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THE ECONOMICAL SIZING OF THE THREE-PHASE POWER LINES HAVING AN EVENLY DISTRIBUTED CHARGE

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ABSTRACT: Usually, the three-phase power lines, with charges evenly distributed along their length, such as the public lighting installations, are sized as if they were a three-phase power line feeding just one consumer. The difference is that the charge of the consumer represents a sum of the evenly distributed charges and the equivalent consumer is concentrated at half the length of the power line to be sized. An economical sizing of such a type of power line is possible if one uses the method of the minimum conductor material volume, also used in designing main three-phase power lines. The paper gives an example of sizing a three-phase power line with evenly distributes charges, using the two methods, and pointing out to the characteristics of application for the economical designing method.
KEYWORDS: three-phase power lines, distributes charges, methods

INTRODUCTION

The sizing of three-phase power lines having an evenly distributed charge is done similarly to that of the DC lines [1-4]. Such power lines are used in street lighting. In order to size them, we consider the charges purely resistive. Such a power line is given in figure1.

The three-phase power line feeds n consumers, having an active power P_r. The total power of the consumers is:

$$P = n \cdot P_r \qquad (1)$$

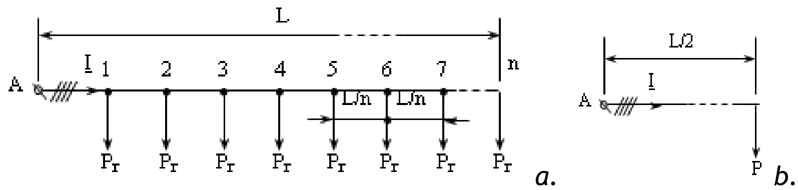

Figure 1. Three-phase power line with an evenly distributes charge a) the single-wire diagram; b) the equivalent single-wire diagram.

The charges on the three phases are equal. The power evenly distributed along length L is:

$$p = \frac{P}{L} \qquad (2)$$

Since the consumers are considered resistive (ΔU_{lr} = 0), the voltage loss is equal to the active voltage loss:

$$\Delta U_l = \Delta U_{la}; \quad \Delta U_l = \frac{\Delta U_l\%}{100} \cdot U_{\ln} \qquad (3)$$

The cross section of the power line is calculated by means of [2-4]:

$$s = \frac{\sqrt{3} \cdot \rho \cdot I}{\Delta U_l} \cdot \frac{L}{2} \qquad (4)$$

or according to the total power P:

$$s = \frac{\rho \cdot P}{\Delta U_l \cdot U_{\ln}} \cdot \frac{L}{2} \qquad (5)$$

In these relations, I is the total current of the n consumers:

$$\underline{I} = n \cdot \underline{I}_r \qquad (6)$$

where I_r is receptor current and $\Delta U\%$ = 3 (for lighting installations).

In this case too, the power line with an evenly distributed charge, having length L, (fig.1.a) can be replaced by a simple, three-phase line of length L/2, which has the entire charge concentrated at its end (figure 1.b)

CHARACTERISTICS OF THE SIZE CALCULATION

In reality, the power line of length L_2, having the consumers evenly distributed, is connected to the power source by the section of length L_1 (figure 2.a) [1]. Using the same procedure as in sizing the single-wire line, given in fig.1, for a constant cross-section, the relations we used are:

$$s = \frac{\sqrt{3} \cdot \rho \cdot I}{\Delta U_l} \cdot \left(L_1 + \frac{L_2}{2} \right) \tag{7}$$

or:

$$s = \frac{\rho \cdot P}{\Delta U_{\ln} \cdot U_{\ln}} \cdot \left(L_1 + \frac{L_2}{2} \right) \tag{8}$$

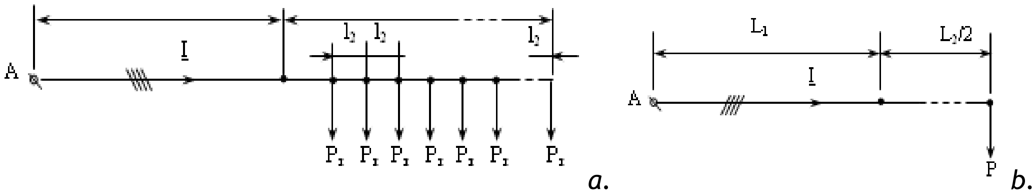

Figure 2. Real three-phase power line with the charge evenly distributed
a) the single-wire diagram; b) the equivalent single-wire diagram

