THE ECONOMICAL SIZING OF THE THREE-PHASE POWER LINES HAVING AN EVENLY DISTRIBUTED CHARGE

1. Iosif POPA

DEPARTMENT OF ELECTROTECHNICAL ENGINEERING AND INDUSTRIAL INFORMATICS, FACULTY OF ENGINEERING HUNEDOARA, POLITEHNICA UNIVERSITY TIMIȘOARA, STR. REVOLUȚIEI, NO.5, HUNEDOARA, ROMANIA

ABSTRACT: Usually, the three-phase power lines, with charges evenly distributed along their length, such as the public lighting installations, are sized as if they were a three-phase power line feeding just one consumer. The difference is that the charge of the consumer represents a sum of the evenly distributed charges and the equivalent consumer is concentrated at half the length of the power line to be sized. An economical sizing of such a type of power line is possible if one uses the method of the minimum conductor material volume, also used in designing main three-phase power lines. The paper gives an example of sizing a three-phase power line with evenly distributed charges, using the two methods, and pointing out to the characteristics of application for the economical designing method.

KEYWORDS: three-phase power lines, distributes charges, methods

INTRODUCTION

The sizing of three-phase power lines having an evenly distributed charge is done similarly to that of the DC lines [1-4]. Such power lines are used in street lighting. In order to size them, we consider the charges purely resistive. Such a power line is given in figure 1.

The three-phase power line feeds n consumers, having an active power \( P_r \). The total power of the consumers is:

\[ P = n \cdot P_r \] (1)

The charges on the three phases are equal. The power evenly distributed along length \( L \) is:

\[ p = \frac{P}{L} \] (2)

Since the consumers are considered resistive (\( \Delta U_r = 0 \)), the voltage loss is equal to the active voltage loss:

\[ \Delta U_f = \Delta U_{\text{in}}; \quad \Delta U_f = \frac{\Delta U_{\text{in}} \cdot \%}{100} U_{\text{in}} \] (3)

The cross section of the power line is calculated by means of [2-4]:

\[ s = \frac{\sqrt{3} \cdot p \cdot I}{\Delta U_f} \cdot L \] (4)

or according to the total power \( P \):

\[ s = \frac{p \cdot P}{\Delta U_f \cdot U_{\text{in}}} \cdot L \] (5)

In these relations, \( I \) is the total current of the \( n \) consumers:

\[ I = n \cdot I_r \] (6)

where \( I_r \) is receptor current and \( \Delta U_{\%} = 3 \) (for lighting installations).

In this case too, the power line with an evenly distributed charge, having length \( L \), (fig.1.a) can be replaced by a simple, three-phase line of length \( L/2 \), which has the entire charge concentrated at its end (figure 1.b).

CHARACTERISTICS OF THE SIZE CALCULATION

In reality, the power line of length \( L_2 \), having the consumers evenly distributed, is connected to the power source by the section of length \( L_1 \), (figure 2.a) [1]. Using the same procedure as in sizing the single-wire line, given in fig.1, for a constant cross-section, the relations we used are:
\[ s = \frac{\sqrt{3} \cdot \rho \cdot I}{\Delta U} \left( L_1 + \frac{L_2}{2} \right) \]  

or:

\[ s = \frac{\rho \cdot P}{\Delta U \cdot U_{\text{in}}} \left( L_1 + \frac{L_2}{2} \right) \]

Figure 2. Real three-phase power line with the charge evenly distributed
a) the single-wire diagram; b) the equivalent single-wire diagram

After having determined the cross-section, we choose for it an immediately superior normalized value: \( s_n \geq s \). Further on, we proceeded to checking it when warm. For section \( s_n \), the tables given in the reference books point to a maximal admitted current \( I_{\text{max}} \). The cross section of the line is correct as to the warming criterion if:

\[ I \leq I_{\text{max}} \]  

