ANNALS OF FACULTY ENGINEERING HUNEDOARA
– International Journal of Engineering
Tome XI (Year 2013) – FASCICULE 3 (ISSN 1584 – 2673)

[1.] R. MUTHUCUMARASWAMY, [2.] V. LAKSHMI

ON FLOW PAST A PARABOLIC STARTED ISOTHERMAL VERTICAL PLATE WITH VARIABLE MASS DIFFUSION IN THE PRESENCE OF THERMAL RADIATION

[1.] DEPT. OF APPLIED MATHEMATICS, SRI VENKATESWARA COLLEGE OF ENGINEERING, SRIPERUMBUDUR, INDIA
[2.] DEPT. OF MATHEMATICS, RAJALAKSHMI ENGINEERING COLLEGE, THANDALAM, SRIPERUMBUDUR, INDIA

ABSTRACT: Theoretical solution of thermal radiation effects on unsteady flow past a parabolic starting motion of the infinite isothermal vertical plate with variable mass diffusion has been studied. The plate temperature as well as concentration level near the plate are raised uniformly. The dimensionless governing equations are solved using Laplace-transform technique. The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium. The effect of velocity profiles are studied for different physical parameters like thermal radiation parameter, thermal Grashof number, mass Grashof number and Schmidt number. It is observed that the velocity increases with increasing values the thermal Grashof number or mass Grashof number. The trend is just reversed with respect to the thermal radiation parameter.
KEYWORDS: Parabolic, radiation, isothermal, vertical plate, heat and mass transfer

INTRODUCTION

Radiative heat and mass transfer play an important role in manufacturing industries for the design of fins, steel rolling, nuclear power plants, gas turbines and various propulsion device for aircraft, combustion and furnace design, materials processing, energy utilization, temperature measurements, remote sensing for astronomy and space exploration, food processing and cryogenic engineering, as well as numerous agricultural, health and military applications. If the temperature of the surrounding fluid is rather high, radiation effects play an important role and this situation does exist in space technology. In such cases, one has to take into account the combined effect of thermal radiation and mass diffusion.

Natural convection on flow past an linearly accelerated vertical plate in the presence of viscous dissipative heat using perturbation method by Gupta et al [1]. Kafousias and Raptis [2] extended this problem to include mass transfer effects subjected to variable suction or injection. Soundalgekar [3] studied the mass transfer effects on flow past a uniformly accelerated vertical plate. Mass transfer effects on flow past an accelerated vertical plate with uniform heat flux was analyzed by Singh and Singh [4]. Free convection effects on flow past an exponentially accelerated vertical plate was studied by Singh and Naveen Kumar [5]. The skin friction for accelerated vertical plate has been studied analytically by Hossain and Shayo [6]. Mass transfer effects on exponentially accelerated infinite vertical plate with constant heat flux and uniform mass diffusion was studied by Basant Kumar Jha et al [7]. Agrawal et al [8] studied free convection due to thermal and mass diffusion in laminar flow of an accelerated infinite vertical plate in the presence of magnetic filed. Agrawal et al [9] further extended the problem of unsteady free convective flow and mass diffusion of an electrically conducting elasto-viscous fluid past a parabolic starting motion of the infinite vertical plate with transverse magnetic plate. The governing equations are tackled using Laplace transform technique.

It is proposed to study the effects of on flow past an infinite isothermal vertical plate subjected to parabolic motion with variable mass diffusion, in the presence of thermal radiation. The dimensionless governing equations are solved using the Laplace-transform technique. The solutions are in terms of exponential and complementary error function.

MATHEMATICAL FORMULATION

Here the unsteady flow of a viscous incompressible fluid past an infinite isothermal vertical plate with variable mass diffusion, in the presence of thermal radiation has been considered. The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium. The x' -axis is taken along the plate in the vertically upward direction and the y -axis is taken normal to the plate.

At time t' ≤ 0, the plate and fluid are at the same temperature T_∞ and concentration C'_∞. At time t'> 0 , the plate is started with a velocity $u = u_0 . t'^2$ in its own plane against gravitational field and the temperature from the plate is raised to T_w and the concentration level near the plate is raised linearly with time. The plate is infinite in length all the terms in the governing equations will be independent of x' and there is no flow along y-direction. Then under usual Boussinesq's approximation for unsteady parabolic starting motion is governed by the following equations:

$$\frac{\partial u}{\partial t'} = g\beta(T - T_\infty) + g\beta^*(C' - C'_\infty) + \nu\frac{\partial^2 u}{\partial y^2} \quad (1)$$

$$\rho C_p \frac{\partial T}{\partial t'} = k\frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} \quad (2)$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y^2} \quad (3)$$

With the following initial and boundary conditions:

$$u = 0, \quad T = T_\infty, \quad C' = C'_\infty \text{ for all } y, t' \le 0$$
$$t' > 0: \ u = u_0 . t'^2, \quad T = T_w, \quad C' = C'_\infty + (C'_w - C'_\infty)At' \quad \text{at } y = 0 \quad (4)$$
$$u \to 0 \quad T \to T_\infty, \ C' \to C'_\infty \quad \text{as } y \to \infty$$

where $A = \frac{u_0^2}{\nu}$.

