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HEAT AND MASS TRANSFER EFFECTS ON MHD BOUNDARY LAYER FLOW OVER A MOVING VERTICAL POROUS PLATE

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ABSTRACT: We study a two-dimensional free convection effects on the steady incompressible laminar MHD heat and mass transfer characteristics of a linearly started porous vertical plate, the velocity of the fluid far away from the plate surface is assumed zero for a quiescent state fluid. The variations of surface temperature and concentration are linear. All the fluid properties are assumed to be constant except for the density variations in the buoyancy force term of the linear momentum equation. The magnetic Reynolds number is assumed to be small, so that the induced magnetic field is neglected. No electrical field is assumed to exist and both viscous and magnetic dissipations are neglected. The Hall effects, the viscous dissipation and the joule heating terms are also neglected. The governing equations are solved numerically by using shooting method. Dimensionless velocity, temperature and concentration profiles are displayed graphically for different values of suction parameter (fw), magnetic parameter (M), permeability parameter (K), local temperature Grashof number (Gr), local concentration Grashof number (Gc), Prandtl number (Pr) and Schmidt number (Sc). The values of skin-friction, Nusselt number and Sherwood number for different physical parameters are also presented through tables.
KEYWORDS: Porous medium, MHD, Heat transfer and Mass transfer, Vertical plate

INTRODUCTION

Magnetohydrodynamic (MHD) flows have applications in meteorology, solar physics, cosmic fluid dynamics, astrophysics, geophysics and in the motion of earth's core. In addition from the technological point of view, MHD free convection flows have significant applications in the field of stellar and planetary magnetospheres, aeronautical plasma flows, chemical engineering and electronics. An excellent summary of applications is given by Huges and Young (1996). Raptis (1986) studied mathematically the case of time varying two dimensional natural convective flow of an incompressible, electrically conducting fluid along an infinite vertical porous plate embedded in a porous medium. Helmy (1998) analyzed MHD unsteady free convection flow past a vertical plate embedded in a porous medium. Elabashbeshy (1997) studied heat and mass transfer along a vertical plate in the presence of magnetic field. Chamkha and Khaled (2001) investigated the problem of coupled heat and mass transfer by magnetohydrodynamic free convection from an inclined plate in the presence of internal heat generation or absorption. Transport processes through porous media play important roles in diverse applications, such as in geothermal operations, petroleum industries, thermal insulation, design of solid-matrix heat exchangers, chemical catalytic reactors, and many others. Bejan and Khair (1985) reported on the natural convection boundary layer flow in a saturated porous medium with combined heat and mass transfer. Vafai and Tien (1981) have discussed the importance of inertia effects for flows in porous media. Makinde (2009) considered the MHD boundary-layer flow and mass transfer past a vertical plate in a porous medium with constant heat flux. Raptis et al. (1981) constructed similarity solutions for boundary layer near a vertical surface in a porous medium with constant temperature and concentration. Many transport processes exist in nature and in industrial applications in which the simultaneous heat and mass transfer occur as a result of combined buoyancy effects of thermal diffusion and diffusion of chemical species. A few representative fields of interest in which combined heat and mass transfer plays an important role are designing of chemical processing equipment, formation and dispersion of fog, distribution of temperature and moisture over a agricultural fields and groves of fruit trees, crop damage due to freezing, and environmental pollution. In this context, Soundalgekar (1979) studied the effects of mass transfer and free convection on the flow past an impulsively started vertical flat plate. Erickson et al. (1966) have discussed the effects of heat and Mass transfer in the laminar boundary layer flow of a moving flat surface with constant surface velocity and temperature focusing on the effects of *suction/injection. Callahan and Marner (1976) considered the transient free convection flow past a semi-infinite vertical plate with mass transfer. Unsteady free convective flow on taking into account the mass transfer phenomenon past an infinite vertical plate was studied by Soundalgekhar and Wavre (1977). Yih (1999) studied free convection effect on MHD coupled heat and mass transfer of a moving permeable vertical surface. Ibrahim and Makinde (2010) have discussed the chemically reacting MHD boundary layer flow of heat and mass transfer over a moving vertical plate with suction.*

In the problem formulation, the continuity, momentum, energy and concentration equations are reduced to some parameter problem by introducing suitable transformation variables. The equations that governing the flow are coupled and solved numerically using by shooting method. The effects of various flow controlling parameters such as velocity, temperature and concentration are presented graphically and discussed quantitatively. The local skin-friction coefficient and the heat and mass transfer results are illustrated for representative values of the major parameters.

