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MHD SLIP FLOW OF VISCOUS FLUID OVER AN ISOTHERMAL REACTIVE STRETCHING SHEET

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ABSTRACT: *Heat and mass transfer in the slip flow of a viscous incompressible fluid over flat isothermal linearly stretching sheet, in presence of transverse magnetic field, is investigated. The sheet reacts with the flowing fluid and inert specie is produced that diffuses inside the boundary layer. The mass flux of the specie at the plate is taken directly proportional to specie concentration at the plate. The governing equations of continuity, momentum, energy and specie diffusion are transformed into ordinary differential equation using suitable transformations. The system of non-linear ordinary differential equations is obtained and solved numerically using Runge-Kutta method along with shooting technique. The velocity, heat and specie concentration distribution are obtained for different parameters and presented through figures. Skin-friction coefficient, Nusselt number and Sherwood number at plate for various physical parameters are discussed numerically and presented through tables.*
KEYWORDS: *Slip flow, Stretching surface, Reactive surface, MHD*

INTRODUCTION

The boundary layer flow over a stretching sheet in the quiescent fluid is studied in fluid dynamics due to its numerous engineering applications. At micro-scale level the fluid flow on stretching sheet is involved in processing of micro electro mechanical systems (MEMS). One of the building blocks in MEMS processing is the ability to deposit or selectively remove specie at or from the surface. These processes involve fluid flow, heat and mass transfer at micro-scale level and the flows are dominated by fluid surface interaction and belong to slip flow regime [Gad-el-Hak (1999)]. Studies concerning fluid flow with slip boundary condition have been addressed in literature, but the interplay of such a flow with heat and mass transfer are in scanty. Andersson (2002) presented the exact solution of slip flow past a stretching sheet. Martin and Boyd (2006) studied the momentum and heat transfer in boundary layer flow of fluid along a stationary plate in uniform stream employing the Maxwell slip condition. Martin and Boyd (2009) analyzed the stagnation point heat transfer with slip flow. Aziz (2010) modified the work of Martin and Boyd (2006) with constant heat flux. Makinde and Chinyoka (2010) presented a numerical solution for the effect of Navier slip on MHD transient flows and heat transfer of dusty fluid in a channel with variable physical properties. Makinde (2011) investigated the thermal boundary layer problem over a vertical moving surface with internal internal heat generation and a convective boundary condition. Ali et al. (2011) studied the effect of Hall current on hydromagnetic mixed convection boundary layer flow over a stretched vertical flat plate. Makinde (2009) obtained the similarity solution for the MHD boundary-layer flow and mass transfer past a vertical plate in a porous medium with constant heat flux. Fang et al. (2009) presented the exact solution of MHD slip flow over a stretching sheet in quiescent fluid. Earlier, Chiam (1997, 1998) studied the fluid on the stretching with various thermal boundary conditions and variable physical properties. Murray and Carey (1998) studied the growth of metal oxide on the reactive surface in chemical transport phenomena where the specie concentration is proportional to mass flux at the surface. Succi et al. (2002) analyzed reactive microflows over catalytic surface using Lattice Boltzmann Simulation. Liu et al. (2008) investigated the boundary layer modeling of reactive flow over porous surface with angled injection.

In the present paper MHD slip flow due to linearly stretching sheet in the quiescent fluid is studied. The sheet surface is isothermal, reactive to the fluid and produces inert specie, which diffuses inside the boundary layer. The specie concentration at the plate is taken proportional to mass flux at the plate [Murray and Carey (1998)] as the boundary condition. The partial differential equations governing the flow are equations of continuity; momentum, energy and specie diffusion, which are transformed into ordinary differential equations using the similarity transformation and solved numerically. The interplay between various obtained dimensionless parameters is explained using tables and graphs

