MHD SLIP FLOW OF VISCOUS FLUID OVER AN ISOTHERMAL REACTIVE STRETCHING SHEET

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ABSTRACT: Heat and mass transfer in the slip flow of a viscous incompressible fluid over flat isothermal linearly stretching sheet, in presence of transverse magnetic field, is investigated. The sheet reacts with the flowing fluid and inert specie is produced that diffuses inside the boundary layer. The mass flux of the specie at the plate is taken directly proportional to specie concentration at the plate. The governing equations of continuity, momentum, energy and specie diffusion are transformed into ordinary differential equation using suitable transformations. The system of non-linear ordinary differential equations is obtained and solved numerically using Runge-Kutta method along with shooting technique. The velocity, heat and specie concentration distribution are obtained for different parameters and presented through figures. Skin-friction coefficient, Nusselt number and Sherwood number at plate for various physical parameters are discussed numerically and presented through tables.

KEYWORDS: Slip flow, Stretching surface, Reactive surface, MHD

INTRODUCTION

The boundary layer flow over a stretching sheet in the quiescent fluid is studied in fluid dynamics due to its numerous engineering applications. At micro-scale level the fluid flow on stretching sheet is involved in processing of micro electro mechanical systems (MEMS). One of the building blocks in MEMS processing is the ability to deposit or selectively remove specie at or from the surface. These processes involve fluid flow, heat and mass transfer at micro-scale level and the flows are dominated by fluid surface interaction and belong to slip flow regime [Gad-el-Hak (1999)]. Studies concerning fluid flow with slip boundary condition have been addressed in literature, but the interplay of such a flow with heat and mass transfer are in scanty. Andersson (2002) presented the exact solution of slip flow past a stretching sheet. Martin and Boyd (2006) studied the momentum and heat transfer in boundary layer flow of fluid along a stationary plate in uniform stream employing the Maxwell slip condition. Martin and Boyd (2009) analyzed the stagnation point heat transfer with slip flow. Aziz (2010) modified the work of Martin and Boyd (2006) with constant heat flux. Makinde and Chinyoka (2010) presented a numerical solution for the effect of Navier slip on MHD transient flows and heat transfer of dusty fluid in a channel with variable physical properties. Makinde (2011) investigated the thermal boundary layer problem over a vertical moving surface with internal heat generation and a convective boundary condition. Ali et al. (2011) studied the effect of Hall current on hydromagnetic mixed convection boundary layer flow over a stretched vertical flat plate. Makinde (2009) obtained the similarity solution for the MHD boundary-layer flow and mass transfer past a vertical plate in a porous medium with constant heat flux. Fang et al. (2009) presented the exact solution of MHD slip flow over a stretching sheet in quiescent fluid. Earlier, Chiam (1997, 1998) studied the fluid on the stretching with various thermal boundary conditions and variable physical properties. Murray and Carey (1998) studied the growth of metal oxide on the reactive surface in chemical transport phenomena where the specie concentration is proportional to mass flux at the surface. Succi et al. (2002) analyzed reactive microflows over catalytic surface using Lattice Boltzmann Simulation. Liu et al. (2008) investigated the boundary layer modeling of reactive flow over porous surface with angled injection.

In the present paper MHD slip flow due to linearly stretching sheet in the quiescent fluid is studied. The sheet surface is isothermal, reactive to the fluid and produces inert specie, which diffuses inside the boundary layer. The specie concentration at the plate is taken proportional to mass flux at the plate [Murray and Carey (1998)] as the boundary condition. The partial differential equations governing the flow are equations of continuity; momentum, energy and specie diffusion, which are transformed into ordinary differential equations using the similarity transformation and
solved numerically. The interplay between various obtained dimensionless parameters is explained using tables and graphs.

**PROBLEM FORMULATION**

Consider a steady 2D laminar slip flow over a flat isothermal linearly stretching sheet in a viscous incompressible electrically conducting quiescent fluid. The x-axis is taken along the sheet and y-axis is normal to it, the flow is confined in half plane \( y > 0 \). The sheet stretches in its own plane with velocity \( u_w = ax \). The magnetic field with strength \( B \) is applied in y direction and the induced magnetic field is neglected. The sheet surface is considered to react with the fluid and produces insert specie that would diffuse into the fluid. The flux of the specie at the plate is taken directly proportional to specie concentration at the plate. Following Martin and Boyd (2006), Fang (2009) and Aziz (2010), the governing equations of continuity, momentum, energy and specie are given by

\[
\begin{align*}
\frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y} &= 0, \\
\frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} &= \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2}{\rho} u, \\
\frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} &= \frac{k}{\rho C_p} \frac{\partial^2 T}{\partial y^2}, \\
\frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} &= D \frac{\partial^2 C}{\partial y^2}.
\end{align*}
\]

