INVENTORY MANAGEMENT IN A BI-ECHELON MULTI-ITEM SUPPLY CHAIN USING AN INTEGRATED MATHEMATICAL MODEL WITH A FUZZY-ANP APPROACH

ABSTRACT: In this paper, an inventory management framework and deterministic costs models within the context of this framework are structured for effective supply chain under deterministic and fuzzy environment. In this regard, we propose a bi-echelon multi-product supply chain in which a central manufacturing firm (lower echelon) with warehouse space limitation is fed through different suppliers (upper echelon). In order to develop operational measures of supply chain flexibility, a cost structure which captures some different operational cost factors such as inventory holding costs, backordering cost, ordering costs is defined. Moreover, a fuzzy-ANP framework is developed to evaluate suppliers agility to achieve a more agile and responsible network. Further, computational experiment is augmented to examine the supposed model validation as well.

KEYWORDS: Bi-echelon supply chain, Integrated approach, Inventory control and management, Fuzzy analytic network process

INTRODUCTION

Researchers as well as manufacturing industries’ practitioners have recently taken account into inventory control problems’ development in supply chain management (Sana 2011). Since, 20 to 60 percent of the total assets of manufacturing firms belong to inventory (Gumus and Guneri 2007). The two basic questions when and how much to order/product must be answered in the context of inventory control systems. To do so, a numerous papers and books have been published based upon various conditions and assumptions. After Harris’s classic square root economic order quantity (EOQ) model development, the researchers and practitioners have taken into account the analysis and modeling inventory systems with respect to different operating parameters and assumptions (Routroy and Kodali 2005; Pentico and Drake 2009). Further, some authors have extended the classic EOQ model to relax one or more of its traditional assumptions; such as Jagieta and Michenzi (1982), and Khouja and Park (2003) (Pentico and Drake 2009). Thereafter, many models have been developed to identify optimal lot sizes and safety stocks in a multi-echelon framework (Gumus and Guneri 2007). This paper is extended to relax some relative assumptions of classical production/inventory control models including single-product, and infinite availability of warehouse space. Whereas, a multi-product two-echelon supply chain including many suppliers and one central manufacturing plant with warehouse space limitation is supposed. In addition, in order to elicit suitable suppliers to provide firm’s demand in an agile manner, a fuzzy-ANP framework is developed.

The rest of the paper is organized as follows. In Section 2, we briefly review the previously performed researches. Next, Section 3 presents a bi-echelon multi-item supply chain. The supposed fuzzy-ANP framework and the models of the proposed inventory policies are introduced as well. Section 4 presents the experimental results in order to validate the proposed model. Finally, we provide some conclusions and future research directions in the last section.

LITERATURE REVIEW

The analysis of multi-echelon inventory systems with a long history has been introduced by Clark and Scarf (1960). Inventory control is a challenging research area in multi-echelon systems. The first multi-echelon inventory model has been developed by Sherbrooke (1968) in order to manage the inventory of service parts. Thereafter, many researchers produced numerous models that generally seek to identify optimal lot sizes and safety stocks in a multi-echelon framework (e.g. Moinzadeh and Lee 1986; Grahovac and Chakravarty 2001). Mathematical modeling technique has been applied to manage multi-echelon inventory in SCs (Rau et al. 2003; Diks and de Kok 1998; Mitra and Chatterjee 2004; Axsater and Zhang 1999; Nozick and Turnquist 2001). Diks and de Kok (1998) studied a divergent multi-echelon inventory system with a fixed lead time.
Some authors studied a two-stage inventory system, whereas the retailers face stationary and independent Poisson demand (Axssater and Zhang 1999; Nozick and Turnquist 2001). Mitra and Chatterjee (2004) developed De Bodt and Graves’ model (1985) as ‘Continuous-review policies for a multi-echelon inventory problem with stochastic demand’. Recently, Ben-Daya and Al-Nassar (2008) developed Lu’s (1995) idea to make shipments from a production lot as it is being produced to the three stage multi-customers supply chain applying the integer multiplier of the cycle time mechanism. Thereafter, Ben-Daya et al. (2010) have developed Ben-Daya and al-Nassar’s (2008) model in three-echelon SCs. Their model consists of raw material ordering cost for both supplier and manufacturer in a network including a supplier, a manufacturer and multi-retailers. In addition, in the context of perfect and imperfect quality items, Sana (2011) has presented an integrated production-inventory model in a three-layer SC having supplier, manufacturer and retailer.