If this condition is not met, then we gradually passed on to higher conductor cross sections until the criterion of maximal admitted warming is met. With the help of the maximal admitted warming criterion one can achieve an economic sizing of the three-phase power line with an evenly distributed charge. For this, we needed to determine the values of the currents in the \( n \) sections of the power line of length \( l_i \) (\( i = 1, 2, ..., n \)):

\[ n \sum \frac{l_i}{2} s = \left( \frac{1}{n} \sum \frac{l_i}{2} \right) s \]  

These currents have the values:

\[ L_1 = l_1 / n \times L_2 ; \quad L_2 = (n - 1) \times L_2 ; \quad L_k = \left[ n - (k - 1) \right] L_2 , \ldots \]

We looked in the tabled the cross-sections for which:

\[ I_{\text{max}} > I_k, \quad k = 1 \ldots n \]  

and thus the power line with evenly distributed charges has sections with different cross sections:

\[ s_1 \geq s_2 \geq s_3 \geq \ldots \geq s_n \]

It is compulsory that after having determined these sections, one should calculate the total voltage loss along the line (fig. 1.a) for a normal functioning of the consumers:

\[ \Delta U = \sqrt{3} \cdot \rho \cdot \frac{L}{n} \left( \frac{I_{1}}{s_{1}} + \frac{I_{2}}{s_{2}} + \ldots + \frac{I_{n}}{s_{n}} \right) \Delta U = \sqrt{3} \cdot \rho \cdot \frac{L}{n} \sum_{i=1}^{n} \frac{I_i}{s_{in}} \]  

The power line sections are well chosen if the voltage loss along the line, calculated by means of (14) does not exceed the maximal admitted loss, determined by means of (3).

The power line can also be sized according to the criterion of the minimum volume of conductor material, as in the case of the DC power line with an evenly distributed charge.

We have to point out that the receiver with power \( P_r \) of figure 1 and figure 2 is a three-phase one. The consumers of the lighting installations are incandescent or mercury-vapor lamps, therefore they are mono-phase receivers having the power \( P_L \) (figure 3). The lighting installations are constructed in such a way that on each phase there be the same number \( n \) of mono-phase receivers (figure 3). If the lamps mounted on light poles \( S_1, S_4, S_7, \ldots \) are fed by phase \( R \), those on light poles \( S_2, S_5, S_8, \ldots \) from phase \( S \), those on light poles \( S_3, S_6, S_9, \ldots \) from phase \( T \), and if the lamps have the same power \( P_L \), figure 2 and figure 3 show that:

\[ \frac{L_2}{n} = 3 \cdot l_1; \quad \frac{L_2}{n} = l_2 \]

and

\[ P_r = 3 \cdot P_L \]
EXAMPLE OF SIZING THREE PHASE POWER LINES WITH EVENLY DISTRIBUTES CONSUMERS

We are supposed to size the three-phase lighting system network (the overhead distribution line) in fig.3, considering that the lamps are equally distributed upon the three phases, each pole carrying one lamp having the power \( P_L = 500 \text{ W} \) and \( U_n = 220 \text{ V} \) the distance between two consecutive poles being 60m. The number of lamps is 18 and the distance between the feeding point and the line of evenly distributed consumers is 140 m. The conductors of the line are made of copper (\( \rho = 0.017 \ \Omega \cdot \text{mm}^2/\text{m} \)). The mean value of the environment temperature is \( \theta_{02} = 10^\circ\text{C} \).

**Solution:** The three-phase power line with evenly distributed consumers can be brought to the configuration given in fig.2.a. Can calculate the number of three-phase receivers:

\[
\frac{n}{3} = \frac{18}{3} = 6
\]

The distance between the three-phase receivers (fig.2.b) is:

\[
l_2 = 3 \cdot l_s = 3 \cdot 60 = 180 \text{ m}
\]

The power of the receivers has the value:

\[
P_r = 3 \cdot P_L = 3 \cdot 500 = 1500 \text{ W}
\]

The line of evenly distributed consumers has the length:

\[
L_2 = n \cdot l_2 = 6 \cdot 180 = 1080 \text{ m}
\]

The length of the three-phase power line (used in calculations) feeding the line with three-phase consumers, evenly distributed, has the value:

\[
L_1 = l_1 + 2 \cdot l_s = 140 + 2 \cdot 60 = 260 \text{ m} , \ (l_s=140\text{m})
\]