FORMULATION OF THE PROBLEM

Consider a two-dimensional free convection effects on the steady incompressible laminar MHD heat and mass transfer characteristics of a linearly started porous vertical plate, the velocity of the fluid far away from the plate surface is assumed zero for a quiescent state fluid. The variations of surface temperature and concentration are linear. All the fluid properties are assumed to be constant except for the density variations in the buoyancy force term of the linear momentum equation. The magnetic Reynolds number is assumed to be small, so that the induced magnetic field is neglected. No electrical field is assumed to exist and both viscous and magnetic dissipations are neglected. The Hall effects, the viscous dissipation and the joule heating terms are also neglected. Under these assumptions, along with Boussinesq approximations, the boundary layer equations describing this flow as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty)+g\beta^*(C-C_\infty)-\frac{\sigma B_o^{\,2}}{\rho}u-\frac{\nu}{k^{'}}u \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2} \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2} \tag{4}$$

The boundary conditions for the velocity, temperature and concentration fields are

$$\begin{aligned}&u=Bx,\ v=V,\ T=T_w=T_\infty+ax, C=C_w=C_\infty+bx \quad \text{at} \quad y=0\\ &u\to 0,\ T\to T_\infty,\ C\to C_\infty \quad as \quad y\to\infty\end{aligned} \tag{5}$$

where x and y represent the coordinate axes along the continuous stretching surface in the direction of motion and normal to it, respectively, u and v are the velocity components along the x and y axes respectively, ν is the kinematics viscosity, β, β are the thermal and concentration expansion coefficient respectively, σ electric conductivity, B_0 is the uniform magnetic field, ρ is the density k' is the permeability of the porous medium, T is the temperature inside the boundary layer, T_∞ is the temperature for away from the plate, C is the species concentration in the boundary layer, C_∞ Species concentration of the ambient fluid, α is the thermal diffusivity, D is the is the molecular diffusivity of the species concentration, B is a constant, a and b denotes the stratification rate of the gradient of ambient temperature and concentration profiles.*

We introduce the following non-dimensional variables:

$$\begin{aligned}&\eta=\sqrt{\frac{B}{\nu}}y, u=\frac{\partial\psi}{\partial y}=xBF^{'}, v=-\frac{\partial\psi}{\partial x}=-\sqrt{B\nu}F, M=\frac{\sigma B_0^2}{\rho B}, K=\frac{\nu}{k^{'}B}\\ &Gr=\frac{g\beta\left(T_w-T_\infty\right)}{xB^2}, Gc=\frac{g\beta^*\left(C_w-C_\infty\right)}{xB^2}, \theta=\frac{T-T_\infty}{T_w-T_\infty}, \phi=\frac{C-C_\infty}{C_w-C_\infty}, \Pr=\frac{\nu}{\alpha}, Sc=\frac{\nu}{D}\end{aligned} \tag{6}$$

In view of (6), the Equations (2) - (4) take the form

$$F^{'''}+FF^{''}-F^{'2}+Gr\theta+Gc\phi-\left(M+K\right)F^{'}=0 \tag{7}$$

$$\theta''+\Pr\left[F\theta'-F^{'}\theta\right]=0 \tag{8}$$

$$\phi''+Sc\left[F\phi'-F^{'}\phi\right]=0 \tag{9}$$

where the primes denote the differentiation with respect to η, M *is the magnetic parameter,* K *is the permeability parameter,* Gr *is the local temperature grashof number,* Gc *is the local concentration Grashof number,* $\Pr$ *is the Prandtl number and* Sc *is the Schmidt number.*

The corresponding boundary conditions are

$$F' = 1, F = -F_w,\ \theta = 1,\ \phi = 1 \quad \text{at} \quad \eta = 0$$
$$F' = 0,\ \theta = 0,\ \phi = 0 \quad \text{as} \quad \eta \to \infty \qquad (10)$$

where $F_w = \frac{V}{\sqrt{\nu B}}$ *is the suction parameter.*

SOLUTION OF THE PROBLEM

The governing boundary layer equations (7) - (9) subject to boundary conditions (10) are solved numerically by using shooting method. First of all higher order non-linear differential equations (7) - (9) are converted into simultaneous linear differential equations of first order and they are further transformed into initial value problem by applying the shooting technique. From the process of numerical computation, the skin-friction coefficient, the Nusselt number and Sherwood number which are respectively proportional to $F''(0)$, $-\theta'(0)$ *and* $-\phi'(0)$ *are also sorted out and their numerical values are presented in a tabular form.*

