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FREE CONVECTIVE MHD FLOW OF A VISCO-ELASTIC (WALTERS MODEL - B') DUSTY FLUID THROUGH A POROUS MEDIUM AND CONSTANT HEAT SOURCE

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ABSTRACT: *The present paper is concerned with heat and mass transfer in MHD free convective flow of a visco-elastic (Walters Model – B') dusty fluid through a porous medium with constant heat source induced by motion of a semi infinite flat vertical plate moving with velocity decreasing exponentially with time. The effect of various parameters on velocity, temperature and concentration of dusty fluid and dust particle are discussed graphically.*
KEYWORDS: *heat and mass transfer, MHD, Walters Model*

INTRODUCTION

In many transport processes and industrial applications, transfer of heat and mass simultaneously as a result of combined buoyancy effects of thermal diffusion and diffusion of chemical species. Unsteady natural-convection of heat and mass transfer is great importance in designing control systems for modern free convection heat exchangers. Soundalgekar and Akolkar [1] studied the effect of mass transfer and free convection currents on the flow past an impulsively started infinite vertical plate and observed that the presence of gasses in the flow domain leads to reduce the shear stress and rate of mass transfer significantly.

In view of importance of the diffusion- thermo effect Kafoussias and Williams [2] have studied the effects of thermal diffusion and diffusion-thermo on mixed free and forced convective and mass transfer boundary layer flow with temperature dependent viscosity. Jha and Singh [3], Hajji and Worek [4], El- Din [5], Kafoussias [6], and Alam et al.[7] have contributed significantly to this field of study. Beg et al. [8] studied chemically reacting mixed convective heat and mass transfer along inclined and vertical plates considering Soret and Dufour effect.

Using the formulation of Saffman [9], several authors gave the exact solution of various dusty fluid problems. Michael and Norey [10], Verma and Mathur [11] studied the problem of circular cylinders under various conditions. Singh [12] has studied the MHD flow of a dusty gas through a porous medium induced by the motion of a semi-infinite flat plate moving with velocity decreasing exponentially with time. Varshney and Prakash [13] have discussed the MHD free convective flow of a visco-elastic dusty gas through a porous medium induced by the motion of a semi-infinite flat plate moving with velocity decreasing exponentially with time. P.Singh et.al [14] has studied the heat and mass transfer in MHD free convective flow of a visco-elastic (Walters Model – B') dusty fluid through porous medium.

In the present study our aim is to investigate the heat and mass transfer in MHD free convective flow of a visco-elastic (Walters Model – B') dusty fluid through a porous medium induced by the motion of a semi-infinite flat plate moving with velocity decreasing exponentially with time in presence of heat source.

MATHEMATICAL FORMULATION

We assume that the dusty fluid be confined in the space y > 0 and the flow is produced by the motion of the semi-infinite flat plate moving with a velocity $ve^{-\lambda^2 t}$ in x - direction. Axis of x is taken along the plate and y is normal to it. Since the plate is semi-infinite, all the physical quantities will be functions of y and t only. The governing equations of flow and heat transfer of visco-elastic (Walters Model – B') dusty fluid and dusty particles in the presence of magnetic field, uniform porous matrix are given by

$$\frac{\partial u}{\partial t} = \upsilon\frac{\partial^2 u}{\partial y^2} + \frac{K_0 N_0}{\rho}(v-u) - \left[\frac{\sigma B_0}{\rho} + \frac{\upsilon}{K_p}\right]u + g\beta(T-T_\infty) + g\beta^*(C-C_\infty) - \frac{k}{\rho}\frac{\partial^3 u}{\partial t \partial y^2} \quad (1)$$

$$\frac{\partial v}{\partial t}=\frac{K_0}{m}(u-v) \tag{2}$$

$$\frac{\partial T}{\partial t}=\frac{K_T}{\rho C_p}\frac{\partial^2 T}{\partial y^2}+S(T-T_\infty) \tag{3}$$

$$\frac{\partial C}{\partial t}=D\frac{\partial^2 C}{\partial y^2} \tag{4}$$

where $\theta = T - T_\infty$, $\Phi = C - C_\infty$, more over u and v denote, the fluid and particle velocity; respectively υ is the kinematics coefficient of viscosity of the fluid, K_0 is the Stoke's resistance coefficient, N_0 is the density of the dust particle which is taken as constant, ρ is the density of fluid and m is the mass of the dust particle. K_T is the thermal conductivity, C_p is the specific heat at constant pressure and k is the coefficient of elasticity.

