant velocity in the entire flow domain.*

*b) Unsteady case:*

- *Unsteady flow enhances the resultant velocity nearly twice in magnitude preserving the other characteristics intact.*
- *Reduction of phase angle occurs irrespective of presence or absence of rotation and relative positions of magnetic fields.*
- *Higher Prandtl number fluid in the presence of injection is conducive for the growth of thermal boundary layer where as suction is not.*
- *Frequency of oscillation favours the enhancement of temperature at all points.*
- *Larger amplitude of the shear stress at the stationary plate is experienced due to increasing rotation, magnetic intensity and suction.*
- *Presence of porous matrix and oscillatory motion are unfavourable for reduction of shearing stress.*
- *The increasing frequency of oscillation in case of a high Prandtl number fluid in presence of injection do not favour the high rate of heat transfer.*

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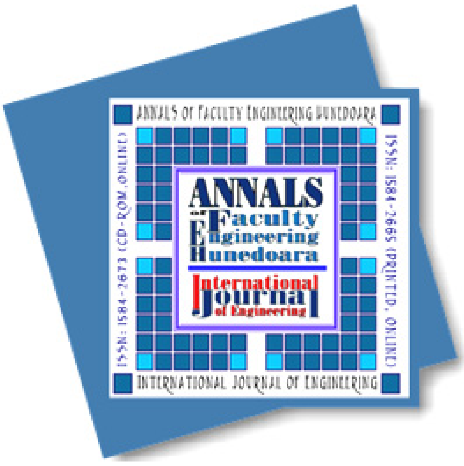


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