RESULTS AND DISCUSSION

In order to get a physical insight into the problem, a representative set of numerical results is shown graphically in Figs.1-18, to illustrate the influence of physical parameters viz., suction parameter fw, magnetic parameter M, *permeability parameter K, temperature Grashof number Gr, concentration Grashof number Gc, Prandtl number* $\Pr$ *and Schmidt number Sc on the velocity* $f'(\eta)$, *temperature* $\theta(\eta)$ *and concentration* $\phi(\eta)$.

The Prandtl number was taken to be Pr = 0.72, which corresponds to air, the values of Schmidt number (Sc) were chosen to be Sc = 0.24, 0.62, 0.78, 2.62, representing diffusing chemical species of most common interest in air like H_2, H_2O, NH_3, *and Propyl Benzene, respectively. Attention is focused on positive values of the buoyancy parameters, that is, Grashof number Gr > 0 (which corresponds to the cooling problem) and solutal Grashof number Gc > 0 (which indicates that the chemical species concentration in the free stream region is less than the concentration at the boundary surface).*

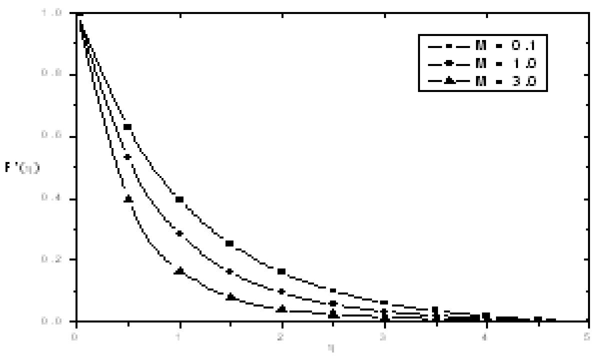

Fig.1. Velocity profile for different values of M when K = 0.1, Gr = Gc = 0.1, Pr = 0.72, Sc = 0.60 and $f_w = 0.1$

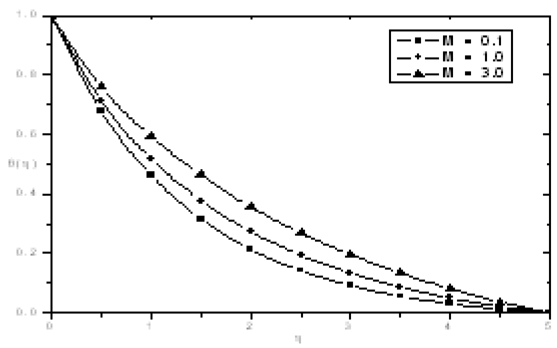

Fig.2. Temperature profile for different values of M when K = 0.1, Gr = Gc = 0.1, Pr = 0.72, Sc = 0.60 and $f_w = 0.1$

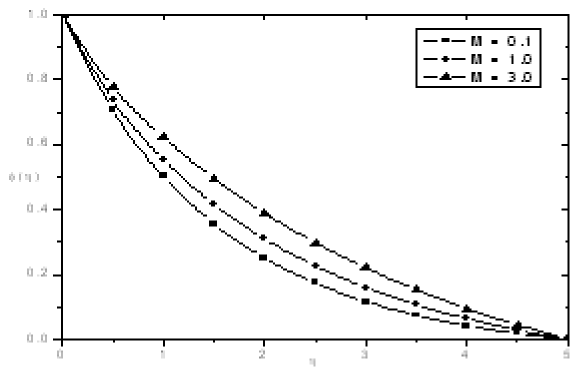

Fig.3. Concentration profile for different values of M when K = 0.1, Gr = Gc = 0.1, Pr = 0.72, Sc = 0.60 and $f_w = 0.1$