PROBLEM FORMULATION

Consider a steady 2D laminar slip flow over a flat isothermal linearly stretching sheet in a viscous incompressible electrically conducting quiescent fluid. The x-axis is taken along the sheet and y-axis is normal to it, the flow is confined in half plane $y > 0$. The sheet stretches in its own plane with velocity, $u_w = ax$. The magnetic field with strength B is applied in y direction and the induced magnetic field is neglected. The sheet surface is considered to react with the fluid and produces insert specie that would diffuse into the fluid. The flux of the specie at the plate is taken directly proportional to specie concentration at the plate. Following Martin and Boyd (2006), Fang (2009) and Aziz (2010), the governing equations of continuity, momentum, energy and specie are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 , \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2}{\rho}u , \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\kappa}{\rho C_p}\frac{\partial^2 T}{\partial y^2} , \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} . \tag{4}$$

The appropriate boundary conditions are

$$y = 0 : \quad u = ax + L\frac{\partial u}{\partial y}, v = 0, T = T_w, -D\frac{\partial C}{\partial y} = RC ,$$
$$y \to \infty : u_\infty = 0,\ T \to T_\infty,\ C \to C_\infty . \tag{5}$$

Introducing the stream function ψ (x, y) such that

$$u = \frac{\partial \psi}{\partial y} \text{ and } v = -\frac{\partial \psi}{\partial x} , \tag{6}$$

where $\psi(x, y) = \sqrt{a\nu}\, x\, f(\eta)$ and the dimensionless variable $\eta = y\left(\frac{a}{\nu}\right)^{1/2}$, (7)

the system of equations (1) to (4) is reduced to system of ordinary differential equations, given as

$$f''' + ff'' - f'^2 - M^2 f' = 0 , \tag{8}$$

$$\theta'' + Pr\, f\theta' = 0 , \tag{9}$$

and

$$\phi'' + Scf\phi' = 0 . \tag{10}$$

The boundary conditions (5) are reduced to

$$f(0) = 0, f'(0) = 1 + \gamma f''(0), f'(\infty) = 0, \theta(0) = 1,\ \theta(\infty) = 0,$$
$$\phi'(0) = -Da\,(1 + \phi(0)), \phi(\infty) = 0. \tag{11}$$

The equations (8)-(10) together with boundary conditions (11) form a system of coupled non-linear ordinary differential equations. The system is solved using Runge-Kutta fourth order method along with shooting technique [Conte and Boor (1981)]. From the process of numerical computation, we obtained the expressions for the skin-friction coefficient C_f , the Nusselt number (Nu) and the local Sherwood number (Sh) as

$$C_f = 2(Re)^{-1/2} f''(0),\ Nu = -(Re)^{1/2}\theta'(0),\ Sh = -(Re)^{1/2}\phi'(0). \tag{12}$$

In order to validate the numerical scheme, the results obtained for $f''(0)$ were compared with Andersson (2002) in the absence of magnetic field i.e. M = 0 and with Fang et al. (2009) for M = 0.5, which are shown in table 1 below;

Table 1. Comparison of the numerical values of $f''(0)$ with Andersson (2002) and Fang et al. (2009).

γ	Andersson (2002)	Present Paper	γ	Fang et al. (2009)	Present Paper
0.1	-0.8721	-0.872082	0.1	-0.64951	-0.649513
0.5	-0.5912	-0.591195	0.5	-0.46912	-0.469122
1.0	-0.4302	-0.430159	1.0	-0.23051	-0.2305131
2.0	-0.2840	-0.283979	2.0	-0.08617	-0.086171

RESULTS AND DISCUSSION

In the case of no-slip boundary condition the fluid sticks to sheet and so fluid velocity is same as sheet velocity, however in slip flow regime this ceases to be true. It is seen from Table 2 that with increase in velocity slip parameter γ, the drag ($-f''(0)$) experienced by the fluid, due to sheet velocity, decreases and therefore the surface fluid velocity ($f'(0)$) decreases. It is also observed that the rate heat transfer i.e. flux ($-\theta'(0)$) decreases while the mass flux ($-\phi'(0)$) increases with the increase in parameter γ.

