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GENERALIZED MODEL OF THE PRESSURE EVOLUTION WITHIN THE BRAKING SYSTEM OF A VEHICLE

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ABSTRACT: There are lots of methods to issue mathematical models that can cover better or worse the real life phenomenon. Most of the mathematical models are using parameters that are rather estimated than real. Hence, a higher or a lower degree of accuracy can be expected. Since most of the input variables are estimated, low accuracy level is usually achieved. In this respect, the authors of this paper tried to offer an alternative model of a real-life mechanical amount simulation. This model starts over a larger amount of tests, where one could get a database, large enough to backup the further behavior of the system. In this respect, we have measured the pressure variation of the brake cylinders of a vehicle, as a function of the input pressure developed by the servomechanism of the system. Based on this large amount of data, we used a parametric mathematical model based on polynomial methods to estimate the generalized model. We can strongly confirm that the model is suitable - within a reasonable error margin - for all the braking systems of that specific kind of vehicles.
KEYWORDS: braking system, parametric models, data-based models

INTRODUCTION

We've used a Romanian-made reconnaissance armored personnel carrier to perform the required tests (figure 1). These tests were part of a bigger program, aiming at updating the braking system of the vehicle.

The braking system of the vehicle is a hydraulic one, assisted by an air-compressed section. So, the brake cylinders are acted by brake fluid. At its turn, the liquid is sent into the cylinders by a brake pump, pressed by the foot's force and helped by compressed air (fig. 2). The system is quite simple and already well known; hence no further details are needed.

Figure 1 - TABC-79, 4x4 Reconnaissance APC

Also, the braking mechanism at each wheel's level is a classic one, consisting of a pair of brake shoes, bolted on a brake plate, acting inside a brake drum and using a liquid-acted cylinder as a power actuator.

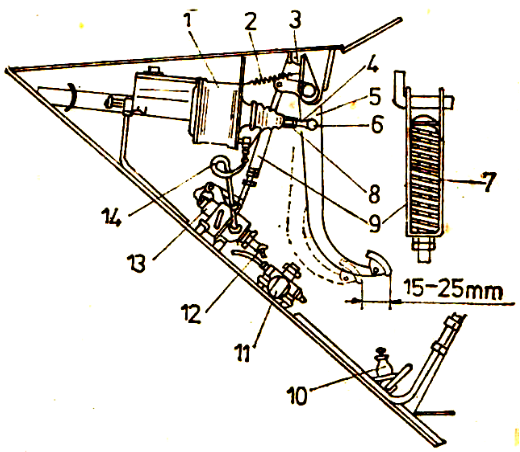

Figure 2 - Structure of the braking system's control (cross-section)

The working principle of the servomechanism is quite simple (figure 2). When pressing the brake pedal 5 its pushing rod acts on the piston of the master brake cylinder (a twin one, serving a dual circuit).

The master twin cylinder sends the fluid into the brake cylinders that, at their turn, act the brake shoes. When pressing the brake pedal, the pedal's lever pushes the connecting rod 9, which acts on a double chamber, unbalanced, compressed air distributor 13 (that mainly works as a "faucet" or a "tap", opening and closing the way for the compressed air and also regulating the pressure).

The regulated, compressed air is then sent through pipe 14 into the pneumatic chamber 1 (or item 8 in figure 3) that assists the master hydraulic pump (item 1, figure. 3), supplementing the force acting on its cylinder and helping the driver with the braking effort.

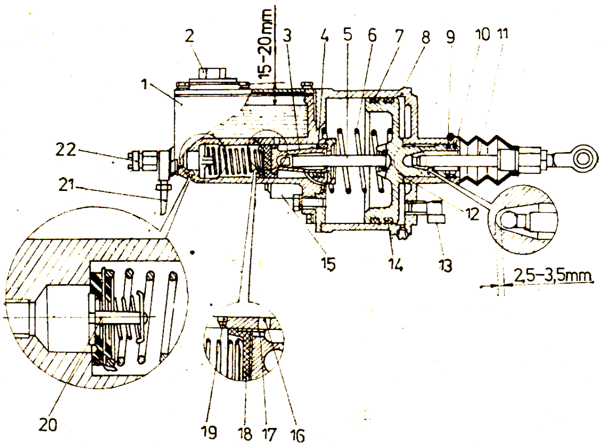

Figure 3 - Hydro-pneumatic master cylinder (assisted brake pump)