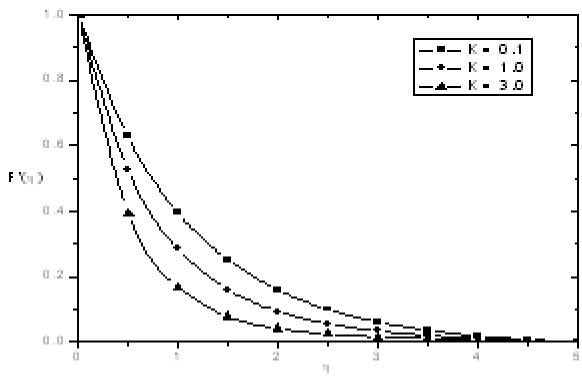

Fig.4. Velocity profile for different values of k when M = 0.1, Gr = Gc = 0.1, Pr = 0.72, Sc = 0.60 and $f_w = 0.1$

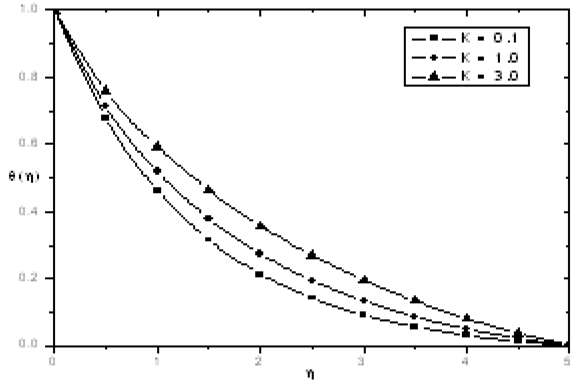

Fig.5. Temperature profile for different values of k when M = 0.1, Gr = Gc = 0.1, Pr = 0.72, Sc = 0.60 and f_w = 0.1

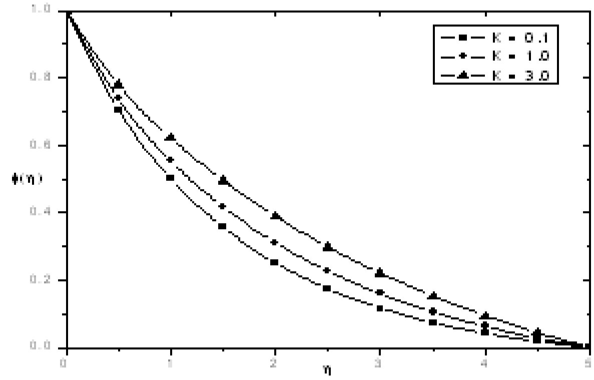

Fig.6. Concentration profile for different values of k when M = 0.1, Gr = Gc = 0.1, Pr = 0.72, Sc = 0.60 and f_w = 0.1

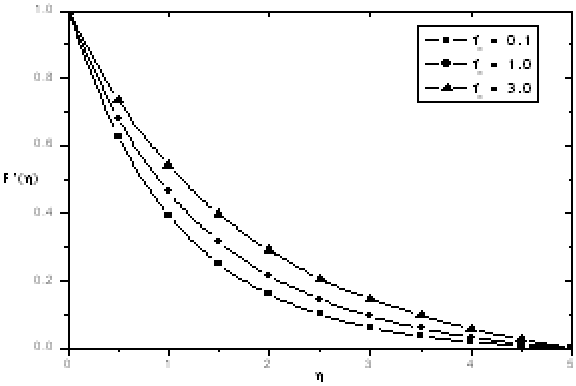

Fig.7. Velocity profile for different values of f_w when M = K = 0.1, Gr = Gc = 0.1, Pr = 0.72 and Sc = 0.60

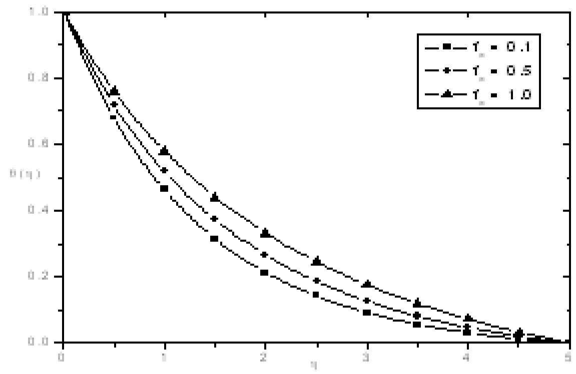

Fig.8. Temperature profile for different values of f_w when M = K = 0.1,Gr = Gc = 0.1, Pr = 0.72 and Sc = 0.60