Table 2. Numerical values of $f''(0)$, $f'(0)$, $-\theta'(0)$, $-\phi'(0)$ and $\phi(0)$ for different values of parameter γ and M.

	Pr = 1.0, Sc = 0.5, Da = 0.2, M = 0.25				
	$f''(0)$	$f'(0)$	$-\theta'(0)$	$-\phi'(0)$	$\phi(0)$
γ = 0.1	-0.897729	0.910227	0.548585	0.502126	0.502126
γ = 0.5	-0.606874	0.696562	0.478160	0.648009	2.240045
γ = 1.0	-0.440885	0.559145	0.426795	0.891978	3.459892
γ = 2.0	-0.290581	0.418836	0.367059	2.091622	9.458112
	Sc = 2.0, Da = 0.2, M = 0.25, γ = 0.1				
M = 0.25	-0.897729	0.910227	0.548585	0.260378	0.301890
M = 0.5	-0.969671	0.903032	0.527848	0.262567	0.312836
M = 0.75	-1.076350	0.892364	0.497594	0.266149	0.330745
M = 1.0	-1.205640	0.879435	0.762071	0.271098	0.355494
M = 1.5	-1.497150	0.850284	0.388523	0.285300	0.426503

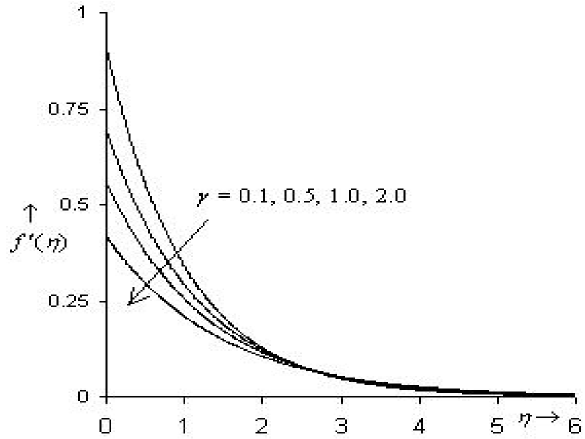

Figure 1. Velocity distribution versus η when Pr = 1.0, Sc = 0.5, M = 0.25, Da = 0.2.

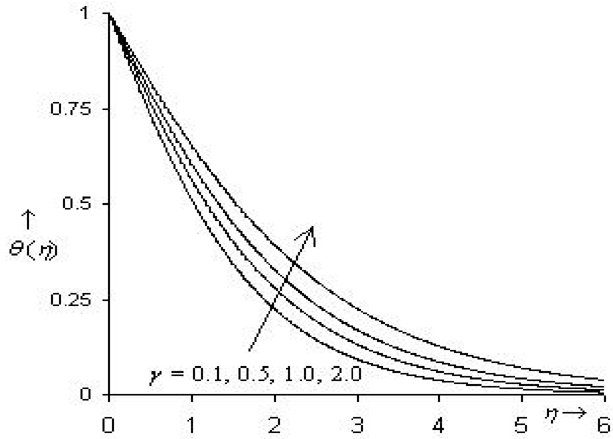

Figure 2. Temperature distribution versus η when Pr = 1.0, Sc = 0.5, M = 0.25, Da = 0.2

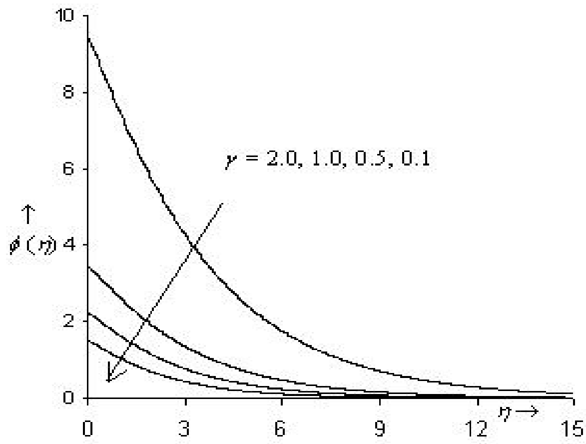

Figure 3. Concentration distribution versus η when Pr = 1.0, Sc = 0.5, M = 0.25, Da = 0.2