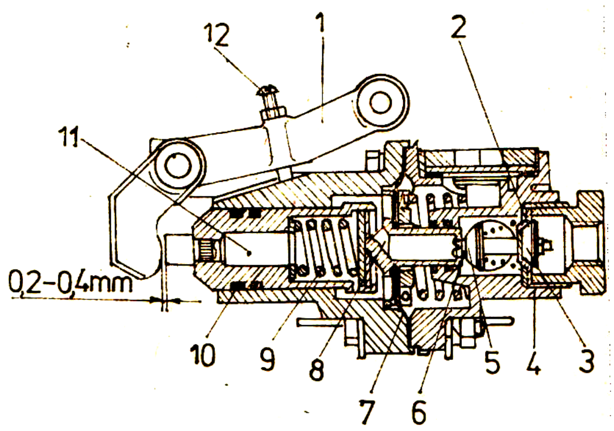

Figure 4 - Compressed air distributor

MEASURED SIGNALS

As we mentioned, the testing program was rather complex. But as far as this paper is concerned, the following amounts were needed:

- ☐ *the input air pressure on the brake distributor;*
- ☐ *the output air pressure on the brake distributor;*
- ☐ *the input liquid pressure on the brake cylinders of both wheels of the real axle (the braking pressure).*

THE PRINCIPLE OF THE MODEL

We are now looking for a mathematical model based on the measured signals that describes the braking pressure evolution at the wheel's level as a function of the input air pressure on the brake distributor. As can be seen, the process involves two stages.

First, determining a transfer function for the brake distributor and getting a mathematical model to describe the time history of the output air pressure as a function of the input air pressure.

Second, determining a transfer function for the rest of the braking system and getting a mathematical model to describe the time history of the liquid braking pressure as a function of the distributor's output pressure. It is quite easy to notice that the transfer function of the liquid section can be a first order one (within reasonable margins of error), since the liquid can be considered uncompressible. On the other hand, for the pneumatic section, previous analysis gave us a third order transfer function to describe the phenomena. Nevertheless, it is quite useless to have a too high degree of accuracy while pushing too much the computational means; hence, a second order transfer function is just perfect for our needs.

PARAMETRIC MODELS. IDENTIFICATION PROCEDURE

In the most cases, when describing a dynamic process, parametric models are used having vectors as arguments. If the vector is θ then its model will be known as $M(\theta)$. This approach suggests that, when the vector θ takes a set of possible values, a set of models is obtained and its structure will be M .

Therefore, if the mathematical model of the process is "parameterized " by the vector θ, the problem of the identification resides in determining or assessing the model's parameters on the basis of the experimental data of the input and output variables of the system. Usually, the procedure uses only half of the whole amount of data, the other half being used to confirm the elaborated model. The mostly used checking method is the "predicted value method ". The minimized objective function is given by $f = \arg\sum_{t=1}^{N} e^2(t)$, where $e(t)$ is the error.

*There are a lot of parametric models. Considering the characteristic features of our mechanical system and taking into account the above mentioned reasons, we decided to use a SISO (**S**ingle **I**nput **S**ingle **O**utput) model; its general form is given by:*

$$A(q)y(t) = \frac{B(q)}{F(q)}x(t-nk) + \frac{C(q)}{D(q)}e(t) \qquad (1)$$

where $y(t)$ is the system's output, $x(t)$ is the system's input, $e(t)$ is the noise (that can be interpreted as an error) and t is the independent variable (actually the time, usually given in discrete domain). Eventually, nk is the number of the delaying elements along the system's input-output chain.

The polynomials featuring the SISO model are given by:

$$\begin{cases} A(q) = 1 + a_1 q^{-1} + a_2 q^{-2} + \ldots + a_{na} q^{-na} \\ B(q) = b_1 + b_2 q^{-1} + b_3 q^{-2} + \ldots + b_{nb} q^{-nb+1} \\ C(q) = 1 + c_1 q^{-1} + c_2 q^{-2} + \ldots + c_{nc} q^{-nc} \\ D(q) = 1 + d_1 q^{-1} + d_2 q^{-2} + \ldots + d_{nd} q^{-nd} \\ F(q) = 1 + f_1 q^{-1} + f_2 q^{-2} + \ldots + f_{nf} q^{-nf} \end{cases} \tag{2}$$

where q is known as the delay operator and is given like: $q^{-i}x(t) = x(t-i)$, while na, nb, nc, nd and nf are the polynomials' orders.

