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[1.] S.L.CHAMPATI, [2.] V.V.RAMANA RAO

LAMINAR FLOW OF TWO IMMISCIBLE INCOMPRESSIBLE VISCOUS LIQUIDS IN A SATURATED POROUS MEDIUM THROUGH A ROTATING STRAIGHT PIPE

[1.] SIKSHA 'O' ANUSANDHAN UNIVERSITY, (I.T.E.R), BHUBANESWAR, INDIA
[2.] ANDHRA UNIVERSITY, ANDHRA PRADESH, INDIA

ABSTRACT: In this paper, the steady laminar flow of two incompressible immiscible liquids under the action of a constant pressure gradient through a channel of circular cross - section, rotating with a uniform angular velocity about an axis perpendicular to the channel in saturated porous medium based on Brinkman's Model (1947) has been studied. It is assumed the two angular velocities about the axis of rotation and the porous parameters are small, to obtain the mathematical solution using perturbation technique.
KEYWORDS: Laminar flow, porous medium, viscous, rotating flow

INTRODUCTION

Survey reveals that the velocity profile due to the flow of two incompressible immiscible liquids occupying equal heights between two parallel plates was first obtained by Bird, Stewart and Lightfoot (1960).This problem was further generalized by Kapur and Shukla (1964) to the case of flow of a number of incompressible immiscible liquids occupying different heights. The stability analysis of two superposed fluids between parallel planes was formulated by Yih (1967) and later extended by Nakaya and Hasegawa (1974) to include the effects of gravity and surface tension. Santowski, Seidal and Ames (1969) studied the stability analysis by considering the stratified gas over a liquid under the assumption of inviscid and incompressible flow. This assumption is only an approximation which is true only when the fluid velocities are low. Rudraiah (1975) modified this work by considering the superposed flow of a compressible fluid over an incompressible fluid but under the assumption that the compressibility of fluid is taken as an isothermal atmosphere where the density changes with height.

Ramana Rao and Narayana (1981) Studied the flow of two incompressible immiscible liquids occupying equal heights between two parallel plates in a rotating system under the action of constant pressure gradient. They also studied the associated thermal distribution, assuming equal and different plate temperatures. By immiscible fluids, we mean, superposed fluids of different densities and viscosities. Ramana Rao and Narayana (1981) suggested that olive-oil and water can be taken as the two immiscible liquids.

The uniqueness for two immiscible fluids in a one dimensional porous medium was studied by Baiocchi, Claudio, Evans, Lawrence C., Frank, Leonid, Friedman, Avner (1980).

MATHEMATICAL FORMULATION

The equations of motion in steady state flow for two incompressible immiscible viscous fluids in a saturated porous media based on Brinkmann's model in rotating straight pipe are

$$\rho_i\left(\vec{V}_i'\cdot\nabla'\right)\vec{V}_i' - 2\rho_i\vec{V}_i'\times\vec{\Omega}' = -\vec{\nabla}'\pi_i' + \rho_i\upsilon_i\vec{\nabla}'^2\vec{V}_i' - \frac{\rho_i\upsilon_i}{K_i'}\vec{V}_i' \tag{1}$$

where

$$\pi' = P' - \frac{1}{2}\rho_i\left|\vec{\Omega}'\times\vec{r}'\right|^2 \qquad (i=1,2) \tag{2}$$

$i=1$ corresponds to the upper liquid, $i=2$ the lower liquid.

The equation of continuity is

$$\vec{\nabla}'\cdot\vec{V}_i' = 0 \tag{3}$$

In the above equations, $\vec{V}_i'$ are the velocities of the immiscible liquids, P' the pressure, ρ_i the density and υ_i the kinematic coefficient of viscosities of the two liquids, K_i the perembilities of the *porous media each having the dimension of* $(\text{length})^2$*, we choose the* z'*- direction to be parallel to the axis of the channel and* x'*-direction to be parallel to the axis of rotation. In this thesis, the cross section is taken to be circular. It is convenient to use the cylindrical polar system of coordinates* (r',θ,z') *where* θ *is the angle between the radius and the axis of rotation and* z' *is measured from the axis of pipe. Let* (u_i',v_i',w_i') *be the components of* $\vec{V}_i'$ *in the direction of* (r',θ,z')*. For fully developed laminar flow, those will be functions of* r' *and* θ *only.*