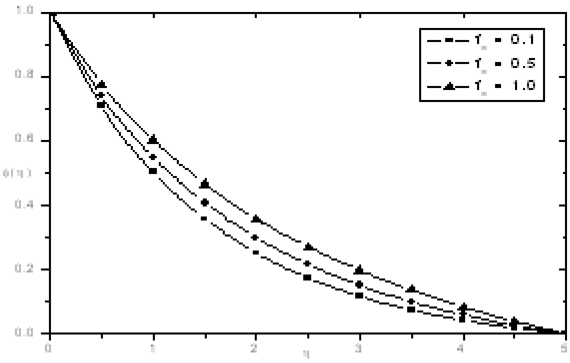

Fig.9. Concentration profile for different values of fw when M = K = 0.1, Gr = Gc = 0.1, Pr = 0.72 and Sc = 0.60

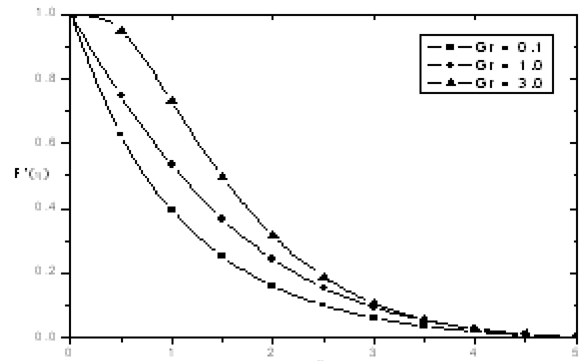

Fig.10. Velocity profile for different values of Gr when M = K = 0.1, Gc = 0.1, Pr = 0.72, Sc = 0.60 and f_w = 0.1

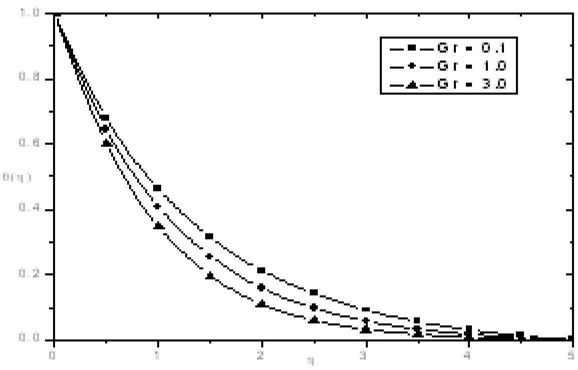

Fig.11. Temperature profile for different values of Gr when M = K = 0.1,Gc = 0.1, Pr = 0.72, Sc = 0.60 and f_w = 0.1

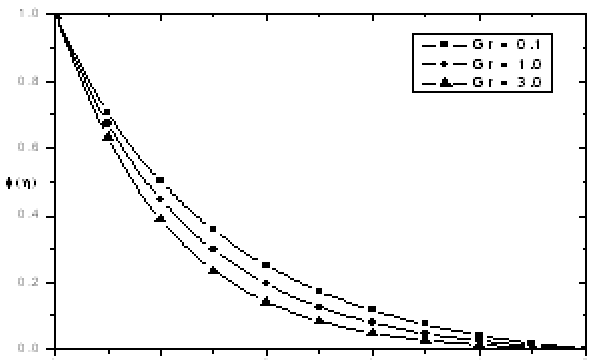

Fig.12. Concentration profile for different values of Gr when M = K = 0.1, Gc = 0.1, Pr = 0.72, Sc = 0.60 and f_w = 0.1

The effects of the magnetic parameter M on the velocity, temperature and concentration fields are shown in Figs. 1-3. It is obvious that an increase in the magnetic parameter M results in a decrease in the velocity. It is observed that the temperature and concentration profiles increase with the increasing of magnetic parameter M. Figs. 4-6 show the dimensionless velocity, temperature and concentration profiles for different values of permeability parameter K. It can be seen that the velocity profiles decrease with the increase of permeability parameter K. It is noticed that the temperature and concentration profiles increase with the increase of permeability parameter K.