The local radiant for the case of an optically thin gray gas is expressed by

$$\frac{\partial q_r}{\partial y} = -4a^*\sigma(T_\infty^4 - T^4) \quad (5)$$

It is assumed that the temperature differences within the flow are sufficiently small such that T^4 may be expressed as a linear function of the temperature. This is accomplished by expanding T^4 in a Taylor series about T_∞ and neglecting higher-order terms, thus

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \quad (6)$$

By using equations (5) and (6), equation (2) reduces to

$$\rho C_p \frac{\partial T}{\partial t'} = k\frac{\partial^2 T}{\partial y^2} + 16a^*\sigma T_\infty^3 (T_\infty - T) \quad (7)$$

On introducing the following non-dimensional quantities:

$$U = u\left(\frac{u_0}{\nu^2}\right)^{1/3}, \quad t = \left(\frac{u_0^2}{\nu}\right)^{1/3} t', \quad Y = y\left(\frac{u_0}{\nu^2}\right)^{1/3}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad C = \frac{C' - C'_\infty}{C'_w - C'_\infty}$$

$$Gr = \frac{g\beta(T - T_\infty)}{(\nu . u_0)^{1/3}}, \quad Gc = \frac{g\beta(C' - C'_\infty)}{(\nu . u_0)^{1/3}}, \quad R = \frac{16a^*\sigma . T_\infty^3}{k}\left(\frac{\nu^2}{u_0}\right)^{\frac{2}{3}}, \quad Pr = \frac{\mu C_p}{k}, \quad Sc = \frac{\nu}{D} \quad (8)$$

in equations (1), (3) and (7), reduces to

$$\frac{\partial U}{\partial t} = Gr\theta + GcC + \frac{\partial^2 U}{\partial Y^2} \quad (9)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial Y^2} - \frac{R}{Pr}\theta \quad (10)$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2} \quad (11)$$

The initial and boundary conditions in non-dimensional quantities are

$$U = 0, \ \theta = 0, \ C = 0 \ \text{for all } Y, t \le 0$$
$$t > 0: \quad U = t^2, \ \theta = 1, \ C = t \quad \text{at } Y = 0 \quad (12)$$
$$U \to 0, \ \theta \to 0, \ C \to 0 \ \text{as } Y \to \infty$$

SOLUTION PROCEDURE

The dimensionless governing equations (6) to (8) and the corresponding initial and boundary conditions (9) are tackled using Laplace transform technique.

$$\theta = \frac{1}{2}\left[exp(2\eta\sqrt{Rt})\,erfc(\eta\sqrt{Pr} + \sqrt{at}) + exp(-2\eta\sqrt{Rt})\,erfc(\eta\sqrt{Pr} - \sqrt{at})\right] \quad (10)$$

$$C = t\left[(1 + 2\eta^2 Sc)\,erfc(\eta\sqrt{Sc}) - \frac{2\eta\sqrt{Sc}}{\sqrt{\pi}}\,exp(-\eta^2\, Sc)\right] \quad (11)$$

$$\begin{aligned} U = {} & \frac{t^2}{3}\left[(3 + 12\eta^2 + 4\eta^4)\,erfc(\eta) - \frac{\eta}{\sqrt{\pi}}(10 + 4\eta^2)\,exp(-\eta^2)\right] \\ & + 2c\,erfc(\eta) - c\,exp(bt)\left[exp(2\eta\sqrt{bt})\,erfc(\eta + \sqrt{bt}) + exp(-2\eta\sqrt{bt})\,erfc(\eta - \sqrt{bt})\right] \\ & - d\left[(3 + 12\eta^2 + 4\eta^4)\,erfc(\eta) - \frac{\eta}{\sqrt{\pi}}(10 + 4\eta^2)\,exp(-\eta^2)\right] \\ & - c\left[exp(2\eta\sqrt{Rt})\,erfc(\eta\sqrt{Pr} + \sqrt{at}) + exp(-2\eta\sqrt{Rt})\,erfc(\eta\sqrt{Pr} - \sqrt{at})\right] \\ & + c\,exp(bt)\left[\begin{aligned} & exp(-2\eta\sqrt{Pr(b+a)t})\,erfc(\eta\sqrt{Pr} - \sqrt{(b+a)t}) \\ & + exp(2\eta\sqrt{Pr(b+a)t})\,erfc(\eta\sqrt{Pr} + \sqrt{(b+a)t}) \end{aligned}\right] \\ & + d\left[(3 + 12\eta^2 Sc + 4\eta^4 (Sc)^2)\,erfc(\eta\sqrt{Sc}) - \frac{\eta\sqrt{Sc}}{\sqrt{\pi}}(10 + 4\eta^2 Sc)\,exp(-\eta^2 Sc)\right] \end{aligned} \quad (12)$$

