ABSTRACT: An analysis has been carried out to investigate the effects of the heat source, chemical reaction and radiation absorption on unsteady MHD flow with heat and mass transfer of an incompressible, viscous, electrically conducting fluid past an infinite vertical moving plate with constant temperature in the presence of transverse applied magnetic field through porous medium. An exact solution for the flow problem has been obtained by solving the governing equations using Laplace-transform technique. At time $t' > 0$, the plate is given an impulsive motion with a constant velocity $u_0$. At the same time, the plate temperature and concentration levels near the plate are raised to $T'_w$ and $C'_w$ respectively. The velocity, temperature, concentration and the rate of mass transfer are discussed through graphs while the numerical values of Nusselt number are presented in a table.

KEYWORDS: MHD, Heat and mass transfer, Radiation absorption, Heat source, chemical reaction, infinite, vertical plate

INTRODUCTION

The study of first order chemical reaction with combined heat and mass transfer is attracted by the many researchers and received a considerable amount of attention in recent years. In many processes such as energy transfer in a wet cooling tower in a desert cooler flow, evaporation at the surface of a water body and in heat and mass transfer occur simultaneously. Some applications of this type of flow can be found in many industries such as in power industry, among the methods of generating electric power is one in which electrical energy is extracted directly from a moving conducting fluid. The study of heat generation or absorption in moving fluids is important in problems dealings with chemical reactions dissociating fluids. Possible heat generation effects may alter the temperature distribution and consequently, the particle deposition rate in nuclear reactors, electronic chips and semi-conductor wafers. Since some fluids can also emit and absorb thermal radiation, it is of interest to study the effects of magnetic field on the temperature distribution and heat transfer when the fluid is not only an electrical conductor but also when it is capable of emitting and absorbing radiation. Hence, heat transfer by thermal radiation is becoming of greater importance when we concerned with space applications and higher operating temperatures.

The study of Magneto hydro-dynamics with heat and mass transfer in the presence of radiation and diffusion has attracted the attention of a large number of scholars due to diverse applications. In astrophysics and geophysics, it is applied to study the stellar and solar structures, radio propagation through the ionosphere etc. In engineering we find its applications like in MHD pumps, MHD bearings, etc. The phenomenon of mass transfer is also very common in theory of stellar structure and observable effects are detectable on the solar surface. In free convection flow the study of effects of magnetic field play a major role in liquid metals, electrolytes and ionized gases. In power engineering, the thermal physics of hydro magnetic problems with mass transfer have enormous applications. Radiative flows are encountered in many industrial and environment processes. e.g. heating and cooling chambers, fossil fuel combustion energy processes, evaporation from large open water reservoirs, astrophysical flows, solar power technology and space vehicle re-entry.

Bestman [1] investigated the natural convection boundary layer with suction and mass transfer in a porous medium. He found that suction stabilizes the boundary layer and affords the most

The aim of the present work is to investigate the effects of radiation absorption, chemical reaction, mass diffusion and heat source parameter of heat generating fluid on unsteady MHD natural convection flow with heat and mass transfer past an impulsively started infinite vertical plate with constant temperature in the presence of transverse applied magnetic field through porous medium. Initially, it is assumed that the plate and fluid are at the same temperature $T_w'$ in the stationary condition with concentration level $C_w'$ at all the points. The $x'$-axis is taken along the plate in vertical upward direction and $y'$-axis is taken normal to it. At time $t'>0$ the plate is given an impulsive motion with a constant velocity $u_0$. And at the same time, the temperature from the plate is raised to $T_w'$ and the concentration level near the plate is also raised to $C_w'$. A transverse magnetic field of uniform strength $B_0$ is assumed to be applied normal to the plate. It is also assumed that:

(i) the fluid properties are constant except for the density variation that induces the buoyancy force.
(ii) The induced magnetic field is assumed to be negligible as the magnetic Reynolds number of the flow is taken to be very small.
(iii) The viscous dissipation is neglected in the energy equation.
(iv) The effects of variation in density ($\rho$) (with temperature) and species concentration are considered only on the body force term, in accordance with usual Boussinesq approximation.
(v) The fluid considered here is gray, absorbing / emitting radiation but a non-scattering medium.
(vi) Since the flow of the fluid is assumed to be in the direction of $x'$- axis, so the physical quantities are functions of the space co-ordinate $y'$ and $t'$ only. Under these assumptions, the equations that describe the physical situation are given by

$$\frac{\partial u'}{\partial t'} = g\beta(T' - T_w') + g\beta' (C' - C_w') + v \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma \beta u'}{\rho} - \frac{u'}{k'}$$  \hspace{1cm} (1)

$$\frac{\partial T'}{\partial t'} = \frac{\kappa}{\rho c_p} \frac{\partial^2 T'}{\partial y'^2} - \frac{Q_0}{\rho c_p} (T' - T_w') + Q'_1 (C' - C_w')$$  \hspace{1cm} (2)

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y'^2} - K_i (C' - C_w')$$  \hspace{1cm} (3)

With the following initial and boundary conditions

$$t' \leq 0 : u' = 0, \quad T' = T_w', \quad C' = C_w' \, \text{for all} \, y'$$

$$t' > 0 : \quad u' = u_0' , \quad T' = T_w' , \quad C' = C_w' \, \text{at} \, y' = 0$$

and $u' = 0, \quad T' \rightarrow T_w', \quad C' \rightarrow C_w' \quad \text{as} \, \quad y' \rightarrow \infty$  \hspace{1cm} (4)
where $y'$ and $t'$ are the dimensional distance along the perpendicular plate and dimensional time respectively. $u'$ is the dimensional velocity along $x'$ direction, $T'$ is the dimensional temperature, $C'$ is the dimensional concentration, $C_w$ and $T_w$ are the concentration and temperature at the wall, $T_r$ and $C_r$ are the dimensional temperature and concentration for away from the plate respectively. $\rho$ is the density, $v$ is the kinematic viscosity, $c_p$ is the specific heat at constant pressure, $\sigma$ is the fluid electrical conductivity, $B_0$ is the magnetic induction, $Q_0$ is the dimensional heat absorption coefficient, $Q'$, is the coefficient of proportionality for the absorption of radiation, $D$ is the mass diffusivity, $g$ is the acceleration due to gravity, $K_{l}$ is the chemical reaction parameter, finally, $B$ and $B^*$ are the thermal and concentration expansion coefficients respectively.

On introducing the following non-dimensional quantities

$$
\frac{u}{u_0}, \quad t' = \frac{t'u^2}{v}, \quad y' = \frac{y'u_0}{v}, \quad \theta = \frac{T' - T_w}{T_r - T_w}, \quad C = \frac{C' - C_w}{C_r - C_w},
$$

$$
G_r = \frac{g\beta v(T'_r - T'_w)}{u_0^3}, \quad G_m = \frac{g\beta^* v(C'_r - C'_w)}{u_0^3}, \quad Pr = \frac{\mu C_p}{K}, \quad k = \frac{vK_{i}}{u_0^2}, \quad Sc = \frac{v}{D}, \quad M = \frac{\sigma B_0^2 v}{\mu u_0^2},
$$

$$
\phi = \frac{Q_0 v}{\rho c_p u_0^2}, \quad Q_l = \frac{\nu Q(C'_r - C'_w)}{(T'_r - T'_w)u_0^2}, \quad K = \frac{u_0^2 k'}{v^2}
$$

we get the following governing equations which are dimensionless.

$$
\frac{\partial u}{\partial t} = G_r \theta + G_m C + \frac{\partial^2 u}{\partial y^2} - Mu - \frac{u}{K}, \quad (6)
$$

$$
\frac{\partial \theta}{\partial t} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} - \phi \theta + Q_l C, \quad (7)
$$

$$
\frac{\partial C}{\partial t} = \frac{1}{Sc} \frac{\partial^2 C}{\partial y^2} - k C, \quad (8)
$$