For different values of the suction parameter f_w, the velocity, temperature and concentration fields are shown in Figs. 7-9. It is seen that the velocity, temperature and concentration increases with an increase in the suction parameter f_w. The effects of the Grashof number Gr on the velocity, temperature and concentration fields are shown in Figs 10-12. It is obvious that an increase in the Grashof number Gr results in a decrease in the velocity. It is seen that the temperature and concentration decreases with an increase in the Grashof number Gr.

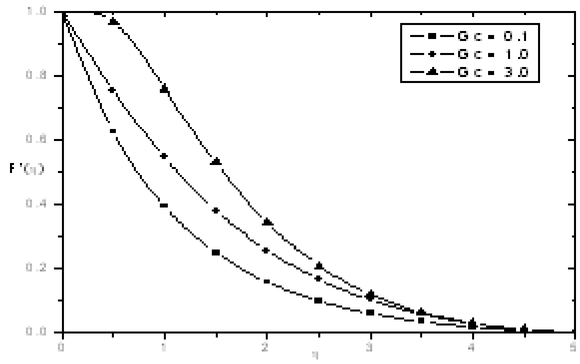

Fig.13. Velocity profile for different values of Gc when M = K = 0.1, Gr = 0.1, Pr = 0.72, Sc = 0.60 and f_w = 0.1

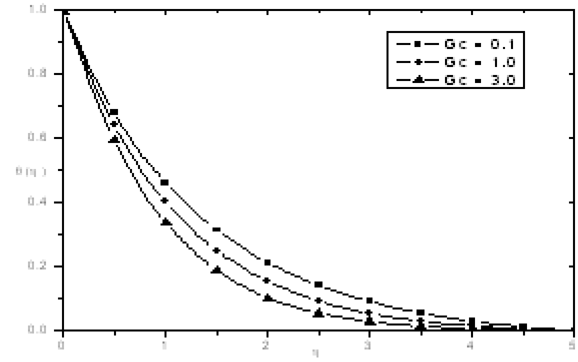

Fig.14. Temperature profile for different values of Gc when M = K = 0.1, Gr = 0.1, Pr = 0.72, Sc = 0.60 and f_w = 0.1

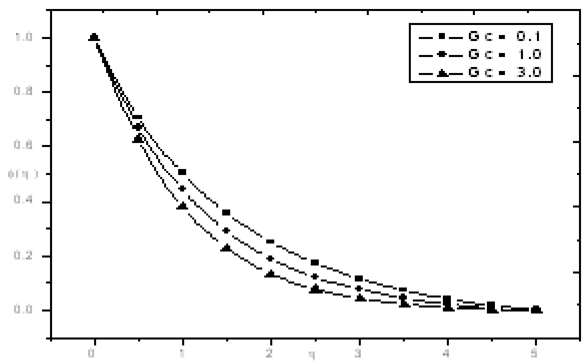

Fig.15. Concentration profile for different values of Gc when M = K = 0.1, Gr = 0.1, Pr = 0.72, Sc = 0.60 and f_w = 0.1

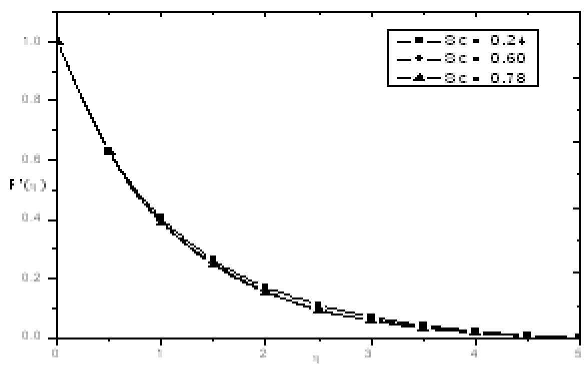

Fig.16. Velocity profile for different values of Sc when M = K = 0.1, Gr = Gc = 0.1, Pr = 0.72, and f_w = 0.1

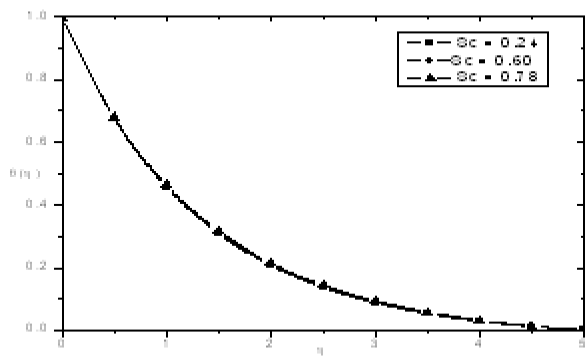

Fig.17. Temperature profile for different values of Sc when M = K = 0.1, Gr = Gc = 0.1, Pr = 0.72, and f_w = 0.1