The appropriate boundary conditions are

\[
y = 0 : \quad u = ax + L \frac{\partial u}{\partial y}, v = 0, T = T_w, -D \frac{\partial C}{\partial y} = RC,
\]
\[
y \to \infty : u_w = 0, T \to T_w, C \to C_\infty.
\]

Introducing the stream function \( \psi(x, y) \) such that

\[
\begin{align*}
u &= \frac{\partial \psi}{\partial y}, \\
v &= -\frac{\partial \psi}{\partial x},
\end{align*}
\]

where \( \psi(x, y) = \sqrt{u/v} f(\eta) \) and the dimensionless variable \( \eta = \frac{a}{v} \sqrt{x} \),

the system of equations (1) to (4) is reduced to system of ordinary differential equations, given as

\[
\begin{align*}
f'''' + f(f' - f'^2 - M^2 f') &= 0, \\
\theta'' + Pr f \theta' &= 0, \\
\phi'' + Scf \phi' &= 0.
\end{align*}
\]

The boundary conditions (5) are reduced to

\[
\begin{align*}
f(0) &= 0, f'(0) = 1 + \gamma f''(0), f'(\infty) = 0, \theta(0) = 1, \theta(\infty) = 0, \\
\phi'(0) &= -Da (1 + \phi(0)), \phi(\infty) = 0.
\end{align*}
\]

The equations (8)-(10) together with boundary conditions (11) form a system of coupled non-linear ordinary differential equations. The system is solved using Runge-Kutta fourth order method along with shooting technique [Conte and Boor (1981)]. From the process of numerical computation, we obtained the expressions for the skin-friction coefficient \( C_f \), the Nusselt number (Nu) and the local Sherwood number (Sh) as

\[
\begin{align*}
C_f &= 2(Re)^{1/2} f''(0), \\
Nu &= -(Re)^{1/2} \theta'(0), \\
Sh &= -(Re)^{1/2} \phi'(0).
\end{align*}
\]

In order to validate the numerical scheme, the results obtained for \( f''(0) \) were compared with Andersson (2002) in the absence of magnetic field i.e. \( M = 0 \) and with Fang et al. (2009) for \( M = 0.5 \), which are shown in table 1 below;

<table>
<thead>
<tr>
<th>( \gamma )</th>
<th>Andersson (2002)</th>
<th>Present Paper</th>
<th>( \gamma )</th>
<th>Fang et al. (2009)</th>
<th>Present Paper</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>-0.8721</td>
<td>-0.872082</td>
<td>0.1</td>
<td>-0.64951</td>
<td>-0.649513</td>
</tr>
<tr>
<td>0.5</td>
<td>-0.5912</td>
<td>-0.591195</td>
<td>0.5</td>
<td>-0.46912</td>
<td>-0.469122</td>
</tr>
<tr>
<td>1.0</td>
<td>-0.4302</td>
<td>-0.430159</td>
<td>1.0</td>
<td>-0.23051</td>
<td>-0.2305131</td>
</tr>
<tr>
<td>2.0</td>
<td>-0.2840</td>
<td>-0.283977</td>
<td>2.0</td>
<td>-0.08617</td>
<td>-0.086171</td>
</tr>
</tbody>
</table>
RESULTS AND DISCUSSION

In the case of no-slip boundary condition the fluid sticks to sheet and so fluid velocity is same as sheet velocity, however in slip flow regime this ceases to be true. It is seen from Table 2 that with increase in velocity slip parameter $\gamma$, the drag ( $-f''(0)$) experienced by the fluid, due to sheet velocity, decreases and therefore the surface fluid velocity ( $f'(0)$) decreases. It is also observed that the rate heat transfer i.e. flux ( $-\theta'(0)$) decreases while the mass flux ( $-\phi'(0)$) increases with the increase in parameter $\gamma$.

Table 2. Numerical values of $f''(0)$, $f'(0)$, $-\theta'(0)$, $-\phi'(0)$ and $\phi(0)$ for different values of parameter $\gamma$ and $M$.