Concluding from the literature review, no attention has been dedicated to different inventory control policies in a multi-echelon multi-item supply chain. Therefore, the present study attempts to model diverse inventory policies in a multi-item bi-echelon SC to imply the task of inventory planning for the first time. The proposed supply chain consists of multi suppliers and a central manufacturing firm which is offered N-type product to the customers arrived to the network. The aim of the paper is to increase supply chain’s flexibility and agility in such away to provide economic order/product during optimum date to satisfy customers’ demands. In this regard, manufacturing firm’s demand is supplied through different suppliers in the SC. In order to determine each supplier’s agility in providing economic lot size, a fuzzy-ANP model is implemented based upon the agility indices. Having ANP model used to estimate supplier power, we achieve a more responsible, flexible, and more agile network for the customers’ demands. In the next section, the supposed problem is elaborated.

**PROPOSED BI-ECHELON MULTI-PRODUCT SUPPLY CHAIN**

In this paper, it is attempted to propose a bi-echelon supply chain having a central manufacturing plant with warehouse space limitation and many suppliers in which N product types are presented to the customers. In the supply chain, manufacturing plant is fed from upper echelon (suppliers) and feed lower echelon (customers), as illustrated in Figure 1. As, the entire customers’ demand is forwarded to the firm. Next, is delivered in economic batch size assuring agility in terms of flexibility and performance. Since, raw materials of diverse productions made at the manufacturing plant are supplied through different suppliers (i.e. the order quantity for raw materials at manufacturing plant split to smaller lot sizes). Next, they are forwarded to the suitable suppliers. A fuzzy-ANP approach is devoted to evaluate and rank suppliers in supplying raw materials. It must be noted, requirements of the proposed suppliers in the SC are provided through the external suppliers without any restriction.

**Inventory planning at the manufacturing plant level**

The inventory control system corresponding to the central firm is managed applying EOQ and EPQ policies. The deterministic EPQ with full-backordering is dedicated to analyze the relevant inventory system of the final products which are produced in the proposed company. And, the deterministic EOQ with full-backordering is devoted for the control of the inventory system of raw materials that are consumed in processing the final productions. The supposed models are integrated so that the total network costs are minimized. Whereas, some operational cost factors such inventory holding costs, backordering cost, ordering costs are introduced to assess supply chain flexibility. The situation by which the suppliers and the manufacturing firm interact with each other is defined as following; the manufacturer produces all of the demanded products with known and constant rate, the customers demand of each product is known with a constant rate, lead times is zero, the suppliers send the orders to the manufacturing plant in equal lot sizes, the warehouse space of manufacturing plant for raw materials is limited, the shortage, ordering and holding costs are known.

The developed EOQ and EPQ models are dedicated to determine how much and when product should be placed as an order and a production at the manufacturing plant. In this regard, the notations are used in this study are as following:

- \( i \)  Index for final product (\( i = 1, \ldots, n \));
- \( i' \)  Index for raw material (\( i' = 1, \ldots, n' \));
- \( j \)  Index for supplier nodes (\( j = 1, \ldots, m \));
- \( D_i \)  demand per year for final product \( i \) at central manufacturing plant;
- \( P_i \)  production rate per year for final product \( i \) at central manufacturing plant;
- \( D_{i'} \)  demand per year for raw material \( i' \) at manufacturing plant;
- \( C_i \)  the fixed cost of receiving an order for final product \( i \) at manufacturing plant;
C_{ij}^{f}$: the fixed cost of placing an order for raw material $i$ at manufacturing plant towards supplier $j$;

$C_{ij}^{p}$: the cost to hold a unit of final product $i$ at manufacturing plant in inventory for a cycle;

$C_{ij}^{b}$: the cost to hold a unit of raw material $i$ at manufacturing plant in inventory for a cycle;

$C_{ij}^{c}$: the cost to keep a unit backordered of final product $i$ at manufacturing plant for a cycle;

$C_{ij}^{d}$: the cost to keep a unit backordered of raw material $i$ at manufacturing plant for a cycle;

$cc_{ij}$: the consumption coefficient of product $i$ from raw material $i'$ at manufacturing plant;

$Q_i$: the order quantity for raw material $i$ at manufacturing plant;

$T_i$: the length of order cycle of raw material $i$ at manufacturing plant;

$F_{ij}$: the fill rate for raw material $i$ at manufacturer from supplier $j$ that will be filled from stock;

$L_i$: the maximum inventory level of final product $i$ at manufacturing plant;

$I_i$: the maximum inventory level of final product $i$ at manufacturing plant;

$B_i$: the maximum backorder position of final product $i$ at manufacturing plant;

$B_i'$: the maximum backorder position of raw material $i$ at manufacturing plant;

$T_{C1}$: total related cost with EPQ per year at manufacturing plant;

$T_{C2}$: total related cost with EOQ per year at manufacturing plant;

$TC$: total cost per year for the whole of the supply chain ($T_{C1} + T_{C2}$).