where, $a = \frac{R}{Pr}$, $b = \frac{R}{1 - Pr}$, $c = \frac{Gr}{2b(1 - Pr)}$, $d = \frac{Gct^2}{6(1 - Sc)}$ and $\eta = \frac{Y}{2\sqrt{t}}$.

RESULTS AND DISCUSSION

For physical understanding of the problem numerical computations are carried out for different physical parameters Gr, Gc, Sc and t upon the nature of the flow and transport. The value of the Schmidt number Sc is taken to be 0.6 which corresponds to water-vapor. Also, the value of Prandtl number Pr is chosen such that they represent air (Pr = 0.71). The numerical values of the velocity are computed for different physical parameters like Prandtl number, thermal Grashof number, mass Grashof number, Schmidt number and time.

Figure 1 represents the effect of concentration profiles for different Schmidt number ($Sc = 0.16, 0.6, 2.01$) and time $t = 0.4$. The effect of concentration is important in concentration field. The profiles have the common feature that the concentration decreases in a monotone fashion from the surface to a zero value far away in the free stream. It is observed that the wall concentration increases with decreasing values of the Schmidt number.

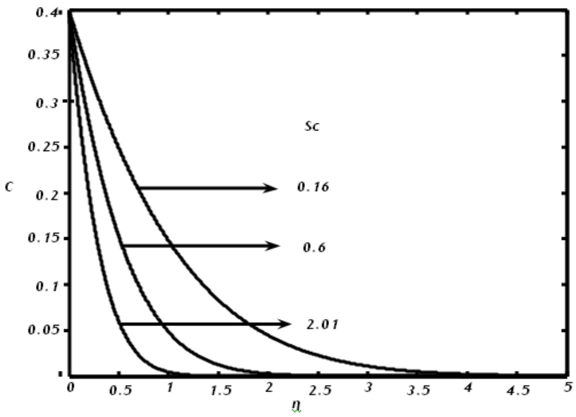

Figure 1: Concentration Profiles for different values of Sc

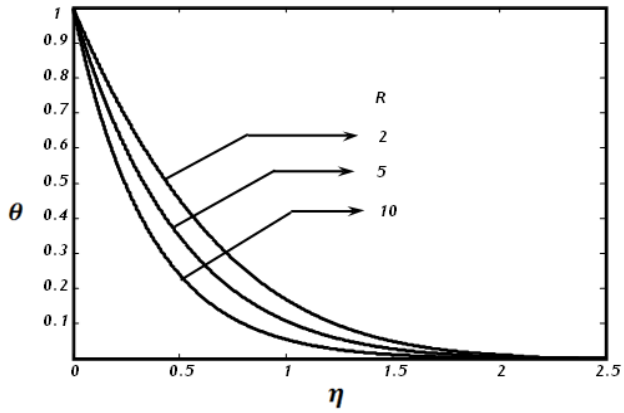

Figure 2: Temperature profiles for different values of R

The temperature profiles are calculated for different values of thermal radiation parameter ($R = 0.2, 2, 5$) are shown in Figure 2, for air ($Pr = 0.71$). The effect of thermal radiation parameter is important in temperature profiles. It is observed that the temperature increases with decreasing radiation parameter.

The effect of velocity for different values of the radiation parameter ($R = 2, 5, 10$), $Gr = 5 = Gc$ and $t = 0.4$ are shown in figure 3. The trend shows that the velocity increases with decreasing radiation parameter. It is observed that the velocity decreases in the presence of high thermal radiation.