The initial and boundary conditions in dimensionless form are as follows:

$$
t' \leq 0 : \quad u = 0, \quad \theta = 0, \quad C = 0 \quad \text{for all} \quad y,\n$$

$$
t' > 0 : \quad u = 1, \quad \theta = 1, \quad C = 1 \quad \text{at} \quad y = 0,\n$$

$$
u \to 0, \quad c \to 0 \quad \text{as} \quad y \to \infty. \quad (9)
$$

where $G_r$, $G_m$ are the thermal and mass Grashof numbers respectively. $P_r$ is the Prandtl number, $k$ is the chemical reaction parameter, $K$ is the permeability parameter, $M$ is the magnetic field parameter, $\phi$ is the heat source parameter, $Sc$ is the Schmidt number and $Q_l$ is radiation absorption parameter.

The dimensionless governing equations from (6) to (8), subject to the boundary conditions (9) are solved by usual Laplace transform technique and the solutions for velocity, temperature and concentration fields are obtained as follows in terms of exponential and complementary error functions.

$$
C(y,t) = \frac{1}{2} \left[ \exp(y\sqrt{kSc}) \erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}} + \sqrt{kt}\right) + \exp(-y\sqrt{kSc}) \erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}} - \sqrt{kt}\right) \right], \quad (10)
$$

$$
\theta(y,t) = \frac{1}{2} \left[ \left(1 - \frac{b}{c}\right) \exp(y\sqrt{\phi Pr}) \erfc\left(\frac{y\sqrt{Pr}}{2\sqrt{t}} + \sqrt{\phi t}\right) + \exp(-y\sqrt{\phi Pr}) \erfc\left(\frac{y\sqrt{Pr}}{2\sqrt{t}} - \sqrt{\phi t}\right) \right. \quad + \left. \frac{b}{2c} \exp(ct) \left[ \exp(y\sqrt{\phi Pr + c Pr}) \erfc\left(\frac{y\sqrt{Pr}}{2\sqrt{t}} + (\phi + c)t\right) + \exp(-y\sqrt{\phi Pr + c Pr}) \erfc\left(\frac{y\sqrt{Pr}}{2\sqrt{t}} - \sqrt{(\phi + c)t}\right) \right] \right], \quad (11)
$$

$$
u(y,t) = \left(1 - \frac{A_1 - A_2}{2}\right) \left[ \exp(y\sqrt{M'}) \erfc\left(\frac{y}{2\sqrt{t}} + \sqrt{M't}\right) + \exp(-y\sqrt{M'}) \erfc\left(\frac{y}{2\sqrt{t}} - \sqrt{M't}\right) \right] \quad (12)
$$

The initial and boundary conditions in dimensionless form are as follows:

$$
t' \leq 0 : \quad u = 0, \quad \theta = 0, \quad C = 0 \quad \text{for all} \quad y,\n$$

$$
t' > 0 : \quad u = 1, \quad \theta = 1, \quad C = 1 \quad \text{at} \quad y = 0,\n$$

$$
u \to 0, \quad c \to 0 \quad \text{as} \quad y \to \infty. \quad (9)
$$

where $G_r$, $G_m$ are the thermal and mass Grashof numbers respectively. $P_r$ is the Prandtl number, $k$ is the chemical reaction parameter, $K$ is the permeability parameter, $M$ is the magnetic field parameter, $\phi$ is the heat source parameter, $Sc$ is the Schmidt number and $Q_l$ is radiation absorption parameter.