SPECIFIC FEATURES OF THE USED MODEL

The mathematical model in (1) is the general one. It can be customized, according to the operator's needs. As we've already mentioned, the demands for a model should take into account not a too high level of accuracy, an error level lower than 3% being more than enough for our needs. In this respect, the particular ARX (Auto-Regressive with eXogene inputs) model was used.

This particular model is featured by the following conditions:

$$\begin{cases} nc = nd = nf = 0 \\ C(q) = D(q) = F(q) = 1 \end{cases} \tag{3}$$

that turns equation (1) into the particular one:

$$A(q)y(t) = B(q)x(t-nk) + e(t) \tag{4}$$

We consider necessary to mention that we used a lot of other models. Their accuracy had usually been higher than the one's we mentioned but the computing resources had been too high to keep them as suitable.

Considering the previously mentioned issues, we could use two different models (a first order model for the hydraulic section and a second order one for the pneumatic section) then combine them. As we actually did first time, and the results can be seen in the next sections of the paper.

We noticed that is more complicated to act that way since the signals we obtained and used were very smooth, with no needs for intense filtration. Moreover, the hydraulic section is acting rather "predictable" with no problems in its evolution. So we made the decision to issue a single model, a second order one, delivering the time history of the hydraulic pressure of the brake cylinder as a function of the input air pressure on the air distributor.

ACHIEVED DATA. MODELS

Figure 4 depicts a sample of the measured data. Hundreds of tests were developed and the data were stored and preprocessed (that means sorting, filtering, smoothening and discharging the unsuitable ones). As can be seen, we measured the force on the brake pedal, but it is unusable from the modeling point of view, due to the fact that the driver can press completely random the pedal from test to test. It was however useful to have this signal, since it provides the starting and ending points of the braking process.

Figures 5 and 6 provide two samples of the partial mathematical models based on the SISO structure and using ARX algorithm. For one (but the same test), figure 5 gives the time history of the distributor output pressure as a function of its input pressure. Figure 6 depicts in the same time and for the same test the time history of the pressure within the left wheel's brake cylinder as a function of the distributor's output pressure.

The mathematical model of the time history of the distributor's output pressure as a function of its input pressure (depicted in fig. 5) is given by:

$$\frac{d^2 y}{dt^2} + 230{,}5\frac{dy}{dt} + 6{,}768 \cdot 10^4 y = 95{,}53\frac{dx}{dt} + 8{,}163 \cdot 10^4 x \tag{5}$$

In the equation above $y(t)$ is the distributor's output pressure and $x(t)$ is the distributor's input pressure.

In the same time, the mathematical model of the time history of the brake cylinder's pressure as a function of the distributor's output pressure (depicted in fig. 6) is given by:

$$\frac{dy}{dt} + 32{,}43y = 406{,}8x \tag{6}$$

In the equation above $y(t)$ is the brake cylinder's pressure and $x(t)$ is the distributor's output pressure. Should be mentioned that different models have been issued for left and right brake cylinders.

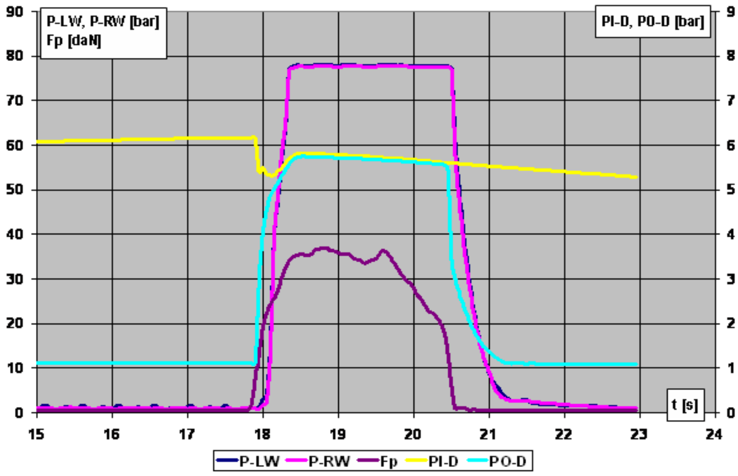

Figure 4 - Test sample. Measured parameters: P-LW: pressure on the left wheel's brake cylinder; P-RW: pressure on the right wheel's brake cylinder; Fp: force on the brake pedal; PI-D: input pressure on the distributor; PO-D: output pressure on the distributor