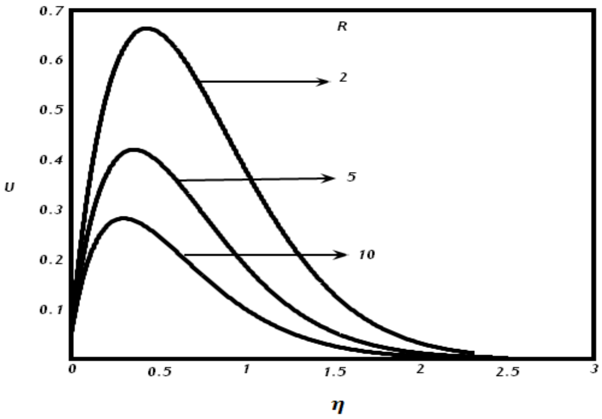

Figure 3: Velocity Profiles
for different values of R

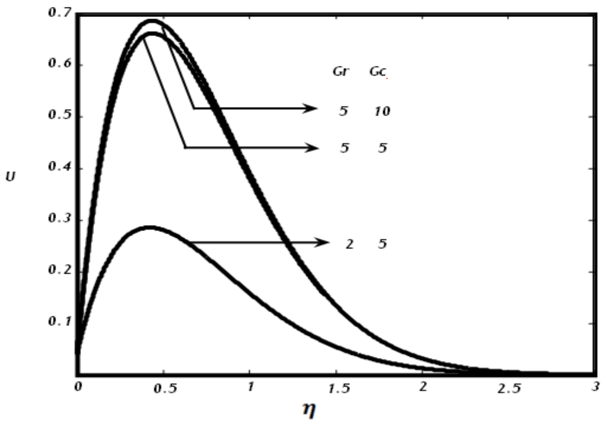

Figure 4: Velocity Profiles
for different values of Gr and Gc

Figure 4 demonstrates the effects of different thermal Grashof number (Gr = 2, 5) and mass Grashof number (Gc = 5,10) on the velocity at t = 0.4. It is observed that the velocity increases with increasing values of the thermal Grashof number or mass Grashof number.

CONCLUSIONS

The theoretical solution of flow past a parabolic starting motion of the infinite isothermal vertical plate with variable mass diffusion, in the presence thermal radiation has been studied. The dimensionless governing equations are solved by the usual Laplace transform technique. The effect of different physical parameters like thermal radiation parameter, thermal Grashof number and mass Grashof number are studied graphically. The conclusions of the study are as follows:

(i) The velocity increases with increasing thermal Grashof number or mass Grashof number, but the trend is just reversed with respect to the thermal radiation parameter.
(ii) The temperature of the plate increases with decreasing values of the thermal radiation parameter.
(iii) The plate concentration increases with decreasing values of the Schmidt number.

REFERENCES

[1] GUPTA A.S., POP I. and SOUNDALGEKAR V.M., 1979. Free convection effects on the flow past an accelerated vertical plate in an incompressible dissipative fluid. Rev. Roum. Sci. Techn.- Mec. Apl., vol.24, pp.561-568, 1979.
[2] KAFOUSIAS N.G. and RAPTIS A.A., Mass transfer and free convection effects on the flow past an accelerated vertical infinite plate with variable suction or injection, Rev. Roum. Sci. Techn.- Mec. Apl., vol.26, pp.11-22, 1981.
[3] SOUNDALGEKAR V.M., Effects of mass transfer on flow past a uniformly accelerated vertical plate, Letters in heat and mass transfer, vol.9. pp.65-72, 1982.
[4] SINGH A.K. and SINGH J., Mass transfer effects on the flow past an accelerated vertical plate with constant heat flux, Astrophysics and Space Science, vol.97, pp.57-61,1983.
[5] SINGH A.K. and NAVEEN KUMAR, Free convection flow past an exponentially accelerated vertical plate, Astrophysics and Space Science, vol.98, pp.245-258, 1984.
[6] HOSSAIN M.A. and SHAYO L.K., The skin friction in the unsteady free convection flow past an accelerated plate, Astrophysics and Space Science, vol.125. pp. 315-324, 1986.
[7] BASANTH KUMAR JHA and RAVINDRA PRASAD, Free convection and mass transfer effects on the flow past an accelerated vertical plate with heat sources, Mechanics Research Communications. vol.17, pp.143-148, 1990.
[8] AGRAWAL A.K. and SAMRIA N.K. and GUPTA S.N, Study of heat and mass transfer past a parabolic started infinite vertical plate, Journal of, Heat and mass transfer, vol.21, pp.67-75, 1999.
[9] AGRAWAL A.K.and SAMRIA N.K. and GUPTA S.N, free convection due to thermal and mass diffusion in laminar flow of an accelerated infinite vertical plate in the presence of magnetic filed, Journal of, Heat and mass transfer, vol.20, pp.35-43, 1998.