The equations (1), (3) referred to this coordinate system become,

$$-2\rho_1\Omega' w_1'\sin\theta+\rho_1\left(u_1'\frac{\partial u_1'}{\partial r'}+\frac{v_1'}{r'}\frac{\partial u_1'}{\partial\theta}-\frac{v_1'^2}{r'}\right)=\frac{-\partial\pi'}{\partial r'}+\rho_1\upsilon_1\left(\nabla'^2u_1'-\frac{u_1'}{r'^2}-\frac{2}{r'^2}\frac{\partial v_1'}{\partial\theta}\right)-\frac{\rho_1\upsilon_1u_1'}{K_1'} \quad (4)$$

$$-2\rho_1\Omega' w_1'\cos\theta+\rho_1\left(u_1'\frac{\partial v_1'}{\partial r'}+\frac{v_1'}{r'}\frac{\partial v_1'}{\partial\theta}+\frac{u_1'v_1'}{r'}\right)=-\frac{1}{r'}\frac{\partial\pi'}{\partial\theta}+\rho_1 v_1\left(\nabla'^2v_1'-\frac{v_1'}{r'^2}+\frac{2}{r'^2}\frac{\partial u_1'}{\partial\theta}\right)-\frac{\rho_1\upsilon_1}{K_1'}v_1' \quad (5)$$

$$2\rho_1\Omega'(u_1'\sin\theta+v_1'\cos\theta)+\rho_1\left(u_1'\frac{\partial w_1'}{\partial r'}+\frac{v_1'}{r'}\frac{\partial w_1'}{\partial\theta}\right)=-\frac{\partial\pi'}{\partial z'}+\rho_1\upsilon_1\nabla'^2w_1'-\frac{\rho_1\upsilon_1}{K_1'}w_1' \quad (6)$$

$$\frac{\partial u_1'}{\partial r'}+\frac{u_1'}{r'}+\frac{1}{r'}\frac{\partial v_1'}{\partial\theta'}=0 \quad (7)$$

where

$$\pi'=P'-\frac{1}{2}\rho_1\Omega'^2\left(r'^2\sin^2\theta+z'^2\right) \quad (8)$$

$$\nabla'^2=\frac{\partial^2}{\partial r'^2}+\frac{1}{r'}\frac{\partial}{\partial r'}+\frac{1}{r'^2}\frac{\partial^2}{\partial\theta^2} \quad (9)$$

For fully developed laminar flow, the form of π' *is restricted to*

$$\pi'=-\mathrm{c}z'+F(r',\theta) \quad (10)$$

where 'c' is a constant and may be termed as the gradient of π' *along the axis of the pipe.*

The above equations are the equations of motion of a viscous incompressible liquid characterised by viscosity υ_1 *and density* ρ_1 *occupying the space between* $r'=a\in$ *and* $r'=a$ *in the circular pipe for the upper liquid. Here* (u_1',v_1',w_1') *are the components of the velocity in the direction of* (r',θ,z') *where '*θ*' is the angle between the radius and the axis of rotation and* z' *is measured from the axis of the pipe.*

For fully developed laminar flow these will be the functions of radial (r') *and azimathul* (θ) *only for the lower liquid occupying the space between* $r'=0$ *to* $r'=a\in$ *of the circular pipe, we have the lower liquid is as follows*

$$-2\rho_2\Omega' w_2'\sin\theta+\rho_2\left(u_2'\frac{\partial u_2'}{\partial r'}+\frac{v_2'}{r'}\frac{\partial u_2'}{\partial\theta}-\frac{v_2'^2}{r'}\right)=\frac{-\partial\pi'}{\partial r'}+\rho_2\upsilon_2\left(\nabla'^2u_2'-\frac{u_2'}{r'^2}-\frac{2}{r'^2}\frac{\partial v_2'}{\partial\theta}\right)-\frac{\rho_2\upsilon_2}{K_2'}u_2' \quad (11)$$

$$-2\rho_2\Omega' w_2'\cos\theta+\rho_2\left(u_2'\frac{\partial v_2'}{\partial r'}+\frac{v_2'}{r'}\frac{\partial v_2'}{\partial\theta}+\frac{u_2'v_2'}{r'}\right)=-\frac{1}{r'}\frac{\partial\pi'}{\partial\theta}+\rho_2 v_2\left(\nabla'^2v_2'-\frac{v_2'}{r'^2}+\frac{2}{r'^2}\frac{\partial u_2'}{\partial\theta}\right)-\frac{\rho_2\upsilon_2v_2'}{K_2'} \quad (12)$$