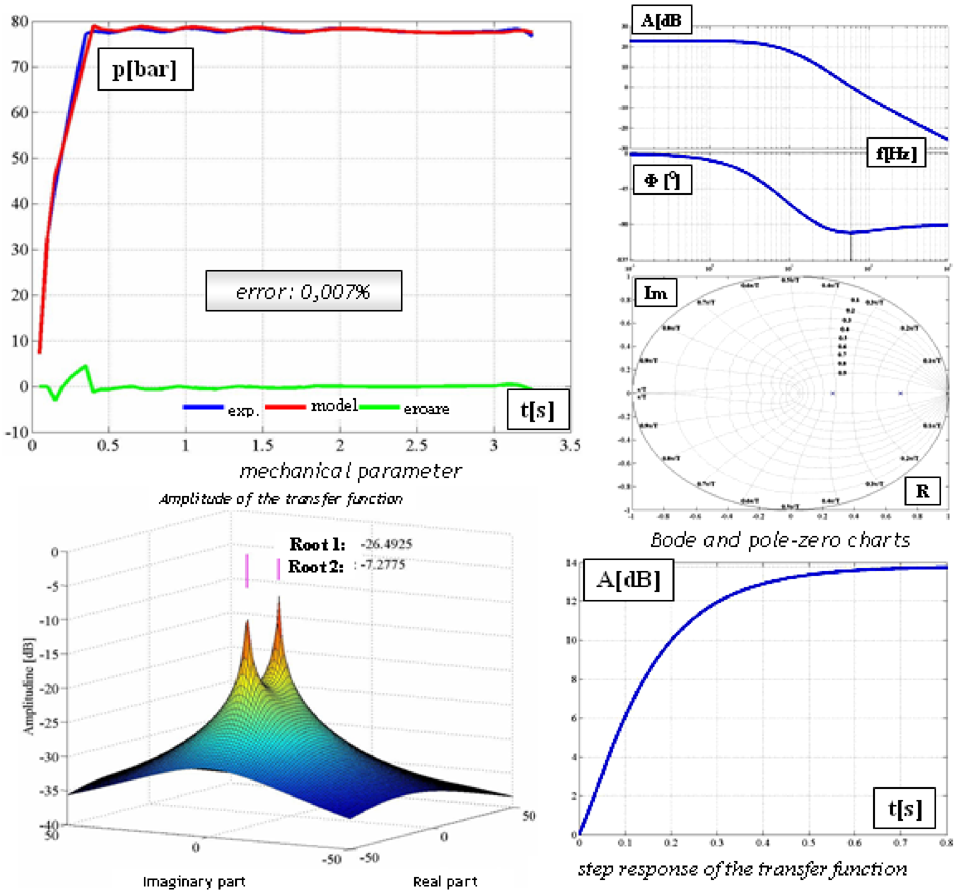

Figure 5 - Characteristic features of the mathematical model, and the time history of the distributor's output pressure as a function of the distributor's input pressure - ARX algorithm

These two models can be lately combined and get a final one. At this stage of our research, we found to be useful to having a global model of the whole braking system instead of modeling every of its sections apart. But it's much more difficult to compose these two models and get a final one than to create from the very beginning a global model. And we still have to keep in mind that this kind of work should be developed for every single test then average the results to get the generalized model.

Instead of acting that way, we chose to use a global model and find (of course, for every single test) the time history of the brake cylinder's pressure as a function of the distributor's input pressure. The result, for one of the tests, is depicted in figure 7. As can be noticed, a second order transfer function was used. For the chosen test, the mathematical model is given by:

$$\frac{d^2y}{dt^2}+33{,}770\frac{dy}{dt}+192{,}280y=50{,}300\frac{dx}{dt}+2660x \qquad (7)$$

In the equation above $y(t)$ is the brake cylinder's pressure and $x(t)$ is the distributor's input pressure. Should also be mentioned that different models have been issued for left and right brake cylinders. As a result, every test of the whole set of tests had two mathematical models of this kind: one for the right wheel's slave cylinder pressure and another for the right wheel's cylinder.