### The Developed EPQ Model

In order to examine the manufacturing firm’s inventory system, a deterministic EPQ with pure backordering is developed. The EPQ model considers a central firm with a constant production rate $P_i$, and demand rate $D_i$, where $P_i > D_i$. Having final product inventory control system modeled, the length of planning cycle time and the fraction of demand to be filled from stock is obtained so that the total inventory costs including production, holding, ordering, and backordering costs are minimized. As depicted in Figure 2, the firm produces $Q_i$ of product $i$ during $T_i$.

As shown in Figure 2, planning cycle of product $i$ at manufacturing firm, which has length $T_i$, could be divided into four sub-interval having the shortage time, the production/demand time, the demand time. Where, $t_i^1 = (1 - F_i)T_i (D_i / P_i)$, $t_i^2 = F_i T_i (1 - (D_i / P_i))$, $t_i^3 = (1 - F_i)T_i (1 - (D_i / P_i))$. Hence, based upon these values, we develop the expression for $I_i$, the maximum inventory level of final product $i$ at manufacturing firm with $I_i = (P_i - D_i) t_i^1 = F_i T_i (1 - (D_i / P_i))$, and $B_i$, the maximum backorder position of product $i$ at manufacturing plant with $B_i = D_i t_i^1 = D_i (1 - F_i) T_i (1 - (D_i / P_i))$. Hence, the production quantity of each product is calculable as (1).

$$Q_i = I_i + B_i = F_i T_i D_i \left(1 - \frac{D_i}{P_i}\right) + \left(1 - F_i\right) T_i D_i \left(1 - \frac{D_i}{P_i}\right).$$

Supposing the length of planning cycle time of each product at manufacturing firm is multiple of each other. Therefore, the total related cost with EPQ per year for the firm yield applying (2) while the constraints are satisfied:

$$\text{Min} \quad T_{C1}(T_i, K_i, F_i) = \sum \left(C_{ij}^{f} / T_i\right) + \sum \left(C_{ij}^{p} + C_{ij}^{b} + C_{ij}^{d} / 2\right) \left(1 - \frac{D_i}{P_i}\right)$$

$$s.t.: \quad K_i T_i = K_{ij} T_{ji} \quad i = 1, 2, ..., n - 1, \quad l = 1, 2, ..., n - i. \quad (3)$$

### The Developed EOQ Model

In this section, a deterministic EOQ with complete backorder is modeled to manage the inventory system of raw materials which are consumed in the production process of the final products.

The extended EOQ model considers a central manufacturing firm with a constant demand rate $D_i$. The order quantity of any raw material $i$ is equal. Although, the order quantity of each raw material placed by the firm might be vary from another one. Therefore, this section is developed to seek economic order quantity of each raw material due to reduce the costs corresponding to inventory...
system to satisfy customers’ demand. In this regard, supposing holding costs, ordering costs and backordering costs of raw materials as inventory system costs.

As depicted in Figure 3, order cycle of raw materials at manufacturing firm from suppliers comprises of \( t^{ii}_{ij} \) which is demand time with \( t^{ii}_{ij} = F_{ij} q_{ij} T_i \) and \( t^{ii}_{ij} \) which is shortage time with \( t^{ii}_{ij} = (1 - F_{ij}) q_{ij} T_i \). Therefore, based upon these values, \( I_{ij} \) is the maximum inventory level of raw material \( i \) from supplier \( j \) with \( I_{ij} = D_i t^i_{ij} \); \( t^{ii}_{ij} = D_i F_{ij} q_{ij} T_i \); \( S_{ij} \) is the maximum stockout quantity of raw material \( i \) from supplier \( j \) with \( S_{ij} = D_i t^{ii}_{ij} = D_i (1 - F_{ij}) q_{ij} T_i \); \( B_{ij} \) is the maximum backorder position of raw material \( i \) from supplier with \( B_{ij} = B_{ij} S_{ij} = D_i (1 - F_{ij}) q_{ij} T_i \). The EOQ model with complete backordering is a particular case of EOQ model with partial backordering in which the backordering rate that will be backordered \((B_{ij})\) equals one. Hence, with respect to these expressions, order quantity of raw material \( i \) toward supplier \( j \) is calculated applying (4). Where; \( Q_{ij} = q_{ij} Q_i \), \( I_{ij} = q_{ij} I_i \), \( B_{ij} = q_{ij} B_i \). Based on equation (5), the relevant average cost of firm’s raw material inventory per year is obtained. In the present model, constraint (6) corresponds to the warehouse space limitation.