$$2\rho_2\Omega'(u_2'\sin\theta+v_2'\cos\theta)+\rho_2\left(u_2'\frac{\partial w_2'}{\partial r'}+\frac{v_2'}{r'}\frac{\partial w_2'}{\partial\theta}\right)=-\frac{\partial\pi'}{\partial z'}+\rho_2\upsilon_2\nabla'^2w_2'-\frac{\rho_2\upsilon_2}{K_2'}w_2' \quad (13)$$

$$\frac{\partial u_2'}{\partial r'}+\frac{u_2'}{r'}+\frac{1}{r'}\frac{\partial v_2'}{\partial\theta'}=0 \quad (14)$$

where

$$\pi'=P'-\frac{1}{2}\rho_2\Omega'^2\left(r'^2\sin^2\theta+z'^2\right) \quad (15)$$

For the upper liquid, we introduce the stream function ϕ_1' *such that*

$$r'u'=-\frac{\partial\varphi_1'}{\partial\theta},\quad v'=\frac{\partial\varphi_1'}{\partial r'} \quad (16)$$

where ϕ_1' *is a function of* r' *and* θ *only. Eliminating* π' *from (4) and (5) and using (16), we get,*

$$-2\Omega'\left(D'_* w'_1\right)+\frac{1}{r'}\partial'\left(\phi'_1,\nabla'^2\phi'_1\right)=\upsilon_1\nabla'^4\phi'_1-\frac{\upsilon_1}{K'_1}\nabla'^2\phi'_1 \tag{17}$$

From equations (6) and (16) we get

$$2\Omega'\left(D'_*\phi'_1\right)+\frac{1}{r'}\partial'\left(\phi'_1,w'_1\right)=\frac{c}{\rho_1}+\upsilon_1\nabla'^2 w'_1-\frac{\upsilon_1 w'_1}{K'_1} \tag{18}$$

where
$$D'_*=\cos\theta\frac{\partial}{\partial r'}-\frac{\sin\theta}{r'}\frac{\partial}{\partial\theta} \tag{19}$$

and $\partial'(X,Y)$ *stands for the Jacobian of X and Y with respect to* r' *and* θ *respectively.*

$$\text{i.e., }\partial'(X,Y)=\frac{\partial(X,Y)}{\partial(r',\theta)} \tag{20}$$

For the lower liquid, following similar analysis, we get

$$-2\Omega'\left(D'_* w'_2\right)+\frac{1}{r'}\partial'\left(\phi'_2,\nabla'^2\phi'_2\right)=\upsilon_2\nabla'^4\phi'_2-\frac{\upsilon_2}{K'_2}\nabla'^2\phi'_2 \tag{21}$$

$$2\Omega'\left(D'_*\phi'_2\right)+\frac{1}{r'}\partial'\left(\phi'_2,w'_2\right)=\frac{c}{\rho_2}+\upsilon_2\nabla'^2 w'_2-\frac{\upsilon_2 w'_2}{K'_2} \tag{22}$$

we seek the solutions of the equations (17),(18),(21)and (22) subject to the boundary conditions for no slip at the walls i.e

$$u'_i=v'_i=w'_i=0 \quad \text{for } i=1 \text{ and } 2$$

or

$$\left.\begin{aligned} w'_1=0,\frac{\partial\varphi'_1}{\partial r'}=0,\frac{\partial\varphi'_1}{\partial\theta}=0\\ w'_2=0,\frac{\partial\varphi'_2}{\partial r'}=0,\frac{\partial\varphi'_2}{\partial\theta}=0\end{aligned}\right\} \tag{23}$$

Further we require that all the velocity componenets i,e $\frac{\partial\phi'_1}{\partial r'},\frac{\partial\phi'_2}{\partial r'},\frac{\partial\phi'_1}{\partial\theta},\frac{\partial\phi'_2}{\partial\theta}$ *must be finite over the cross section.*

In the absence of rotation the problem reduces to the saturated porous flow through a straight pipe under a pressure gradient. When the rotation is present , secondary flow is setup due to the interaction between the pressure gradient and the coriolis forces. The two velocity components of the secondary flow in the plane of the cross section have been expressed in terms of the stream function ϕ' *. Thus the determination of the velocity requires only the determination of the stream function* ϕ' *and the component* w' *(primary flow) of the velocity nomal to the plane of the cross section.A systematic method given below to determine an approximate solution for this flow.*