---

**The determination of the line cross-section by the method of maximal admitted voltage loss [1]**

In this case we use the equivalent diagram given in figure 2.b. The line has a constant cross-section all along its length. If we consider \( \cos\phi = 1 \), the cross section of the line will be calculated by means of:

\[
s = \frac{100 \cdot \rho \cdot n \cdot P_r \cdot (L_1 + \frac{L_2}{2})}{\Delta U^2 \cdot U_{\text{in}}^2} = 28.25 \ \text{mm}^2
\]

We choose a normalized cross section of an immediate superior value: \( s_n = 35 \ \text{mm}^2 \), and further on we check the line on warming. The current passing through the conductors of the main line has the value:

\[
I = \frac{n \cdot P_r}{\sqrt{3} \cdot U_{\text{in}}} = 13.63 \text{ A}
\]

Out of table 1, one can notice that for \( s_n = 35 \ \text{mm}^2 \) the corresponding value is \( I_{\text{max}} = 220 \text{ A} \). The value of this current is to be corrected according to the annual mean temperature [2-3].

Table 1. The maximal admitted values of the currents for various cross-sections of the overhead distribution line with copper conductors, at temperature \( \theta_{01} = +25^\circ\text{C} \).

<table>
<thead>
<tr>
<th>( s \ [\text{mm}^2] )</th>
<th>6</th>
<th>10</th>
<th>16</th>
<th>25</th>
<th>35</th>
<th>50</th>
<th>70</th>
<th>95</th>
<th>120</th>
<th>150</th>
</tr>
</thead>
<tbody>
<tr>
<td>( I_{\text{max}} \ [\text{A}] )</td>
<td>70</td>
<td>95</td>
<td>130</td>
<td>180</td>
<td>220</td>
<td>270</td>
<td>340</td>
<td>415</td>
<td>485</td>
<td>570</td>
</tr>
</tbody>
</table>

\[
I_{\text{max}} = \frac{\theta_{\text{max}} - \theta_{02}}{\theta_{\text{max}} - \theta_{01}} \cdot I_{\text{max}}
\]

So, \( I_{\text{max}}' = 254.03 \text{ A} > 13.63 \text{ A} \) and the temperature of the line does not exceed the maximal temperature \( \theta_{\text{max}} \).

**Sizing the power line by the method of the minimal conductor material volume [1-4]**

In this case, the line has six sections, with the lengths \( L_{1e} = L_1 = 260 \text{ m} \), \( L_{2e} = L_2 = L_{3e} = L_{4e} = L_{5e} = L_{6e} = 3 \cdot l_s = 180 \text{ m} \). These sections have the cross-sections \( s_1, s_2, \ldots s_6 \). The powers and the currents through the six sections have the values:

\[
\begin{align*}
P_1 &= n \cdot P_r = 6 \cdot 1500 = 9000 \text{ W} , \quad P_2 = (n-1) \cdot P_r = 5 \cdot 1500 = 7500 \text{ W} , \quad P_3 = (n-2) \cdot P_r = 4 \cdot 1500 = 6000 \text{ W} \\
P_4 &= (n-3) \cdot P_r = 3 \cdot 1500 = 4500 \text{ W} , \quad P_5 = (n-4) \cdot P_r = 2 \cdot 1500 = 3000 \text{ W} , \quad P_6 = (n-5) \cdot P_r = 1 \cdot 1500 = 1500 \text{ W} \\
l_1 &= l = 13.63 \text{ A}
\end{align*}
\]
The sections of the power line, calculated according to the methods of the minimum volume of conducting material, have the values:

\[ I_2 = \frac{(n-1) \cdot P_r}{\sqrt{3} \cdot U_{in}} = 11.38 \, A \]
\[ I_3 = \frac{(n-2) \cdot P_r}{\sqrt{3} \cdot U_{in}} = 9.10 \, A \]
\[ I_4 = \frac{(n-3) \cdot P_r}{\sqrt{3} \cdot U_{in}} = 6.83 \, A \]
\[ I_5 = \frac{(n-4) \cdot P_r}{\sqrt{3} \cdot U_{in}} = 4.55 \, A \]
\[ I_6 = \frac{(n-5) \cdot P_r}{\sqrt{3} \cdot U_{in}} = 2.27 \, A \]