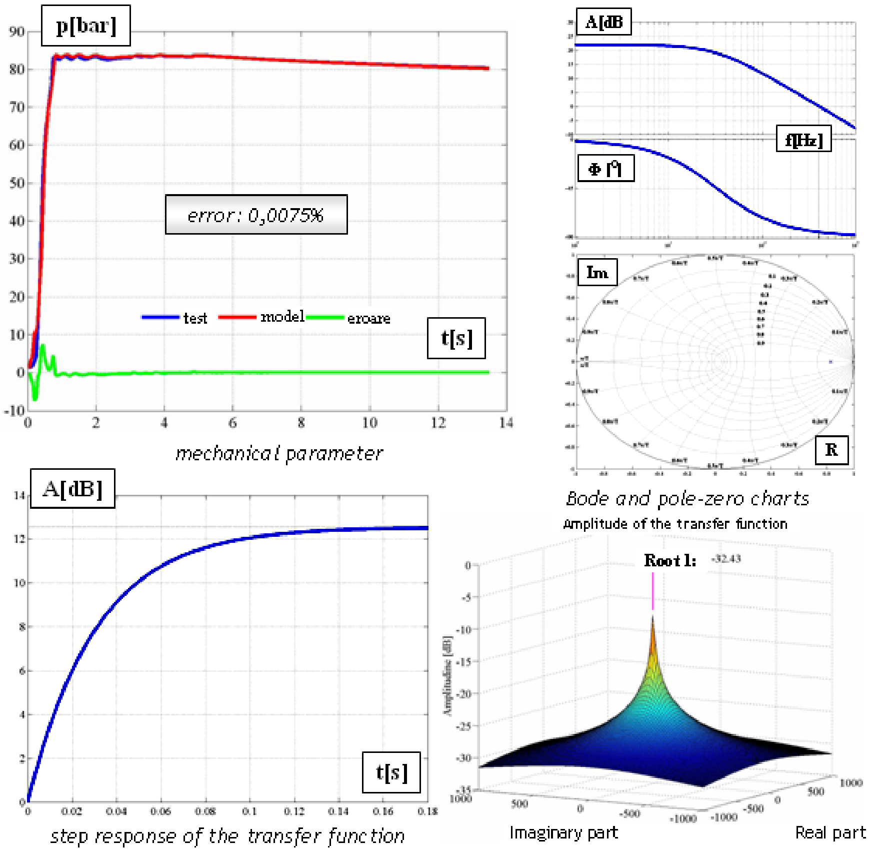

Figure 6 - Characteristic features of the mathematical model, and the time history of the brake cylinder's pressure as a function of the distributor's output pressure - ARX algorithm