In terms of the non-dimensional variables (Dash denotes physical quantity whereas others such as r,w_1,ϕ_1 *, etc. are non-dimensional quantities)*

$$\left.\begin{aligned} &r'=ar,\ w'_1=\frac{c_1a^2w_1}{4\rho_1\upsilon_1},\ \varphi'_1=\frac{c_1a^3\varphi_1}{4\rho_1\upsilon_1},\ R_1=\frac{c_1a^3}{4\rho_1\upsilon_1^2}\\ &T_1=\frac{2\Omega'a^2}{\upsilon_1},\ S_1=\frac{a^2}{K'_1},\ w'_2=\frac{c_2a^2w_2}{4\rho_2\upsilon_2},\varphi'_2=\frac{c_2a^3\varphi_2}{4\rho_2\upsilon_2}\\ &R_2=\frac{c_2a^3}{4\rho_2\upsilon_2^2},\ T_2=\frac{2\Omega'a^2}{\upsilon_2},\ S_2=\frac{a^2}{K'_2}\end{aligned}\right\} \tag{24}$$

where c_1 *and* c_2 *are the pressure gradient,* R_1 *and* R_2 *stands for the Reynolds number of the upper and lower liquids respectively.* T_1 *and* T_2 *are the Taylor numbers for these liquid respectively and* S_1 *and* S_2 *are porous parameters.*

Using the above non-dimensional quantities (24), equations (17-22), we get.

Upper liquid:

$$T_1\left(D_*\phi_1\right)+\frac{R_1}{r}\partial\left(\phi_1,w_1\right)=4+\nabla^2 w_1-S_1 w_1 \tag{25}$$

$$-T_1\left(D_* w_1\right)+\frac{R_1}{r}\partial\left(\phi_1,\nabla^2\phi_1\right)=\nabla^4\phi_1-S_1\nabla^2\phi_1 \tag{26}$$

Lower liquid:

$$T_2\left(D_*\phi_2\right)+\frac{R_2}{r}\partial\left(\phi_2,w_2\right)=4+\nabla^2 w_2-S_2 w_2 \tag{27}$$

$$-T_2\left(D_* w_2\right)+\frac{R_2}{r}\partial\left(\phi_2,\nabla^2\phi_2\right)=\nabla^4\phi_2-S_2\nabla^2\phi_2 \tag{28}$$

where
$$D_*=\cos\theta\frac{\partial}{\partial r}-\frac{\sin\theta}{r}\frac{\partial}{\partial\theta},\ \partial\left(X,Y\right)=\frac{\partial\left(X,Y\right)}{\partial\left(r,\theta\right)} \tag{29}$$

To find the solution for w_1,ϕ_1,w_2,ϕ_2 we use the following boundary conditions for no slip at walls given in non-dimensional form using (23)

$$r=1,w_1=0\ \textit{and}\ \frac{\partial\phi_1}{\partial r}=\phi_1=0 \tag{30}$$

$$r=\in,-\frac{1}{r}\frac{\partial\phi_1}{\partial\theta}=-\frac{1}{r}\frac{\partial\phi_2}{\partial\theta},\frac{\partial\phi_1}{\partial r}=\frac{\partial\phi_2}{\partial r},w_1=w_2$$

$$\frac{\partial w_1}{\partial r}=L\frac{\partial w_2}{\partial r}\ \textit{where}\ L=\frac{\mu_2}{\mu_1} \tag{31}$$

Equation (31) follows from $\mu_1\frac{\partial w_1'}{\partial r}=\mu_2\frac{\partial w_2'}{\partial r'}$

$$r=0,-\frac{1}{r}\frac{\partial\phi_2}{\partial\theta},\frac{\partial\phi_2}{\partial r}\ \textit{and}\ w_2\ \textit{are finite} \tag{32}$$

To solve the equations of motion, we assume

$$\phi_1=T_1\phi_{11}+T_1^2\phi_{12}+\ldots. \tag{33}$$

$$w_1=w_{10}+T_1 w_{11}+T_1^2 w_{12}+\ldots. \tag{34}$$

$$\phi_2=T_2\phi_{21}+T_2^2\phi_{22}+\ldots. \tag{35}$$

$$w_2=w_{20}+T_2 w_{21}+T_2^2 w_{22}+\ldots. \tag{36}$$

where the subscripts 1 and 2 stand for upper $\left(\in\le r\le 1\right)$ *and lower liquids* $\left(0\le r\le\in\right)$ *for* ϕ *and w and further for* $S_1\ll 1$ *and* $S_2\ll 1$*, we expand*