After having determined the cross-section, we choose for it an immediately superior normalized value: $s_n \geq s$. Further on, we proceeded to checking it when warm. For section s_n the tables given in the reference books point to a maximal admitted current I_{max}. The cross section of the line is correct as to the warming criterion if:

$$I \leq I_{max} \tag{9}$$

If this condition is not met, then we gradually passed on to higher conductor cross sections until the criterion of maximal admitted warming is met. With the help of the maximal admitted warming criterion one can achieve an economic sizing of the three-phase power line with an evenly distributed charge. For this, we needed to determine the values of the currents in the n sections of the power line of length l_i (i = 1, 2, …,n):

$$l_i = \frac{L}{n} \tag{10}$$

These currents have the values:

$$\underline{I}_1 = \underline{I} = n \cdot \underline{I}_r; \ \underline{I}_2 = (n-1) \cdot \underline{I}_r; \ \underline{I}_k = [n - (k-1)] \cdot \underline{I}_r; \ \ldots \underline{I}_{n-1} = 2 \cdot \underline{I}_r; \ \underline{I}_n = \underline{I}_r \tag{11}$$

We looked in the tabled the cross-sections for which:

$$I_{maxk} \geq I_k, \qquad k = 1 \ldots n \tag{12}$$

and thus the power line with evenly distributed charges has sections with different cross sections:

$$s_{1n} \geq s_{2n} \geq s_{3n} \geq \ldots \geq s_{nn} \tag{13}$$

It is compulsory that after having determined these sections, one should calculate the total voltage loss along the line (fig.1.a) for a normal functioning of the consumers:

$$\Delta U_l = \sqrt{3} \cdot \rho \cdot \frac{L}{n} \left(\frac{I_1}{s_{1n}} + \frac{I_2}{s_{2n}} + \ldots + \frac{I_n}{s_{nn}} \right); \Delta U_l = \sqrt{3} \cdot \rho \cdot \frac{L}{n} \sum_{i=1}^{n} \frac{I_i}{s_{in}} \tag{14}$$

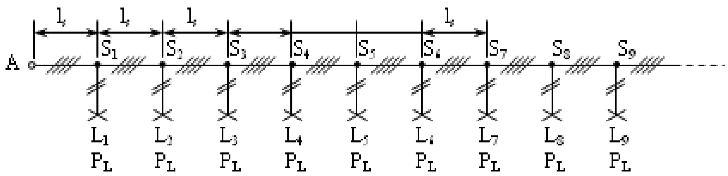

Figure 3. The single-wire diagram of a street lighting installation

The power line sections are well chosen if the voltage loss along the line, calculated by means of (14) does not exceed the maximal admitted loss, determined by means of (3).

The power line can also be sized according to the criterion of the minimum volume of conductor material, as in the case of the DC power line with an evenly distributed charge.

We have to point out that the receiver with power P_r of figure 1 and figure 2 is a three-phase one. The consumers of the lighting installations are incandescent or mercury-vapor lamps, therefore they are mono-phase receivers having the power P_L (figure 3). The lighting installations are constructed in such a way that on each phase there be the same number n of mono-phase receivers (figure 3). If the lamps mounted on light poles S_1, S_4, S_7 … are fed by phase R, those on light poles S_2, S_5, S_8… from phase S, those on light poles S_3, S_6, S_9 … from phase T, and if the lamps have the same power P_L, figure 2 and figure 3 show that:

$$\frac{L_2}{n} = 3 \cdot l_s; \qquad \frac{L_2}{n} = l_2 \tag{15}$$

and

$$P_r = 3 \cdot P_L \tag{16}$$

where l_s is the distance between the light poles. In the same way one can pass from the single-wire installation in figure 3 to the one given in figure 2.

EXAMPLE OF SIZING THREE PHASE POWER LINES WITH EVENLY DISTRIBUTES CONSUMERS

We are supposed to size the three-phase lighting system network (the overhead distribution line) in fig.3, considering that the lamps are equally distributed upon the three phases, each pole carrying one lamp having the power P_L = 500 W and U_n = 220 V the distance between two consecutive poles being 60m. The number of lamps is 18 and the distance between the feeding point and the line of evenly distributed consumers is 140 m. The conductors of the line are made of copper (ρ = 0.017 $\Omega \cdot mm^2/m$). The mean value of the environment temperature is θ_{02} = 10^0C.