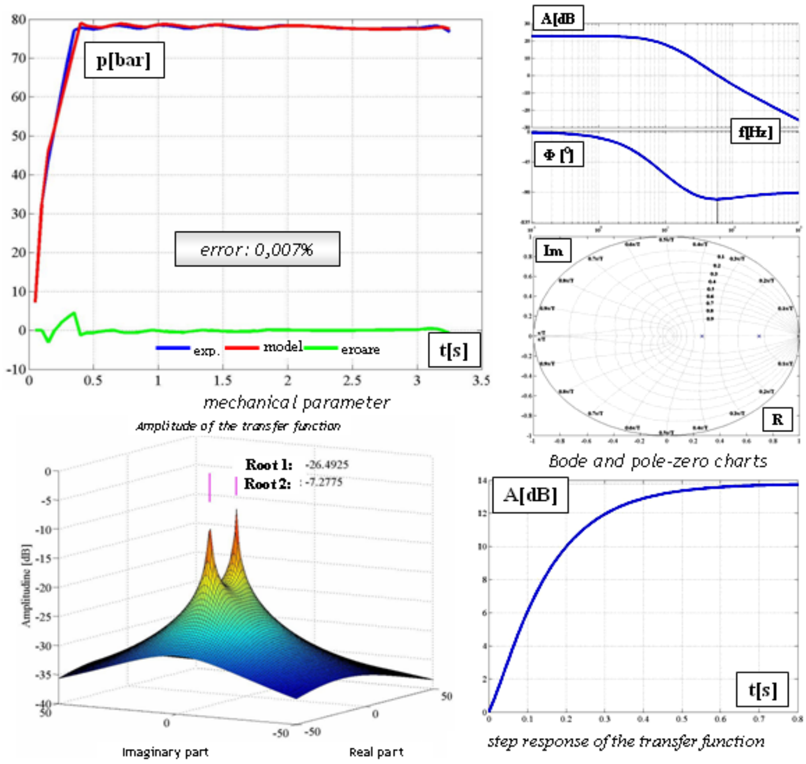

Figure 7 - Characteristic features of the mathematical model, and the time history of the brake cylinder's pressure as a function of the distributor's input pressure (global model) - ARX algorithm

To obtain the global model, each particular model's polynomial coefficients had been set in a table according to their rank. Since the differences between the pressure value and their evolutions in each left and right cylinders weren't significant, we considered that they could be put together in the same table (a sample is given below). Eventually, the values were averaged and they were used to write the final, global, generalized model (8).

Table 1. Averaging the polynomial coefficients (sample)

Test	Brake cylinder	$\frac{d^2y}{dt^2}$	$\frac{dy}{dt}$	y	$\frac{dx}{dt}$	x	Normalized modeling error [%]
....	......	....	....	....	....	....	....
P1_2r_pam	left	1,000	19,640	127,000	37,830	1760	0,001
	right	1,000	52,470	376,500	83,630	5216	0,002
P3_2r_pas	left	1,000	23,970	163,300	46,740	2253	0,011
	right	1,000	28,900	185,200	50,510	2546	0,020
....	......	....	....	....	....	....	....
Average		1,000	30,970	206,483	50,851	2850	0,013

$$\frac{d^2y}{dt^2}+30{,}970\frac{dy}{dt}+206{,}483y=50{,}851\frac{dx}{dt}+2850x \tag{8}$$

To prove the accuracy of the generalized, global model we drew the red curve superimposed over the whole set of tests' curves, as can be seen in figure 8. The average absolute error is lower than the previously considered 3% limit.

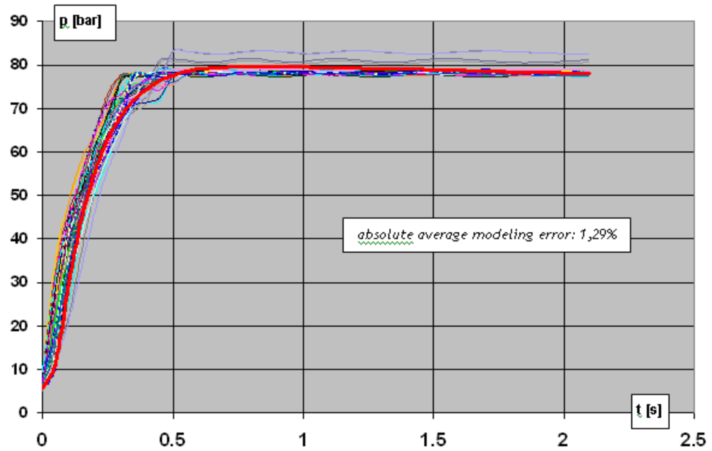

Figure 8 - Generalized, global model of the time history of the brake cylinder's pressure as a function of the distributor's input pressure

CONCLUSIONS

Starting from the test data sets, excellent mathematical models can be obtained when using powerful modeling tools. If only the behavior of some system is need and no further analysis is needed, than this can be the best, the most accurate and the fastest method. The obtained model can be easily used for larger lots of similar products. As a matter of fact, we have proved that the model given by (8) is suitable for another vehicle of the same type and, after confirming it, the error was also less than 3%.

This method can be also used in diagnosing a system, not necessarily the braking system. After averaging the values of a properly working system of several vehicles, for instance, the generalized model can be compared to a malfunctioning one. Our research took further steps and we can even determine "what goes wrong" in a malfunctioning braking system. Of course, we can't find any type of faulty part, but at least we can refine the search at the subsystems' level.

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