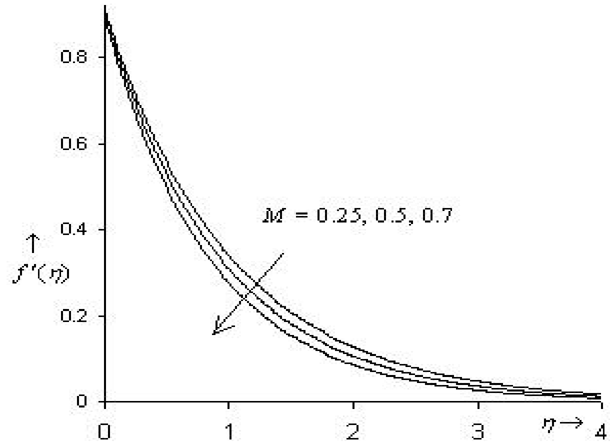

Figure 4. Velocity distribution versus η when γ = 0.1, Sc = 2.0, Da = 0.2, Pr = 1.0

Figure 1 shows that the fluid velocity decreases with increase in parameter γ, which happens because fluid experiences less drag with increase in γ. Now as the fluid velocity decreases, the heat transfer due to convection decreases so the temperature of fluid rises. This is seen in Figure 2. Adding, the decreases in fluid velocity results in accumulation of specie in fluid therefore specie concentration in fluid increases, which is evident from Figure 3. It is also observed that temperature *and concentration boundary thickness decreases with the increase in parameter γ. Table 2 shows that with the increase in magnetic parameter M, the drag experienced by the fluid at sheet increases while the fluid velocity at sheet decreases.*

Figure 4 depicts that fluid velocity decreases with the increase in parameter M. This happens due to setting up of Lorentz force in presence of transverse magnetic field, which impedes fluid velocity. As the fluid velocity decreases the fluid temperature and specie concentration in fluid also increases. This is seen in Figures 5 and 6.

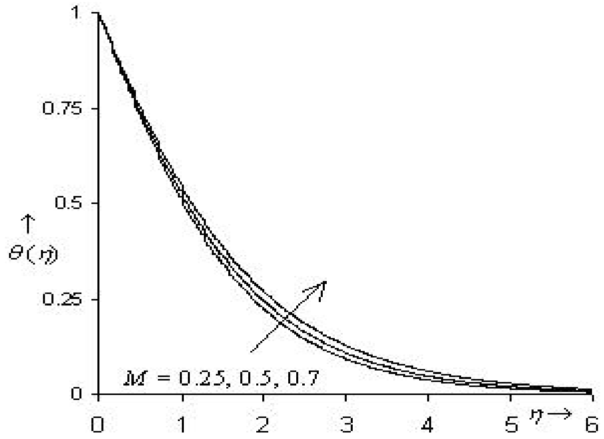

Figure 5. Temperature distribution versus η when γ = 0.1, Sc = 2.0, Da = 0.2, Pr = 1.0

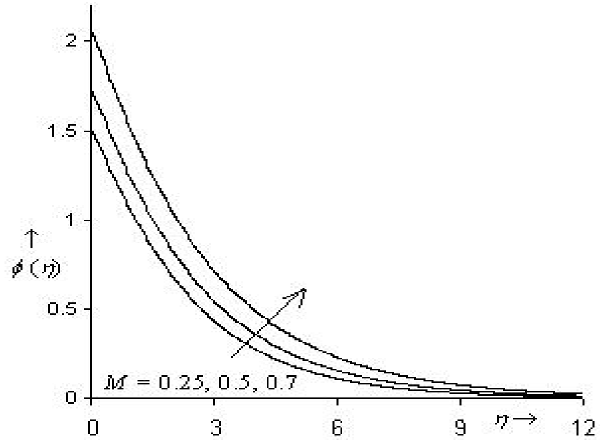

Figure 6. Concentration distribution versus η when γ = 0.1, Sc = 2.0, Da = 0.2, Pr = 1.0