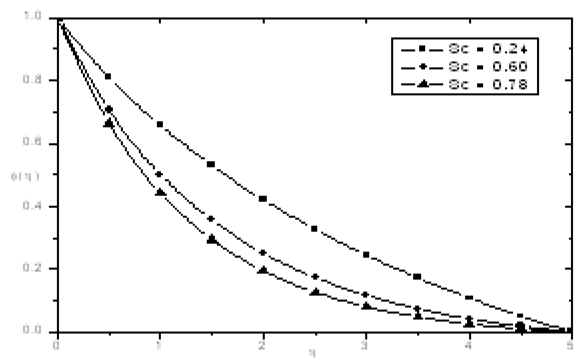

Fig.18. Concentration profile for different values of Sc when M = K = 0.1, Gr = Gc = 0.1, Pr = 0.72, and f_w = 0.1

The influence of the modified Grashof number Gc on the velocity, temperature and concentration fields are shown in Figs. 13-15. It is noticed that the velocity profiles increase with the increase of modified Grashof number Gc. It is obvious that an increase in the modified Grashof number Gc results in a decrease in the temperature and concentration. Figs. 16-18 show the dimensionless velocity, temperature and concentration profiles for different values of Schmidt *number Sc. It can be seen that the velocity profiles decrease with the increase of Schmidt number Sc. It is noticed that the temperature monotonically increases with the increase of Schmidt number Sc. It is seen that the concentration decreases as Schmidt number Sc increases.*

Table 1. Numerical results

M	K	Gr	Gc	f_w	Sc	$f'(0)$	$-\theta(0)$	$-\phi(0)$
0.1	0.1	0.1	0.1	0.1	0.60	0.86431	0.77521	0.70132
1.0	0.1	0.1	0.1	0.1	0.60	1.0235	0.68915	0.62614
0.1	1.0	0.1	0.1	0.1	0.60	1.68143	0.42511	0.50493
0.1	0.1	0.5	0.1	0.1	0.60	0.68947	0.81724	0.74329
0.1	0.1	0.1	0.5	0.1	0.60	0.649267	0.82420	0.75143
0.1	0.1	0.1	0.1	0.5	0.60	0.67056	0.59178	0.54225
0.1	0.1	0.1	0.1	0.1	0.78	0.87435	0.77912	0.812397

Numerical results are reported in the Table 1. From Table 1, it is important to note that the skin friction together with the heat and mass transfer rate at the moving plate surface decreases with increasing magnitude of fluid suction (Fw) at the moving surface. The rate of heat and mass transfer at the plate surface increases with increasing intensity of buoyancy forces (Gr, Gc) and decreases with increasing intensity of magnetic field (M) or permeability parameter (K). Moreover, the skin friction decreases with buoyancy forces and increases with increasing magnetic field intensity and Schmidt number (Sc). Furthermore, the surface mass transfer rate increases, while the surface heat transfer rate decreases with an increase in the Schmidt number (Sc). The effects of various governing parameters on the skin-friction coefficient C_f, Nusselt number Nu and Sherwood number Sh are shown in Table-1. It is observed that the skin-friction, Nusselt number and Sherwood number increases with the increase of unsteadiness parameter A or suction parameter f_w. It is found that the Sherwood number increases as Schmidt number increases. It is noticed that the skin-friction coefficient increases with increasing magnetic parameter M or permeability parameter K, whereas the Nusselt number and Sherwood number decrease with increasing magnetic parameter. It is found that the Nusselt number increases with increasing Prandtl number or heat source parameter.

CONCLUSIONS

In this paper we study the Heat and mass transfer effects on MHD boundary layer flow over a moving vertical porous plate. The expressions for the velocity, temperature and concentration distributions are the equations governing the flow are numerically solved by shooting technique. The skin friction decreases with buoyancy forces and increases with increasing magnetic field intensity and Schmidt number (Sc). The surface mass transfer rate increases, while the surface heat transfer rate decreases with an increase in the Schmidt number (Sc). The skin-friction, Nusselt number and Sherwood number increases with the increase of suction parameter f_w.

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