$$\varphi_{11}=\varphi_{11}^{(0)}+S_1\varphi_{11}^{(1)}+S_1^2\varphi_{11}^{(2)}+o(S_1^{\,3}) \tag{37}$$

$$\varphi_{12}=\varphi_{12}^{(0)}+S_1\varphi_{12}^{(1)}+S_1^2\varphi_{12}^{(2)}+o(S_1^{\,3}) \tag{38}$$

$$w_{10}=w_{10}^{(0)}+S_1 w_{10}^{(1)}+S_1^2 w_{10}^{(2)}+o(S_1^{\,3}) \tag{39}$$

$$w_{11}=w_{11}^{(0)}+S_1 w_{11}^{(1)}+S_1^2 w_{11}^{(2)}+o(S_1^{\,3}) \tag{40}$$

$$w_{12}=w_{12}^{(0)}+S_1 w_{12}^{(1)}+S_1^2 w_{12}^{(2)}+o(S_1^{\,3}) \tag{41}$$

$$\varphi_{21}=\varphi_{21}^{(0)}+S_2\varphi_{21}^{(1)}+S_2^2\varphi_{21}^{(2)}+o(S_2^{\,3}) \tag{42}$$

$$\varphi_{22}=\varphi_{22}^{(0)}+S_2\varphi_{22}^{(1)}+S_2^2\varphi_{22}^{(2)}+o(S_2^{\,3}) \tag{43}$$

$$w_{20}=w_{20}^{(0)}+S_2 w_{20}^{(1)}+S_2^2 w_{20}^{(2)}+o(S_2^{\,3}) \tag{44}$$

$$w_{21}=w_{21}^{(0)}+S_2 w_{21}^{(1)}+S_2^2 w_{22}^{(2)}+o(S_2^{\,3}) \tag{45}$$

$$w_{22}=w_{22}^{(0)}+S_2 w_{22}^{(1)}+S_2^2 w_{22}^{(2)}+o(S_2^{\,3}) \tag{46}$$

The 2nd subscript stands for the order of the Taylor's number where as the super script within the brackets accounts for the order of the porous parameter.

Substituting equations (33) and (34) in equation (25), equation (35) and (36) in equation (27) and to the zeroth power in both T_1 and T_2, we get

$$\nabla^2 w_{10}-S_1 w_{10}=-4\ \ldots \tag{47}$$

$$\nabla^2 w_{20}-S_2 w_{20}=-4\ \ldots \tag{48}$$

$$w_{10}^{(0)} = 1 + 2 \in^2 (1-L)\log r - r^2$$
$$w_{20}^{(0)} = 1 + 2 \in^2 (1-L)\log \in - r^2$$

$$w_{10}^{(1)} = -\frac{3}{16} + \frac{\in^2}{2}(1-L) + \left[\frac{\in^2}{2}(L-1) - \frac{\in^4}{4}(L-1) + \in^4 (1-L)(\log \in -1)(L-1) + \frac{\in^4}{2}(1-L)(L-1)\right]$$
$$\log r + \frac{r^2}{4} - \frac{r^4}{16} + 2\in^2 (1-L)\frac{r^2}{4}(\log r - 1)$$

$$w_{20}^{(1)} = -\frac{3}{16} + \frac{\in^2}{2}(1-L) + \left[\frac{\in^2}{2}(L-1) - \frac{\in^4}{4}(L-1) + \in^4 (1-L)(L-1)(\log \in -1) + \frac{\in^4}{2}(1-L)(L-1)\right]$$
$$\log \in + \frac{r^2}{4} - \frac{r^4}{16} + 2\in^2 (1-L)\frac{r^2}{4}(\log r - 1)$$

$$g_{11}^{(0)} = \frac{1}{192}\left[(A_1 + A_2 \log r)r + \frac{A_3}{r} + A_4 r^3 + 2r^5 - 24\in^2 (1-L)r^3 \log r\right]$$