The boundary conditions are:

$$\begin{aligned} &u=\upsilon e^{-\lambda^2 t},\ \theta=\upsilon e^{-\lambda^2 t},\ \phi=\upsilon e^{-\lambda^2 t}\quad at\quad y=0\\ &u\to 0\ ,\ \ \theta\to 0\ ,\phi\to 0\quad as\quad y\to\infty \end{aligned} \tag{5}$$

Let us introduce the non dimensional variables

$$y'=\frac{y}{(\upsilon\tau)^2},u'=\frac{u}{\upsilon},v'=\frac{v}{\upsilon},t'=\frac{t}{\upsilon},\tau=\frac{m}{K_0},\theta'=\frac{T-T_0}{T_\infty-T_0},\phi'=\frac{C-C_0}{C_\infty-C_0}$$

then omitting dashes, the dimensionless forms of equations (1)-(4), respectively, are

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial t^2}+(v-u)f-\left[M+\frac{1}{K}\right]u+\beta_1\theta+\beta_2\phi-R_C\frac{\partial^3 u}{\partial t\partial y^2} \tag{6}$$

$$\frac{\partial v}{\partial t}=u-v \tag{7}$$

$$\frac{\partial \theta}{\partial t}=\frac{1}{pr}\frac{\partial^2 u}{\partial y^2}+S\theta \tag{8}$$

$$\frac{\partial \phi}{\partial t}=\frac{1}{Sc}\frac{\partial^2 \phi}{\partial y^2} \tag{9}$$

where f the mass concentration of dust particles, M is the magnetic parameter, α_1 is the visco-elastic parameter, P_r is the Prandtl number, K_1 is the permeability parameter, k_1 is the elasticity parameter, β_1 and β_2 are the volumetric expansion parameters. These are given as

$$f=\frac{mN_0}{\rho},M=\frac{m\sigma B_0^{\,2}}{K_0\rho},\alpha_1=\frac{\alpha}{\tau},\Pr=\frac{\rho\upsilon Cp}{K_T},$$

$$Sc=\frac{\upsilon}{D},\frac{1}{K_1}=\frac{\upsilon\tau}{K},\beta_1=g\beta\tau,\beta_2=g\beta^*\tau$$

The boundary conditions (5) are reduced to

$$\begin{aligned} &u=v=e^{-\lambda^2 t},\ \theta=e^{-\lambda^2 t},\ \phi=e^{-\lambda^2 t}\quad at\quad y=0\\ &u\to 0\ ,\ \ \theta\to 0\ ,\phi\to 0\quad as\quad y\to\infty \end{aligned} \tag{10}$$

Let us choose the solution of 6-9 respectively as

$$u=F(y)e^{-\lambda^2 t} \tag{11}$$

$$v=G(y)e^{-\lambda^2 t} \tag{12}$$

$$\theta=H(y)e^{-\lambda^2 t} \tag{13}$$

$$\phi=N(y)e^{-\lambda^2 t} \tag{14}$$

The boundary conditions (10) are transformed to

$$\begin{aligned} &F=1,\quad N=1,\quad H=1\qquad at\quad y=0\\ &F\to 0,\quad N\to 0,\quad H\to 0\quad at\quad y\to\infty \end{aligned} \tag{15}$$

Hence equation (11)-(14), the equations (6)-(9), respectively, transformed to

$$(1+\lambda^2 k_1)\frac{d^2F}{dy^2}+fG+F\left[\lambda^2-f-M-\frac{1}{K_1}\right]=-\beta_1 H-\beta_2 N \tag{16}$$

$$G(1-\lambda^2)=F \tag{17}$$

$$\frac{d^2H}{dy^2}+\Pr(\lambda^2+S)H=0 \tag{18}$$

$$\frac{d^2N}{dy^2}+\lambda^2 ScN=0 \tag{19}$$

Eliminating G from (16) and (17), we get

$$(1+\lambda^2 k_1)\frac{\partial^2 F}{\partial y^2}+\frac{f}{1-\lambda^2}F+F(\lambda^2-f-M-\frac{1}{K_1})=-\beta_1 H-\beta_2 N \tag{20}$$