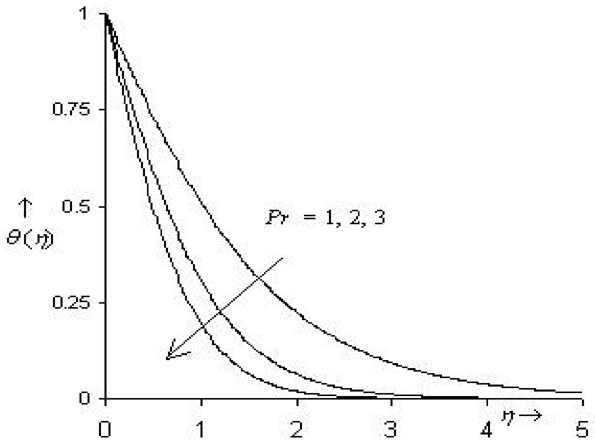

Figure 7. Temperature distribution versus η when γ = 0.1, Sc = 0.5, M = 0.25, Da = 0.2

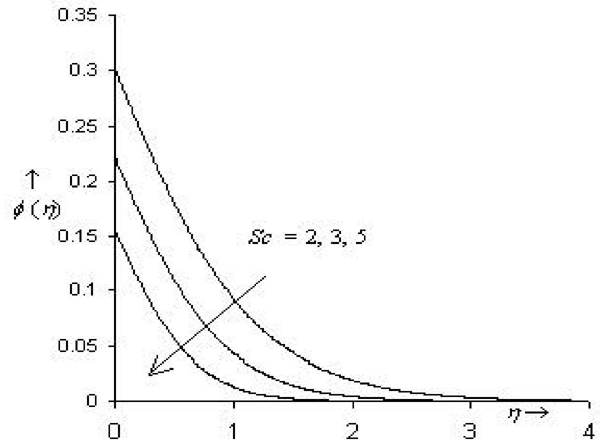

Figure 8. Concentration distribution versus η when γ = 0.1, Pr = 1.0, M = 0.25, Da = 0.2

The change in Prandtl number Pr and Schmidt number Sc does not affect the fluid velocity hence only values of $-\theta'(0)$ and $-\phi'(0)$ are shown in Table 3. The increase in Prandtl number increases the rate of heat transfer while the increase in Schmidt number decreases the mass flux.

Figure 7 shows that the fluid temperature decrease with increase in Prandtl number while figure 8 shows that with the increases in Schmidt number, the specie concentration in fluid decreases along with specie concentration at sheet ($\phi(0)$).

Table 4 shows the affect of increase in specie generation parameter Da on the mass flux and specie concentration at sheet. The increase in Da means that specie generation at sheet increases. So as Da increases the mass flux and specie concentration at the sheet increases. Figure 9 shows that specie concentration in the fluid and concentration boundary layer thickness increases with the increase in parameter Da.

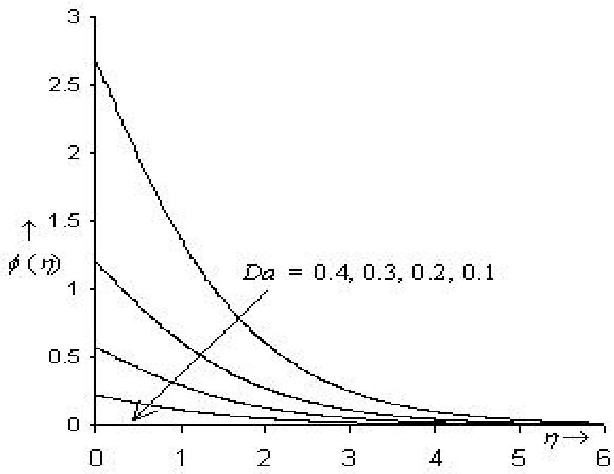

Figure 9. Concentration distribution versus η when γ = 0.1, Sc = 1.0, M = 0.25, Pr = 1.0

Table 3. Numerical values of $-\theta'(0)$, $-\phi'(0)$ and $\phi(0)$ for different values of parameters Pr and Sc.