$$g_{21}^{(0)} = \frac{r}{96}\left(\in^2 - r^2\right)^2$$

$$g_{11}^{(1)} = \frac{1}{192}\left[(C_6 + C_7 \log r)r + \frac{C_8}{r} + C_9 r^3 + A_1\left(\frac{r^3 \log r}{4} + \frac{r^5}{12}\right)\right.$$
$$+\frac{r^7}{24} - \frac{\in^2}{8}(1-L)r^5\left(\log r - \frac{7}{6}\right) - \frac{3}{4}\in^2 (1-L)r^5$$
$$-\left(\frac{\in^2 (L-1)}{2} - \frac{\in^4}{4}(L-1) + \in^4 (1-L)(\log \in -1)(L-1) + \frac{\in^4}{2}(1-L)(L-1)\right)$$
$$\left.\left(12r^3 + \frac{r^5}{2} - \frac{r^7}{24} + \in^2 (1-L)r^5\left(\log r - \frac{7}{6}\right) - \frac{\in^2}{2}(1-L)\right)r^5\right]$$

$$g_{21}^{(1)} = \frac{1}{192}\begin{bmatrix}(D_1 + D_2 \log r)r + \frac{D_3}{r} + D_4 r^3 - \frac{\in^2}{6}r^5 + \frac{r^7}{24} - \frac{r^5}{2} + \frac{r^7}{24} \\ +\frac{\in^2}{2}(1-L)r^5 - \in^2 (1-L)r^5\left(\log r - \frac{7}{6}\right)\end{bmatrix}$$

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$$+\frac{r^7}{24} - \frac{\in^2}{8}(1-L)r^5\left(\log r - \frac{7}{6}\right) - \frac{3}{4}\in^2 (1-L)r^5$$
$$-\left(\frac{\in^2 (L-1)}{2} - \frac{\in^4}{4}(L-1) + \in^4 (1-L)(\log \in -1)(L-1) + \frac{\in^4}{2}(1-L)(L-1)\right)$$
$$\left.\left(12r^3 + \frac{r^5}{2} - \frac{r^7}{24} + \in^2 (1-L)r^5\left(\log r - \frac{7}{6}\right) - \frac{\in^2}{2}(1-L)\right)r^5\right]$$

$$g_{21}^{(1)} = \frac{1}{192}\begin{bmatrix}(D_1 + D_2 \log r)r + \frac{D_3}{r} + D_4 r^3 - \frac{\in^2}{6}r^5 + \frac{r^7}{24} - \frac{r^5}{2} + \frac{r^7}{24} \\ +\frac{\in^2}{2}(1-L)r^5 - \in^2 (1-L)r^5\left(\log r - \frac{7}{6}\right)\end{bmatrix}$$

$$w_{11}^{(1)} = X_1 r + \frac{X_2}{r} + \frac{R_1}{192}\Big[f_1 r\log r + f_2 r^3\log r + f_3 r^5\log r + f_4 r^7\log r + f_5 r(\log r)^2 + f_6 r^3(\log r)^2 + f_7 r^5(\log r)^2 + f_8\frac{\log r}{r} + f_9 r^3 + f_{10} r^5 + f_{11} r^7 + f_{12} r^9\Big]$$

$$w_{21}^{(1)} = Y_1 r + \frac{Y_2}{r} + \frac{R_2}{192}\left[f_{13} r\log r + f_{14} r^3\log r + f_{15} r^5\log r + f_{16} r^7\log r + f_{17} r^3 + f_{18} r^5 + f_{19} r^7 + f_{20} r^9\right]$$

RESULTS AND DISCUSSION

In the Central plane perpendicular to the axis of rotation $\theta = \frac{\pi}{2}$ *or* $\frac{3\pi}{2}$, *it can be seen from the equations (13),* v' *= 0 in either case. So a particle of liquid once in this plane does not leave it in the subsequent motion. The motion in two halves of the pipe is therefore quite distinct from each other.*

The differential equation of the stream line in the central plane of the pipe is

$$\frac{dr'}{u'} = \frac{dz'}{w'}\left(\theta = \frac{\pi}{2}\text{ or }\frac{3\pi}{2}\right)$$

To a sufficient approximation these stream lines for the lower liquid are given by

$$\pm\frac{T_2(z'-z_0')}{48a} = \left(\frac{l_1-\epsilon^2}{\epsilon^3}\right)\left[\frac{r\epsilon}{\epsilon^2-r^2} + \log\tan\left(\frac{\pi}{4}+\frac{\alpha}{2}\right)\right] + \frac{1}{\epsilon}\log\frac{\epsilon+r}{\epsilon-r} +$$