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>$f''(0)$</th>
<th>$f'(0)$</th>
<th>$-\theta'(0)$</th>
<th>$-\phi'(0)$</th>
<th>$\phi(0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>-0.897729</td>
<td>0.910227</td>
<td>0.545855</td>
<td>0.502126</td>
<td>0.502126</td>
</tr>
<tr>
<td>0.5</td>
<td>-0.606874</td>
<td>0.696562</td>
<td>0.478160</td>
<td>0.648009</td>
<td>2.240045</td>
</tr>
<tr>
<td>1.0</td>
<td>-0.440885</td>
<td>0.559145</td>
<td>0.426795</td>
<td>0.891978</td>
<td>3.459892</td>
</tr>
<tr>
<td>2.0</td>
<td>-0.290581</td>
<td>0.418836</td>
<td>0.367059</td>
<td>2.091622</td>
<td>9.458112</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$M$</th>
<th>$f''(0)$</th>
<th>$f'(0)$</th>
<th>$-\theta'(0)$</th>
<th>$-\phi'(0)$</th>
<th>$\phi(0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>-0.897729</td>
<td>0.910227</td>
<td>0.545855</td>
<td>0.260378</td>
<td>0.301890</td>
</tr>
<tr>
<td>0.5</td>
<td>-0.969671</td>
<td>0.903032</td>
<td>0.527848</td>
<td>0.262567</td>
<td>0.312836</td>
</tr>
<tr>
<td>0.75</td>
<td>-1.078350</td>
<td>0.892364</td>
<td>0.497594</td>
<td>0.266149</td>
<td>0.330745</td>
</tr>
<tr>
<td>1.0</td>
<td>-1.205640</td>
<td>0.879435</td>
<td>0.762071</td>
<td>0.270987</td>
<td>0.355494</td>
</tr>
<tr>
<td>1.5</td>
<td>-1.497150</td>
<td>0.850284</td>
<td>0.388523</td>
<td>0.285300</td>
<td>0.428503</td>
</tr>
</tbody>
</table>

Figure 1. Velocity distribution versus $\eta$ when $Pr = 1.0$, $Sc = 0.5$, $M = 0.25$, $Da = 0.2$.

Figure 2. Temperature distribution versus $\eta$ when $Pr = 1.0$, $Sc = 0.5$, $M = 0.25$, $Da = 0.2$.

Figure 3. Concentration distribution versus $\eta$ when $Pr = 1.0$, $Sc = 0.5$, $M = 0.25$, $Da = 0.2$.

Figure 4. Velocity distribution versus $\eta$ when $\gamma = 0.1$, $Sc = 2.0$, $Da = 0.2$, $Pr = 1.0$.

Figure 1 shows that the fluid velocity decreases with increase in parameter $\gamma$, which happens because fluid experiences less drag with increase in $\gamma$. Now as the fluid velocity decreases, the heat transfer due to convection decreases so the temperature of fluid rises. This is seen in Figure 2. Adding, the decreases in fluid velocity results in accumulation of specie in fluid therefore specie concentration in fluid increases, which is evident from Figure 3. It is also observed that temperature
and concentration boundary thickness decreases with the increase in parameter $\gamma$. Table 2 shows that with the increase in magnetic parameter $M$, the drag experienced by the fluid at sheet increases while the fluid velocity at sheet decreases.

Figure 4 depicts that fluid velocity decreases with the increase in parameter $M$. This happens due to setting up of Lorentz force in presence of transverse magnetic field, which impedes fluid velocity. As the fluid velocity decreases the fluid temperature and specie concentration in fluid also increases. This is seen in Figures 5 and 6.

The change in Prandtl number $Pr$ and Schmidt number $Sc$ does not affect the fluid velocity hence only values of $-\theta'(0)$ and $-\phi'(0)$ are shown in Table 3. The increase in Prandtl number increases the rate of heat transfer while the increase in Schmidt number decreases the mass flux.

Figure 7 shows that the fluid temperature decrease with increase in Prandtl number while figure 8 shows that with the increases in Schmidt number, the specie concentration in fluid decreases along with specie concentration at sheet ($\phi(0)$).

Table 4 shows the affect of increase in specie generation parameter $Da$ on the mass flux and specie concentration at sheet. The increase in $Da$ means that specie generation at sheet increases. So as $Da$ increases the mass flux and specie concentration at the sheet increases. Figure 9 shows that specie concentration in the fluid and concentration boundary layer thickness increases with the increase in parameter $Da$. 
and for different values of parameter Da.