Sc = 0.5, Da = 0.2, M = 0.25, γ = 0.1		Sc = 0.5, Da = 0.2, M = 0.25, γ = 0.1		
	$-\theta'(0)$		$-\phi'(0)$	$\phi(0)$
Pr = 1	0.548585	Sc = 0.5	0.502126	1.510631
Pr = 2	0.862491	Sc = 2.0	0.260378	0.301890
Pr = 3	1.104662	Sc = 5.0	0.231032	0.155163

Table 4. Numerical values of $-\phi'(0)$ and $\phi(0)$ for different values of parameter Da.

	Sc = 0.5, Pr = 1.0 M = 0.25, γ = 0.1		Sc = 2.0, Pr = 1.0 M = 0.25, γ = 0.1		Sc = 2.0, Pr = 1.0 M = 1.0, γ = 0.1		Sc = 2.0, Pr = 1.0 M = 0.25, γ = 2.0	
	$-\phi'(0)$	$\phi(0)$	$-\phi'(0)$	$\phi(0)$	$-\phi'(0)$	$\phi(0)$	$-\phi'(0)$	$\phi(0)$
Da = 0.1	0.143031	0.430301	0.113114	0.131148	0.115092	0.150921	0.120846	0.208464
Da = 0.15	0.273358	0.822391	0.181579	0.210528	0.186728	0.244859	0.202362	0.349082
Da = 0.2	0.502126	1.510631	0.260378	0.301890	0.271098	0.355494	0.305346	0.526734
Da = 0.25	1.008541	3.034166	0.352042	0.408168	0.371928	0.487712	0.439567	0.758270
Da = 0.3	3.078211	9.260704	0.460002	0.533341	0.494553	0.648512	0.621775	1.072586

CONCLUSIONS

1. The drag experienced by the fluid decreases with the increase in velocity slip parameter while it increases with the increase in magnetic parameter.
2. The rate of heat transfer decreases with the increase in velocity slip parameter or magnetic parameter while it increases with the increase in Prandtl number.
3. The mass flux increases with the increase in velocity slip parameter, magnetic parameter or specie generation parameter while it decreases with the increase in Schmidt number.
4. The fluid velocity decreases with the increases in velocity slip parameter or magnetic parameter.
5. The fluid temperature increase with the increases in velocity slip parameter or magnetic parameter while it decreases with the increase in Prandtl number.
6. Specie concentration in fluid increases with the increase in velocity slip parameter, magnetic parameter or specie generation parameter while if decreases with the increase in Schmidt number.

APPENDIX : NOMENCLATURE

x, y : Cartesian coordinates
C : concentration of the specie in the fluid
C_∞: concentration of the specie in fluid far away from plate
D : diffusion coefficient
f : dimensionless stream function
R : interfacial reaction rate constant $\{= xk_0\}$
B: magnetic field strength
M: magnetic parameter $\left\{=\left(\frac{\sigma B^2}{\rho a}\right)^{1/2}\right\}$
Nu : Nusselt number
a : positive constant
Pr : Prandtl number $\left\{=\frac{\mu C_p}{\kappa}\right\}$
L: proportionality constant of velocity slip
Da : specie generation parameter $\left\{=\frac{k_o\sqrt{Re}}{\left(\frac{D}{\nu}\right)a}\right\}$
(Damkohler Number)
k_0 : reaction rate
Re : Reynolds number $\left\{=\frac{U_w x}{\nu}\right\}$
Sh : Sherwood number
Sc : Schmidt number $\left\{=\frac{\nu}{D}\right\}$
C_f : skin-friction coefficient
C_p : specific heat at constant pressure
T : temperature of the fluid
T_∞ : temperature of the fluid far away from plate
T_w : temperature of the plate
u, v : velocity components along x- and y-directions, respectively