The sections of the power line, calculated according to the methods of the minimum volume of conducting material, have the values:

\[ s_i = \frac{100 \cdot \rho \cdot \sqrt{P_i}}{\Delta U \cdot U_{in}^2} \sum_{i=1}^{6} L_{ie} \cdot \sqrt{P_i} \]  
\[ s_i = k_i \cdot \sqrt{P_i} \]
\[ k_i = \frac{100 \cdot \rho}{\Delta U \cdot U_{in}^2} \sum_{i=1}^{6} L_{ie} \cdot \sqrt{P_i} \]

So,
\[ s_1 = k_1 \cdot \sqrt{P_1} = 30.94 \, mm^2 \]
\[ s_2 = k_2 \cdot \sqrt{P_2} = 28.58 \, mm^2 \]
\[ s_3 = k_3 \cdot \sqrt{P_3} = 25.56 \, mm^2 \]
\[ s_4 = k_4 \cdot \sqrt{P_4} = 22.14 \, mm^2 \]
\[ s_5 = k_5 \cdot \sqrt{P_5} = 18.07 \, mm^2 \]
\[ s_6 = k_6 \cdot \sqrt{P_6} = 12.78 \, mm^2 \]

We chose the immediately superior normalized sections:
\[ s_{1n} = 35 \, mm^2; \quad s_{2n} = 35 \, mm^2; \quad s_{3n} = 35 \, mm^2; \quad s_{4n} = 25 \, mm^2; \quad s_{5n} = 25 \, mm^2; \quad s_{6n} = 16 \, mm^2. \]

We further performed the warm check of the conductors having these cross-sections. Table 1 gives:

\[ I_{max1} = 1.155 \cdot 180 = 207.9 \, A \]
\[ I_{max5} = 1.155 \cdot 35 = 207.9 \, A \]
\[ I_{max6} = 1.155 \cdot 25 = 150.15 \, A \]

One can notice that:
\[ I_1 < I_{max1} \]
\[ I_2 < I_{max2} \]
\[ I_3 < I_{max3} \]
\[ I_4 < I_{max4} \]
\[ I_5 < I_{max5} \]
\[ I_6 < I_{max6} \]

Therefore, the cross-sections are well chosen from the thermal criterion standpoint, as well.

With these cross-sections, for \( \cos \phi = 1 \), we determined the voltage loss along the three-phase power line with evenly distributed consumers, on a normal functioning:

\[ \Delta U_i = \sqrt{3} \cdot \rho \left[ \frac{L_1}{S_{in}} \cdot I_1 + \frac{L_2}{S_{2n}} \cdot (I_2 + I_3) + \frac{L_4}{S_{4n}} \cdot (I_4 + I_5) + \frac{L_6}{S_{6n}} \cdot I_6 \right] \]

\[ \Delta U_i = 9.25 \, V \]

The maximal loss admitted by the voltage of the line has the value:

\[ \Delta U_{max} = \frac{\Delta U \cdot U_{n}}{100} = 11.4 \, V \]

We thereby conclude that the cross-sections have been well chosen, as:

\[ \Delta U_i < \Delta U_{max}; \quad (9.25 \, V < 11.4 \, V). \]

**Conclusions**

The paper introduces the means of economically sizing of the three-phase electric power lines with evenly distributed consumers. In order to reach this goal, we used the method of the minimal conductor material volume for the sizing example we gave. Obviously, it results from the comparison to the classical method that it is advantageous to use the method we suggested.

An economical sizing of the three-phase power lines with evenly distributed consumers can also be achieved if the cross-sections of the sections are determined by means of the criterion of the maximal admitted heating. In this case, it is compulsory that the maximal voltage loss on the line be less than the maximal loss admitted by the voltage.

**References**