The dimensionless governing equations from (6) to (8), subject to the boundary conditions (9) are solved by usual Laplace transform technique and the solutions for velocity, temperature and concentration fields are obtained as follows in terms of exponential and complementary error functions.
where \( b = \frac{Q_{Pr}}{Sc - Pr}, c = \frac{Pr - kSc}{Sc - Pr}, d = \frac{Gr}{Pr - 1}, e = \frac{M' - \phi Pr}{Pr - 1}, l = \frac{Gr}{Sc - 1}, n = \frac{M' - kSc}{Sc - 1}, r = \frac{Gr}{Sc - 1} \)

\[
A_1 = \frac{Gr}{M' - kSc} \left[ Pr - kSc \right] (Pr - 1) - \left[ Pr - kSc \right] (Sc - Pr)
\]

\[
A_2 = \frac{Gr}{M' - kSc} \left[ Pr - kSc \right] (Sc - Pr)
\]

\[
A_3 = \frac{Gr}{M' - kSc} \left[ Pr - kSc \right] (Sc - Pr)
\]

\[
A_4 = \frac{Gr}{M' - kSc} \left[ Pr - kSc \right] (Sc - Pr)
\]

\[
A_5 = \frac{Gr}{M' - kSc} \left[ Pr - kSc \right] (Sc - Pr)
\]

\[
A_6 = \frac{Gr}{M' - kSc} \left[ Pr - kSc \right] (Sc - Pr)
\]

**THE RATE OF HEAT TRANSFER**

Form temperature field, now we study the rate of heat transfer which is given in non-dimensional form as:

\[
Nu = \left[ \frac{d\theta}{dy} \right]_{y=0}
\]  

(13)

Form equations (11) and (13), we get:

\[
Nu = \left[ 1 - \frac{b}{c} \right] \left[ \frac{Pr}{\pi t} \ exp(-\phi t) + \sqrt{\phi \Pr \Pr \text{erf} \sqrt{\phi t}} \right] + \frac{b}{c} \left[ \frac{Pr}{\pi t} \ exp(-\phi t) + \exp(ct) \sqrt{\phi \Pr + cPr \text{erf} \sqrt{\phi + c t}} \right]
\]

**THE RATE OF MASS TRANSFER**

From concentration field, now study the rate of mass transfer which is given in non-dimensional form as:
\[
Sc = -\frac{dc}{dy} |_{y=0} \tag{14}
\]

From equations (10) and (14), we get:

\[
Sh = \frac{Sc}{\pi t} \exp(-kt) + \sqrt{kSc \text{ erf} \sqrt{kt}}
\]

RESULTS AND DISCUSSION

To get a physical insight into the problem the numerical evaluation of the analytical results reported in the previous section was performed and a set of results is reported graphically in Figures 1-15 for the cases of heating \((Gr < 0, Gm < 0)\) and cooling \((Gr > 0, Gm > 0)\) of the plate. The heating and cooling take place by setting up free-convection current due to temperature and concentration gradient. These results are obtained to illustrate the effects of various physical parameters like magnetic parameter \(M\), absorption radiation parameter \(Q_1\), chemical reaction parameter \(k\), Schmidt parameter \(Sc\), coefficient of heat absorption \(\phi\), thermal Grashof number \(Gr\) and Mass Grashof number \(Gm\) on the velocity, temperature and the concentration profiles.

![Figure 1: Velocity profiles for different values magnetic parameter M](image1)
![Figure 2: Velocity profiles for different values Heat absorption parameter \(\phi\)](image2)

Figure 1 reveals the effect of magnetic field parameter on fluid velocity and we observed that an increase in magnetic parameter \(M\), the velocity decreases in case of cooling and heating of the plate for \(Pr = 0.71\). It is due to the fact that the application of transverse magnetic field will result a resistive type force (Lorentz force) similar to drag force, which tends to resist the fluid flow and thus reducing its velocity. It is observed from Figure 2 and Figure 3, Table 2 and Table 3 that there is a fall in velocity with increase of Heat absorption parameter \(\phi\) or chemical reaction parameter \(k\) or Schmidt number \(Sc\) in case of cooling of the plate while it increases in the case of heating of the plate.

![Figure 3: Velocity for different values Schmidt number Sc](image3)

Figure 4 and Figure 5 show the effects of \(Q_1\) (radiation absorption parameter), \(Gr\) (thermal Grashof number) and \(Gm\) (mass Grashof number) on the velocity field \(u\). From these figures it is found that the velocity \(u\) increases as \(Q_1\) or \(Gr\) or \(Gm\) increases in case of cooling of the plate. It is because that increase in the values of thermal Grashof number and mass Grashof number has the tendency to increase the thermal and mass buoyancy effect. This gives rise to an increase in the induced flow transport and a reverse effect is identified in case of heating of the plate.