Equation (16) can be rewritten as

$$\frac{\partial^2 F}{\partial y^2}+n^2F=-\beta_1' H-\beta_2' N \tag{21}$$

where $n^2=\left[\frac{\lambda^4-\lambda^2(\lambda^2-f-M-\frac{1}{K_1})+M+\frac{1}{K_1}}{(\lambda^2-1)(1+\lambda^2 k_1)}\right]$, $\beta_1'=\beta_1/(1+\lambda^2 k_1)$, $\beta_2'=\beta_2/(1+\lambda^2 k_1)$

From equation (18) and (19) we get

$$H(y)=e^{-i\gamma y} \tag{22}$$

$$N(y)=e^{-iry} \tag{23}$$

where $\gamma=\sqrt{(\lambda^2+S)P_r}$, $r=\lambda\sqrt{Sc}$

The solution of (21), using the boundary conditions (15) is,

$$F(y)=e^{-iny}+\frac{\beta_1'}{n^2-\gamma^2}(e^{-iny}-e^{-i\gamma y})+\frac{\beta_2'}{n^2-r^2}(e^{-iny}-e^{-iry}) \tag{24}$$

From equation (17), we get

$$G(y)=\frac{1}{(1-\lambda^2)}\left[e^{-iny}+\frac{\beta_1'}{n^2-\gamma^2}(e^{-iny}-e^{-i\gamma y})+\frac{\beta_2'}{n^2-r^2}(e^{-iny}-e^{-iry})\right] \tag{25}$$

Hence the velocity of the dusty fluid is

$$u=\left[e^{-iny}+\frac{\beta_1'}{n^2-\gamma^2}(e^{-iny}-e^{-i\gamma y})+\frac{\beta_2'}{n^2-r^2}(e^{-iny}-e^{-iry})\right]e^{-\lambda^2 t} \tag{26}$$

The real part of u is given by

$$u=\left[\cos ny+\frac{\beta_1'}{n^2-\gamma^2}(\cos ny-\cos\gamma y)+\frac{\beta_2'}{n^2-r^2}(\cos ny-\cos ry)\right]e^{-\lambda^2 t} \tag{27}$$

Similarly, the real part of the velocity of the dust particle

$$v=\frac{1}{(1-\lambda^2)}\left[\cos ny+\frac{\beta_1'}{n^2-\gamma^2}(\cos ny-\cos\gamma y)+\frac{\beta_2'}{n^2-r^2}(\cos ny-\cos ry)\right]e^{-\lambda^2 t} \tag{28}$$

The temperature distribution is given by

$$\theta=e^{-i\gamma y}e^{-\lambda^2 t} \tag{29}$$

The concentration distribution is given by

$$\phi=e^{-iry}e^{-\lambda^2 t} \tag{30}$$

The real part of θ and Φ are given by

$$\theta=\cos\gamma y\ e^{-\lambda^2 t} \tag{31}$$

$$\phi=\cos ry\ e^{-\lambda^2 t} \tag{32}$$

RESULTS AND DISCUSSION

The heat and mass transfer in MHD free convective flow of a visco-elastic dusty fluid of Walters B' model in presence of heat transfer is investigated. The expression for velocity, temperature and concentration are obtained by solving the governing equations analytically. During the numerical computation $\lambda = 0.5$, $f = 0.2$, $P_r = 0.71$, $Sc = 0.24$, $t = 1$ are taken for our convenience to compute. The effect of pertinent parameters affecting the flow phenomena like Magnetic parameter (M), permeability parameter (K_p), elastic parameter (R_c), source parameter (S), Pradatl number (P_r), Schmidt number (S_c) etc. are discussed with help of graphs.

From Figure 1 we observed that the magnetic parameter reduces the velocity (curves II and III). It agrees physically due to the presence of Lorentz force. The presence of permeability, elastic and source parameter increases the velocity profile at all points. The Grashof number reduces the velocity profile. Figure 2 is drawn for the effect of particle velocity i.e. velocity due to flow of dust particle in the fluid. On careful observation we notice that the effect of magnetic parameter, source parameter and Grashof number is to reduce the particle velocity at all points of the fluid (Curve II and III). Permeability of the porous medium, elastic parameter enhances the particle velocity.