Solution: The three-phase power line with evenly distributed consumers can be brought to the configuration given in fig.2.a. Can calculate the number of three-phase receivers:

$$n = \frac{n_L}{3} = \frac{18}{3} = 6$$

The distance between the three-phase receivers (fig.2.b) is:

$$l_2 = 3 \cdot l_s = 3 \cdot 60 = 180 \ m$$

The power of the receivers has the value:

$$P_r = 3 \cdot P_L = 3 \cdot 500 = 1500 \ W$$

The line of evenly distributed consumers has the length:

$$L_2 = n \cdot l_2 = 6 \cdot 180 = 1080 \ m$$

The length of the three-phase power line (used in calculations) feeding the line with three-phase consumers, evenly distributed, has the value:

$$L_1 = l_1 + 2 \cdot l_s = 140 + 2 \cdot 60 = 260 \ m, \ (l_1 = 140m)$$

The determination of the line cross-section by the method of maximal admitted voltage loss [1]

In this case we use the equivalent diagram given in figure 2.b. The line has a constant cross-section all along its length. If we consider cosφ = 1, the cross section of the line will be calculated by means of:

$$s = \frac{100 \cdot \rho \cdot n \cdot P_r \cdot \left(L_1 + \frac{L_2}{2} \right)}{\Delta U\% \cdot U_{\ln}^2} = 28.25 \ mm^2$$

We choose a normalized cross section of an immediate superior value: s_n = 35 mm^2, and further on we check the line on warming. The current passing through the conductors of the main line has the value:

$$I = \frac{n \cdot P_r}{\sqrt{3} \cdot U_{\ln}} = 13.63 \ A$$

Out of table 1, one can notice that for s_n = 35 mm^2 the corresponding value is I_{max} = 220 A. The value of this current is to be corrected according to the annual mean temperature [2-3].

Table 1. The maximal admitted values of the currents for various cross-sections of the overhead distribution line with copper conductors, at temperature θ_{01} = $+25^0C$

s [mm^2]	6	10	16	25	35	50	70	95	120	150
I_{max} [A]	70	95	130	180	220	270	340	415	485	570

$$I'_{\max} = c_\theta \cdot I_{\max} = \sqrt{\frac{\theta_{\max} - \theta_{02}}{\theta_{\max} - \theta_{01}}} \cdot I_{\max} \tag{17}$$

$$I'_{\max} = 254.03 \ A$$

So, I'_{max}>I (254.03A>13.63A) and the temperature of the line does not exceed the maximal temperature θ_{max}.

Sizing the power line by the method of the minimal conductor material volume [1-4]

In this case, the line has six sections, with the lengths L_{1e} = L_1 = 260 m, L_{2e} = L_{3e} = L_{4e} = L_{5e} = L_{6e} = 3 · l_s = 180 m. These sections have the cross-sections s_1, s_2, …s_6. The powers and the currents through the six sections have the values:

$$P_1 = n \cdot P_r = 6 \cdot 1500 = 9000 W, \ P_2 = (n-1) \cdot P_r = 5 \cdot 1500 = 7500 W, \ P_3 = (n-2) \cdot P_r = 4 \cdot 1500 = 6000 W$$

$$P_4 = (n-3) \cdot P_r = 3 \cdot 1500 = 4500 W, \ P_5 = (n-4) \cdot P_r = 2 \cdot 1500 = 3000 W, \ P_6 = (n-5) \cdot P_r = 1 \cdot 1500 = 1500 W$$

$$I_1 = I = 13.63 \ A$$

$$I_2 = \frac{(n-1)\cdot P_r}{\sqrt{3}\cdot U_{\ln}} = 11.38\ A \qquad I_3 = \frac{(n-2)\cdot P_r}{\sqrt{3}\cdot U_{\ln}} = 9.10\ A$$

$$I_4 = \frac{(n-3)\cdot P_r}{\sqrt{3}\cdot U_{\ln}} = 6.83\ A \qquad I_5 = \frac{(n-4)\cdot P_r}{\sqrt{3}\cdot U_{\ln}} = 4.55\ A \qquad I_6 = \frac{(n-5)\cdot P_r}{\sqrt{3}\cdot U_{\ln}} = 2.27\ A$$