![Figure 4: Velocity for different values Q1](image4)
![Figure 5: Velocity for different values Gr, Gm](image5)

![Table 1. Nusselt number](image6)

<table>
<thead>
<tr>
<th>(Pr)</th>
<th>(Q_1)</th>
<th>(\phi)</th>
<th>(Sc)</th>
<th>(k)</th>
<th>(t)</th>
<th>Nusselt number</th>
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<td>0.5</td>
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</tr>
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<tr>
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<td>5</td>
<td>4</td>
<td>0.5</td>
<td>0.4</td>
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</tr>
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<td>0.2</td>
<td>0.4</td>
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</tr>
</tbody>
</table>
Table 2: Velocity for different k when M = 3, Ql = 0.5, Sc = 2.01, φ = 5, a = 0.5, Gr = 4, Gm = 2 & t = 0.4

<table>
<thead>
<tr>
<th>Pr</th>
<th>k=0.0</th>
<th>k=0.2</th>
<th>k=0.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2000</td>
<td>0.8785</td>
<td>0.8772</td>
<td>0.8759</td>
</tr>
<tr>
<td>0.4000</td>
<td>0.7163</td>
<td>0.7143</td>
<td>0.7123</td>
</tr>
<tr>
<td>0.6000</td>
<td>0.5510</td>
<td>0.5489</td>
<td>0.5469</td>
</tr>
<tr>
<td>0.8000</td>
<td>0.4034</td>
<td>0.4016</td>
<td>0.3999</td>
</tr>
<tr>
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<td>0.2824</td>
<td>0.2811</td>
<td>0.2798</td>
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<td>0.0269</td>
<td>0.0268</td>
<td>0.0267</td>
</tr>
</tbody>
</table>

Table 3: Velocity for different k when M = 3, Ql = 0.5, Sc = 2.01, φ = 5, a = 0.5, Gr = -4, Gm = -2 & t = 0.4

<table>
<thead>
<tr>
<th>Pr</th>
<th>k=0.0</th>
<th>k=0.2</th>
<th>k=0.4</th>
</tr>
</thead>
<tbody>
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<td>0.2000</td>
<td>0.8785</td>
<td>0.8772</td>
<td>0.8759</td>
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<td>2.0000</td>
<td>0.0269</td>
<td>0.0268</td>
<td>0.0267</td>
</tr>
</tbody>
</table>

Figure 4: Velocity profiles for different values of Radiation absorption parameter Ql

Figure 5: Velocity profiles for different values of Gr and Gm

Figure 6 reveals the velocity variation with time t for the cases of both cooling and heating. It is observed that the velocity increases as time t increases for cooling of the plate, and the trend is just reversed for heating of the plate. From Figure 7 it is seen that the velocity increases with permeability parameter K in both cases of the plate.

Figure 7: Velocity profiles for different values of time t

Figure 8: Temperature profiles for different values of chemical reaction parameter with Pr=0.71, Sc=0.6, Ql =0.5, φ=5 and t=0.4

Figure 9: Temperature profiles for different values of Heat absorption parameter with Pr=0.71, Sc=0.6, Q=0.5, k=0.5 and t=0.4
The influence of various flow parameters on the fluid temperature are illustrated in Figures 8-11. From these figures it is seen that the fluid temperature decreases with an increase in Heat absorption parameter $\phi$ while it increases with increase of Chemical reaction parameter $k$ or Radiation absorption parameter $Q_l$ or time $t$.

The concentration profiles for different values of $Sc$ (Schmidt number), chemical reaction parameter $k$ and time $t$ are presented through Figures 12-14. From these figures it is observed that the concentration decreases with an increase in $Sc$ or $k$ while it increases with time $t$.

REFERENCES