\[
Q_{ij} = I_{ij} + B_{ij} = F_{ij} q_{ij} T_i D_i + (1 - F_{ij}) q_{ij} T_i D_i.
\]

\[
\min TC(f, K, F, T, F, \beta) = \sum_i \left( C_i \sum_j \left( C_{ij} F_{ij} \right) \right) + \sum_i \left( C_i D_i T_i \beta^2 \right) \left( I - \left( D_i / D_i \right) \right) + \sum_i \left( C_i D_i T_i (1 - F_{ij}) \beta^2 \right) \left( I - \left( D_i / D_i \right) \right)
\]

\[
\sum_i \left( C_i D_i q_{ij} T_i (1 - F_{ij}) \beta^2 \right) + \sum_i \left( C_i D_i q_{ij} T_i \beta^2 \right)
\]

\[
\sum_i \left( C_i D_i q_{ij} T_i \beta^2 \right) + \left( D_i / D_i \right) \left( I - \left( D_i / D_i \right) \right)
\]

\[
\sum_i \left( C_i D_i q_{ij} T_i \beta^2 \right) + \left( D_i / D_i \right) \left( I - \left( D_i / D_i \right) \right)
\]

\[
K_i T_i = K_i \max_{l=1,2,...,n-i} \left( \left( D_i / D_i \right) \right), \quad I = 1,2,...,n - 1, \quad \beta = \sum_{c=0} \left( c_i D_i c_i \right); \quad i = 1,2,...,n'.
\]

\[
T_c = \max \left( C_i T_i \right) \left( 1 - \left( D_i / D_i \right) \right), \quad i = 1,2,...,n', \quad i' = 1,2,...,n'.
\]

\[
B_i = \max \left( \left( B_i / c_i \right) \right) \left( 1 - \left( D_i / D_i \right) \right), \quad i = 1,2,...,n.
\]

**Problem formulation**

As we mentioned earlier, the goal is to determine the value of the length of planning cycle times, the multipliers of them and the fractions of demand to be filled from stock of final products; \( T_i, K_i, F_i \), the length of order cycle times and the fractions of demand to be filled from stock of raw material at manufacturing plant; \( T_{ij}, F_{ij} \), so that the total average cost of supply chain per year given in (7) is minimized with respect to constraints satisfaction. Whereas, the constraints are as; the warehouse space of raw materials is limited, the length of order cycle of raw materials is multiple of each other, demand rate for raw materials is multiple of the demand rate for final product \( i \), shortage of final product \( i \) is multiple of the shortage of raw materials which are consumed in producing the product. Hence, the problem is formulated as:

\[
\min TC(f, K, F, T, F, \beta) = \sum_i \left( C_i q_{ij} T_i (1 - F_{ij}) \beta^2 \right) + \sum_i \left( C_i D_i q_{ij} T_i \beta^2 \right)
\]

\[
\sum_i \left( C_i D_i q_{ij} T_i \beta^2 \right) + \left( D_i / D_i \right) \left( I - \left( D_i / D_i \right) \right)
\]

\[
\sum_i \left( C_i D_i q_{ij} T_i \beta^2 \right) + \left( D_i / D_i \right) \left( I - \left( D_i / D_i \right) \right)
\]

\[
K_i T_i = K_i \max_{l=1,2,...,n-i} \left( \left( D_i / D_i \right) \right), \quad i = 1,2,...,n - 1, \quad \beta = \sum_{c=0} \left( c_i D_i c_i \right); \quad i = 1,2,...,n'.
\]

\[
T_c = \max \left( C_i T_i \right) \left( 1 - \left( D_i / D_i \right) \right), \quad i = 1,2,...,n', \quad i' = 1,2,...,n'.
\]

\[
B_i = \max \left( \left( B_i / c_i \right) \right) \left( 1 - \left( D_i / D_i \right) \right), \quad i = 1,2,...,n.
\]