The sections of the power line, calculated according to the methods of the minimum volume of conducting material, have the values:

$$s_1 = \frac{100\cdot\rho\cdot\sqrt{P_1}}{\Delta U\%\cdot U_{ln}^2}\cdot\sum_{i=1}^{6} L_{ie}\cdot\sqrt{P_i} \tag{18}$$

$$s_1 = k_s\cdot\sqrt{P_1} \tag{19}$$

$$k_s = \frac{100\cdot\rho}{\Delta U\%\cdot U_{ln}^2}\sum_{i=1}^{6} L_{ie}\cdot\sqrt{P_i} \tag{20}$$

$$k_s = 0.33\ mm^2\cdot W^{-\frac{1}{2}}$$

So, $s_1 = k_s\cdot\sqrt{P_1} = 30.94\ mm^2 \qquad s_2 = k_s\cdot\sqrt{P_2} = 28.58\ mm^2 \qquad s_3 = k_s\cdot\sqrt{P_3} = 25.56\ mm^2$

$s_4 = k_s\cdot\sqrt{P_4} = 22.14\ mm^2 \qquad s_5 = k_s\cdot\sqrt{P_5} = 18.07\ mm^2 \qquad s_6 = k_s\cdot\sqrt{P_6} = 12.78\ mm^2$

We chose the immediately superior normalized sections:
s_{1n} = 35 mm^2; s_{2n} = 35 mm^2; s_{3n} = 35 mm^2; s_{4n} = 25 mm^2; s_{5n} = 25 mm^2; s_{6n} = 16 mm^2.

We further performed the warm check of the conductors having these cross-sections. Table 1 gives: I_{maxi} i = I, …, n, which is to be corrected according to temperature:

$$I'_{max1} = c_\theta\cdot I_{max1} = 254.03\ A,\ I'_{\max 2} = I'_{\max 3} = I'_{\max 1} = 254.03\ A$$

$$I'_{\max 4} = c_\theta\cdot I_{\max 4} = 1{,}155\cdot 180 = 207.9\ A,\ I'_{\max 5} = I'_{\max 4} = 207.9\ A,\ I'_{\max 6} = c_\theta\cdot I_{\max 6} = 150.15\ A$$

One can notice that:

$$I_1 < I'_{\max 1}(13.63A < 254.03A);\ I_2 < I'_{\max 2}(11.38A < 254.03A);\ I_3 < I'_{\max 3}(9.10A < 254.03A);$$

$$I_4 < I'_{\max 4}(6.83A < 207.9A);\ I_5 < I'_{\max 5}(4.55A < 207.9A);\ I_6 < I'_{\max 6}(2.27A < 150.15A).$$

Therefore, the cross-sections are well chosen from the thermal criterion standpoint, as well. With these cross-sections, for cos φ = 1, we determined the voltage loss along the three-phase power line with evenly distributed consumers, on a normal functioning:

$$\Delta U_l = \sqrt{3}\cdot\rho\cdot\left[\frac{L_1}{s_{1n}}\cdot I_1 + \frac{l_2}{s_{2n}}\cdot(I_2+I_3) + \frac{l_2}{s_{4n}}\cdot(I_4+I_5) + \frac{l_2}{s_{6n}}\cdot I_6\right] \tag{21}$$

$$\Delta U_l = 9.25\ V$$

The maximal loss admitted by the voltage of the line has the value:

$$\Delta U_{l\max} = \frac{\Delta U\%}{100}\cdot U_n = 11.4\ V$$

We thereby conclude that the cross-sections have been well chosen, as:

$\Delta U_l < \Delta U_{lmax}$; (9.25 V < 11.4 V).

CONCLUSIONS

The paper introduces the means of economically sizing of the three-phase electric power lines with evenly distributed consumers. In order to reach this goal, we used the method of the minimal conductor material volume for the sizing example we gave. Obviously, it results from the comparison to the classical method that it is advantageous to use the method we suggested.

An economical sizing of the three-phase power lines with evenly distributed consumers can also be achieved if the cross-sections of the sections are determined by means of the criterion of the maximal admitted heating. In this case, it is compulsory that the maximal voltage loss on the line be less than the maximal loss admitted by the voltage.

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