$$S_2\left[\begin{array}{l}\frac{l_2}{\epsilon^3}\left(\frac{r\epsilon}{\epsilon^2-r^2}+\log\tan\left(\frac{\pi}{4}+\frac{\alpha}{2}\right)\right)+\\ \frac{1-2\epsilon^2(1-L)}{4}\left(-\frac{1}{\epsilon}\log\frac{\epsilon+r}{\epsilon-r}+\frac{1}{r}\left(\frac{r\epsilon}{\epsilon^2-r^2}+\log\tan\left(\frac{\alpha}{2}+\frac{\pi}{4}\right)\right)\right)-\\ \frac{1}{16}\left(\frac{r\epsilon^2}{\epsilon^2-r^2}+\epsilon\log\tan\left(\frac{\alpha}{2}+\frac{\pi}{4}\right)+2r-2\epsilon\log\frac{\epsilon+r}{\epsilon-r}+\epsilon^2(1-L)\int_0^r\frac{r^2\log r}{\left(\epsilon^2-r^2\right)^2}dr\right)\end{array}\right]$$

Table 1 gives the values of $\left(\frac{(z'-z_0')T_2}{48a}\right)$ *from the above equation taking L = 1.1 and* S_2 *= 0.5. It shows the same stream line in the plane of symmetry for* ϵ*=0.5,0.6,0.75,0.9 and* ϵ*=1. We note that no stream line in the central plane can ever reach the edge of the pipe. As the angular velocity* Ω' *is increased , the distance which must be covered by the central stream line to be within a given distance from the edge gets smaller; this result holds good for all values of* ϵ *between 0.5 and 1.*

Table 1 : Values of stream lines for the lower liquid

r ↓	ϵ / S_2 ↓	0.5	0.6	0.75	0.9	1
0.1	0	3.391635	1.6249175	0.6582208	0.3115829	0.20067
	0.5	3.034739	1.4597205	0.588324	0.27667175	0.138658
0.2	0	7.34312	3.415409	1.352609	0.6328003	0.40546
	0.5	6.6032925	3.205653	1.2065365	0.5661673	0.35981375
0.3	0	13.00858	5.640874	2.131843	0.9751172	0.61903
	0.5	11.819994	4.7060745	2.10858705	0.8781637	0.554328
0.4	0	25.24018	8.962895	3.078641	1.3547032	0.84729
	0.5	23.4528235	8.310087	2.762295	1.2367872	0.769416

For a fixed value of T_2 *the effect of the porosity is to decrease the distance that the liquid particle in the central plane travel in going from points near* ϵ *to points* ϵ *= 1, also the effect the porosity is to decrease the same as* ϵ *moves from 0.5 to 1 at any r.*

The differential equation for any stream line is given by

$$\frac{dr'}{u'} = \frac{r'd\theta}{v'} = \frac{dz'}{w'} \tag{49}$$

From the first relation, we obtain for the curves of intersection of constant surface with a section z' *= constant, the polar equation for the lower liquid*

$$96k\sec\theta = g_{21}^{(0)} + S_2 g_{21}^{(1)} \tag{50}$$

$$k\sec\theta = r\left(1-r^2\right)^2\left[1+\frac{S_2}{24}\left(-6+r^2\right)\right]$$

$$\frac{\partial}{\partial r}g_{21}^{(0)} + S_2\frac{\partial}{\partial r}g_{21}^{(1)} = 0$$

$$\frac{\in^4 -6r^2 \in^2 +5r^4}{96} + S_2\frac{(D_1 - \frac{D_3}{r^2} + r^4(\frac{-5}{6}\in^2 + \frac{-5}{2} + \frac{17}{3}\in^2 (1-L)) + r^6\frac{7}{12} - 5\in^2 (1-L)r^4\log r)}{192} = 0$$

Table 2 shows the values of r and k of the above equation (2.65) and (2.66). For $S_2 = 0,\ 0.5;$ $\in = 0.5, 1$ *together with the values of k, obtained by substitution of the values in the previous equation when* $\theta = 0$*. It is concluded that, as porosity increases the point.*

Table 2:- values of r and k

S_2	ε	r	k
0	0.5	0.224	0.00009
0.5	0.5	0.225	0.00008
0	1	0.447	0.29
0.5	1	0.749	0.13