### Table 3. Numerical values of $-\theta'(0)$, $-\phi'(0)$ and $\phi(0)$ for different values of parameters Pr and Sc.

<table>
<thead>
<tr>
<th>Pr = 1</th>
<th>$-\theta'(0)$</th>
<th>$-\phi'(0)$</th>
<th>$\phi(0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Pr = 1$</td>
<td>0.548585</td>
<td>$Sc = 0.5$</td>
<td>0.50276</td>
</tr>
<tr>
<td>$Pr = 2$</td>
<td>0.862491</td>
<td>$Sc = 2.0$</td>
<td>0.280378</td>
</tr>
<tr>
<td>$Pr = 3$</td>
<td>1.104662</td>
<td>$Sc = 5.0$</td>
<td>0.231032</td>
</tr>
</tbody>
</table>

### Table 4. Numerical values of $-\phi'(0)$ and $\phi(0)$ for different values of parameter Da.

<table>
<thead>
<tr>
<th>Da</th>
<th>$-\phi'(0)$</th>
<th>$\phi(0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Da = 0.1$</td>
<td>0.143031</td>
<td>0.430301</td>
</tr>
<tr>
<td>$Da = 0.15$</td>
<td>0.273358</td>
<td>0.822391</td>
</tr>
<tr>
<td>$Da = 0.2$</td>
<td>0.502126</td>
<td>1.510631</td>
</tr>
<tr>
<td>$Da = 0.25$</td>
<td>0.708541</td>
<td>2.04166</td>
</tr>
<tr>
<td>$Da = 0.3$</td>
<td>1.008541</td>
<td>2.960704</td>
</tr>
</tbody>
</table>

### CONCLUSIONS
1. The drag experienced by the fluid decreases with the increase in velocity slip parameter while it increases with the increase in magnetic parameter.
2. The rate of heat transfer decreases with the increase in velocity slip parameter or magnetic parameter while it increases with the increase in Prandtl number.
3. The mass flux increases with the increase in velocity slip parameter, magnetic parameter or specie generation parameter while it decreases with the increase in Schmidt number.
4. The fluid velocity decreases with the increase in velocity slip parameter or magnetic parameter.
5. The fluid temperature increase with the increases in velocity slip parameter or magnetic parameter while it decreases with the increase in Prandtl number.
6. Specie concentration in fluid increases with the increase in velocity slip parameter, magnetic parameter or specie generation parameter while if decreases with the increase in Schmidt number.

### APPENDIX : NOMENCLATURE

- $x, y$: Cartesian coordinates
- $C$: concentration of the specie in the fluid
- $C_\infty$: concentration of the specie in fluid far away from plate
- $D$: diffusion coefficient
- $f$: dimensionless stream function
- $R$: interfacial reaction rate constant $\equiv \frac{a}{k_0}$
- $B$: magnetic field strength
- $M$: magnetic parameter $\left[ \sigma \beta \gamma \rho a \right]^{\frac{1}{2}}$
- $Nu$: Nusselt number
- $a$: positive constant
- $Pr$: Prandtl number $\frac{\mu C_p}{\kappa}$
- $L$: proportionality constant of velocity slip
- $Da$: specie generation parameter $\frac{k_0 \sqrt{Re}}{D \sqrt{a}}$
- $k_0$: reaction rate
- $Re$: Reynolds number $\frac{U_w \gamma}{V}$
- $Sh$: Sherwood number
- $Sc$: Schmidt number $\left[ \frac{\nu}{D} \right]$
- $C_f$: skin-friction coefficient
- $\phi$: dimensionless concentration $\frac{C-C_\infty}{C_\infty}$
- $\theta$: dimensionless temperature $\frac{T-T_w}{T_w}$
- $\eta$: dimensionless variable
- $\sigma$: electrical conductivity of the fluid
- $\nu$: kinematic viscosity $\frac{\mu}{\rho}$
- $\psi$: stream function
- $\gamma$: velocity slip parameter $\left[ \frac{L \sqrt{a}}{V} \right]$

### GREEK LETTERS

- $\mu$: coefficient of viscosity
- $\kappa$: coefficient of thermal conductivity
- $\rho$: density of the fluid
- $\gamma$: velocity of the fluid along $x$- and $y$-directions, respectively

### REFERENCES


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