GREEK LETTERS

μ : coefficient of viscosity
κ : coefficient of thermal conductivity
ρ : density of the fluid
ϕ : dimensionless concentration $\left\{=\frac{C-C_\infty}{C_\infty}\right\}$
θ : dimensionless temperature $\left\{=\frac{T-T_\infty}{T_w-T_\infty}\right\}$
η : dimensionless variable
σ : electrical conductivity of the fluid
ν : kinematic viscosity $\left\{=\frac{\mu}{\rho}\right\}$
ψ : stream function
γ : velocity slip parameter $\left\{=L\sqrt{\frac{a}{\nu}}\right\}$

REFERENCES

[1] Ali F, Nazar R, Arifin N, Pop I. Effect of Hall current on MHD mixed convection boundary layer flow over a stretched vertical flat plate, Meccanica, 46 (5), 1103-1112, 2011.
[2] Andersson H I. Slip flow past a stretching surface, Acta Mechanica, 158, 121-125, 2002

[3] *Aziz A. Hydrodynamic and thermal slip flow boundary layers over a flat plate with constant heat flux boundary condition, Comm. Nonlinear Num. Simu., 15, 573-580, 2010.*

[4] *Chiam T C. Magnetohydrodynamic heat transfer over a non-isothermal stretching sheet, Acta Mechanica, 122, 169-179,1997.*

[5] *Chaim T C (1998). Heat transfer in a fluid with variable thermal conductivity over stretching sheet, Acta Mechanica, 129, 63-72,1998.*

[6] *Conte S D, Boor C. Elementary Numerical Analysis, McGraw-Hill Book Co., New York, 1981.*

[7] *Fang T, Zhang J, Yao S. Slip MHD viscous flow over a stretching sheet- An exact solution, Comm. Nonlinear Num. Simu., 14, 3731-3737, 2009.*

[8] *Gad-el-Hak M. The fluid mechanics of microdevices- The Freeman Scholar Lecture, J Fluid Engg, 121(1), 5-33, 1999.*

[9] *Liu S, Fotache C G, Chao B H, Hautman D J, Ochs SS. Boundary layer modeling of reactive flow over a porous surface with angled injection, Comb. Flame, 154: 378-386, 2008.*

[10] *Makinde O D. On MHD boundary-layer flow and mass transfer past a vertical plate in a porous medium with constant heat flux, Int. J. Num. Methods Heat Fluid Flow, 19(3/4), 546-554, 2009.*

[11] *Makinde O D, Chinyoka T. MHD transient flows and heat transfer of dusty fluid in a channel with variable physical properties and Navier slip condition, Comp. Math. Appl., 60, 660-669, 2010.*

[12] *Makinde O D. Similarity solution for natural convection from a moving vertical plate with internal heat generation and a convective boundary condition, Thermal Sci., 15(1), S137-S143, 2011.*

[13] *Martin M J, Boyd I D. Momentum and heat transfer in a laminar boundary layer with slip flow, J. Thermo. Heat Trans., 20(4): 710-719, 2006.*

[14] *Martin M J, Boyd I D. Stagnation point heat transfers near the continuum limit, AIAA, 47(1), 283-285, 2009.*

[15] *Murray P, Carey G F (1998). Viscous flow and transport with moving free and reactive surfaces, Int. J. Num. Meth. Engg., 30, 1181-1194, 1998.*

[16] *Succi S, Smith G, Kaxiras E. Lattice Boltzmann Simulation of microflows over reactive surfaces, J. Stat. Phy., 107, 343-366, 2002*

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