Corresponding to degenerate stream lines moves away from the dividing surface $\in$*. As r increases the value of k decreases also. However the plots of the polar equation for different values of k have not been attempted, in view of the plots being very close to each other, but the conclusion remain the same. To obtain the relation between* θ *and* z'*, we consider from the equation (49)*

$$\frac{r' d\theta}{v'} = \frac{dz'}{w'}$$

For points r $\to \in$ *and under sufficient approximation, we get for lower liquid*

$$\frac{-(z'-z_1')}{\frac{24a}{T_2}} = \frac{1}{\phi_{21}'^{(0)}}\left[w_{20}^{(0)}(1) + S_2 w_{20}^{(1)}(1)\right]\log\left\{\tan\frac{\pi}{4}+\frac{\theta}{2}\right\}$$

$$z'-z_1' = -\frac{24a}{T_2}\log\left\{\tan\frac{\pi}{4}+\frac{\theta}{2}\right\}\left\{\frac{8\in^2(1-L)\log\in}{5-6\in^2+\in^4} + S_2\frac{i_2+\frac{3}{16}-\frac{\in^2}{2}(1-L)}{5-6\in^2+\in^4}\right\} \tag{51}$$

$z_1' = z'$ *where* $\theta = 0$ *Also* $\in \to 1$*, L* $\to 0$*, we recover the work of Ramana Rao (1970) for the non magnetic and non porous case.*

Table 3 gives the values of $\frac{(z'-z_1')}{\left(\frac{-24a}{T_2}\right)}$ *from equation (51)*

Table 3:- Values of stream lines (central plane)for the lower liquid

θ	ε = 0.5 L = 1.1	ε = 0.6 L = 1.1	ε = 0.75 L = 1.1	ε = 0.9 L = 1.1	ε = 1 L = 1.1
0°	0	0	0	0	0
10°	0.004797815	0.007420925	0.01051724	0.01374082	0.1534998
20°	0.00974682	0.01568470	0.0213659	0.02791257	0.3118312
30°	0.01502335	0.0241763	0.0329325	0.04302661	0.480643
40°	0.02086527	0.0335771	0.0457382	0.0597596	0.667546
50°	0.0276419	0.04448275	0.0605934	0.07916795	0.8843478
60°	0.0360184	0.05796269	0.07895544	0.103159	1.152338
70°	0.0474630	0.07638005	0.10404318	0.1359373	1.518488
80°	0.06663045	0.1072253	0.14606001	0.19083431	2.131715

Table 3 shows the distance which must be covered before a liquid particle at $\theta = \alpha$ *increases and tends to infinite for all values of* S_2 *as* α *tends to* $\frac{\Pi}{2}$*. The effect of the porous parameters is to* *displace the stream lines towards to the edge of the pipe. $S_2 = 0$ (in the non-porous case) we recover the work of Patrudu (2003).*

CONCLUSIONS

- *The motion in two halves of the pipe are therefore quite distinct from each other.*
- *No stream line in the central plane can ever reach the edge of the pipe. As the angular velocity is increased , the distance which must be covered by the central stream line to be within a given distance from the edge gets smaller.*
- *For a fixed value of T_2 the effect of the porosity is to decrease the distance that the liquid particle in the central plane travel in going from points near $\in$ to points $\in = 1$, also the effect the porosity is to decrease the same as $\in$ moves from 0.5 to 1 at any r.*
- *The effect of the porous parameters is to displace the stream lines towards to the edge of the pipe.*

Nomenclature

u_1' Primary velocity of the upper liquid, u_2' Primary velocity of the lower liquid, v_1' Secondary velocity of the upper liquid, v_2' Secondary velocity of the lower liquid, w_1' Axial velocity of the upper liquid, w_2' Axial velocity of the lower liquid, r' Radius of the pipe, μ_2 Viscosity of the lower liquid, θ Angle between the radius and the axis of rotation (azimathul), π' Pressure gradient, Ω' Angular velocity, ρ_1' Density of the upper liquid, ρ_2' Density of the lower liquid, υ_1 Kinematic viscosity of the upper liquid, υ_2 Kinematic viscosity of the lower liquid, T_1 Taylor number of the upper liquid, T_2 Taylor number of the lower liquid, R_1 Reynolds number of the upper liquid, R_2 Reynolds number of the lower liquid, w_1 Axial velocity for the upper liquid. w_2 Axial velocity for the lower liquid, ϕ_1 Stream function for the upper liquid in non-dimensional form, ϕ_2 Stream function for the lower liquid in non-dimensional form, z' Measured from the axis of the pipe, P' Pressure, c Constant and may be termed as the gradient of π' along the axis of the pipe, $\in$ Non-dimensional parameter, μ_1 Viscosity of the upper liquid, r Radius of the pipe in non-dimensional form, K_1' Permeability of porous medium of the upper liquid. K_2' Permeability of porous medium of the lower liquid.

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