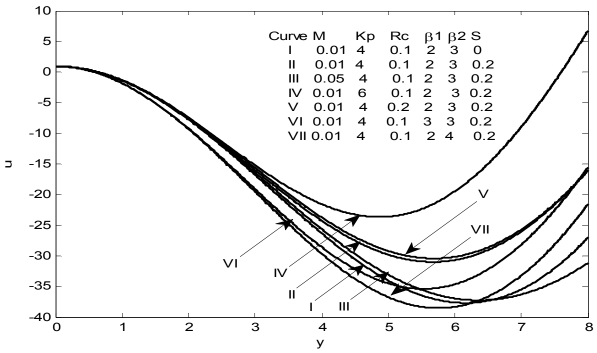

Figure 1. Velocity profile of dusty fluid with Pr = 0.71 and Sc = 0.68

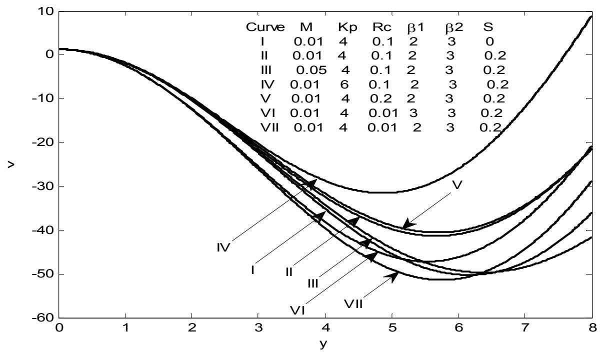

Figure 2. Velocity profile of the dust particle with Pr = 0.71 and Sc = 0.68

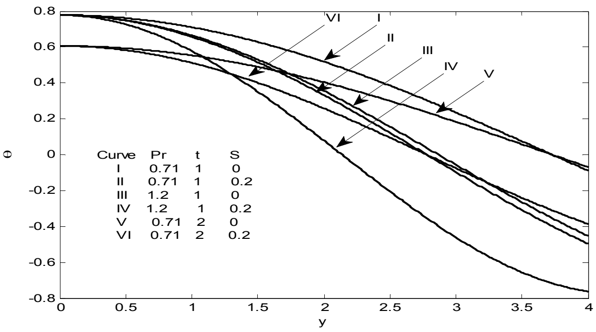

Figure 3. Temperature profile

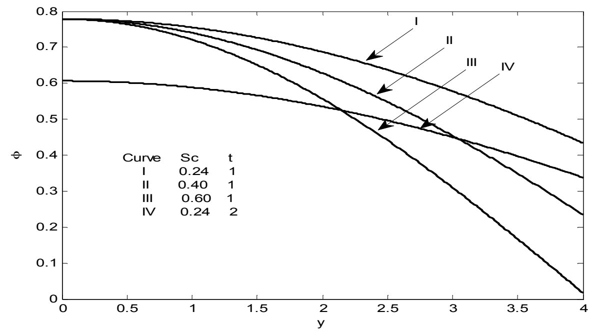

Figure 4. Concentration profile

The temperature profile of visco-elastic fluid (Walters Model – B') dusty fluid in presence of source parameter is plotted in Figure 3 for $\lambda = 0.5$, $f = 0.2$, $Sc = 0.24$, $M = 0.01$, $K_p = 4$, $\beta_1 = 2$, $\beta_2 = 3$ and different values of Pr and t. It is observed that temperature decreases continuously with increase in y. It is interesting to note that in the absence of source parameter our result is an good agreement with the result of P. Singh et.al. [14]. We conclude that the fluid temperature decreases with increase in Prandtl number and time.

In Figure 4 the concentration profile for visco-elastic dusty fluid is plotted for $\lambda = 0.5$, $f = 0.2$, $P_r = 0.71$, $M = 0.01$, $K_p = 4$, $\beta_1 = 2$, $\beta_2 = 3$ and different values of S_c and t. It is observed that concentration decreases continuously with increase in y. It is to note that our result is an good agreement with the result of P.singh et.al.[14]. We conclude that the concentration decreases with increase in Schmidt number and time.

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