**ANF as a Multi-Attribute Decision-Making (MADM) approach has been recently taken into account in numerous fields. The first research towards ANF as a general form of another MADM method, Analytic Hierarchical Process (AHP) (Saaty 1980), dates back to the one by Saaty (1996). In contrary with the AHP in which independency between elements in every level of the associated hierarchy is an axiom, the interrelationship between decision factors as well as alternatives or between them in a more complex manner could be modeled in ANP method. Moreover to cope with the uncertainty and ambiguity of the preference-based comparisons of decision-makers, fuzzy sets theory (Zadeh 1965) is augmented to the ANP. For a comprehensive description about the fuzzy-ANP which is used in the supposed problem, readers are referred to Rafiei and Rabbani (2011).
Proposed model

In the suggested central firm, raw materials which are consumed in the final products are provided from the different suppliers so that the network agility is obtained. The network agility is investigated upon ANP in which the agility indices are adopted from the studies in the context of agility (Gunasekaran 1998; sharifi and zhang 1999; Yusuf et al. 1999; Christian et al. 2001; Tsourveloudis and Valavanis 2002; Lin et al. 2004). Having agility indices applied to develop a fuzzy-ANP framework, suppliers in the SC are evaluated with respect to their agility in providing economic lot size for the firm. In this regard, \( q_{i,j} \) as the percentage of order quantity of raw material \( i \) at manufacturing plant which is provided through supplier \( j \) is estimated applying the supposed ANP model. Whereas, it is larger-equal than 0 and less-equal than 1 (i.e. 0 ≤ \( q_{i,j} \) ≤ 1), and \( \sum_{j} q_{i,j} = 1 \) (i = 1, ..., \( n \)). Supposing \( Q_i \) as the order quantity for raw material \( i \) at manufacturing plant. Therefore, the value of order quantity of raw material \( i \) at manufacturing firm from supplier \( j \) could be found as \( Q_{i,j} = q_{i,j} Q_i \). In order to attain a more flexible and responsible network for the customers, \( q_{i,j} \) is estimated applying agility index in the fuzzy-ANP model. As demonstrated in Figure 4, the ANP model is formed upon the clusters, decision criteria and their relationships (i.e. inner dependency or loop, outer dependency or cycle). The dependency relations in the proposed network are described in detail as following:

Manufacturing speed affects on product design flexibility, product volume flexibility, customer satisfaction, inventory turnover, employee skills utilization; product design flexibility affects on product flexibility, customer satisfaction, inventory turnover; appropriate/new technology, IT skills and knowledge affect on product/service quality, customer satisfaction, inventory turnover, new product introduction responsiveness, product volume flexibility; learning organization affects on employee skills utilization, continuous training and development; multi-skilled and flexible people affects on employee skills utilization, product volume flexibility, product/service quality.

EXPERIMENTAL RESULT

This section is provided to illustrate the validation of the developed model. To do so, we propose a numerical sample of a bi-echelon SC having two suppliers, and one manufacturing plant in which two product types are suggested to the customers. Whereas, two types of raw materials are consumed in producing the final products at the proposed firm. To perform the test problem, a twofold structure is developed. Firstly, the proposed ANP framework is implemented to estimate the value of order coefficients of each raw material from each supplier, leading to have a more agile and efficient network.

After implementing the ANP model, the resulted weights have converted to values with summation one applying some mathematical estimation. While, the values of \( q_{i,j} \) have obtained as: 0.4, 0.6 for raw material 1 from supplier 1, and 2 and 0.5, and 0.5 for raw material 2 from supplier 1 and 2, respectively. Next, the supposed non-linear model is solved while the relevant data of the final products and raw materials at the central manufacturing firm are randomly generated, as depicted in Table 1 and 2, respectively. The results of the supposed numerical sample are reported in Table 3. It must be noted, the proposed problem is coded by LINGO11 software and run on a PC with 2 Ghz processor.

CONCLUSION AND FUTURE RESEARCH DIRECTION

In this paper, a bi-echelon supply chain including multi suppliers and a central firm with storage space limitation which produces N-type product has been presented. The corresponding inventory system to the firm was managed upon the deterministic EOQ and EPQ policies. While, suppliers agility...
in supplying firm's demand have been evaluated applying a fuzzy-ANP model. Moreover, a cost structure which captures some different operational cost factors has been defined to measure total costs of the deterministic proposed model. Finally, the extended non-linear model has been optimally solved applying LINGO software.

For future research, some suggestions are provided. First, other methodology solutions such as exact and heuristic algorithm could be developed to achieve compact mathematical equations for decision variables. Further, other assumptions could be considered to make the problem more practical; inventory discount models and uncertain conditions. In addition, other decision criteria could be defined in agile environments to make the ANP model more practical.

REFERENCES