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1. R. N. BARIK

MHD MIXED CONVECTION FLOW AND HEAT TRANSFER IN A POROUS MEDIUM

1. DEPARTMENT OF MATHEMATICS, TRIDENT ACADEMY OF TECHNOLOGY, INFOCITY, BHUBANESWAR-751024, ODISHA, INDIA

ABSTRACT: The effects of steady two dimensional laminar MHD mixed convection flow and heat transfer against a heated vertical semi-infinite permeable surface in a porous medium has been discussed. The coupled nonlinear partial differential equations describing the conservation of mass, momentum and energy are solved by perturbation technique. The results are presented to illustrate the influence of Hartmann number(M), Prandtl number(Pr), permeability parameter(K_p), suction/blowing parameter(f_w), heat generation/absorption co-efficient(ϕ) and mixed convection or buoyancy parameter(γ). The effects of different parameters on the velocity and temperature as well as the skin friction and wall heat transfer are discussed with the help of figures.
KEYWORDS: MHD flow; Mixed convection; Porous medium; Heat transfer; Stagnation point; Skin-friction

INTRODUCTION

Stagnation point flow has become an interesting area of research due to its varied applications both in industrial and scientific applications such as extrusion of polymers, cooling of metallic plates, aerodynamics plastic extrusion, glass blowing and fiber spinning etc. The two-dimensional flow of a fluid near a stagnation point is a classical problem in fluid dynamics. The plane and axisymmetric flow near a stagnation point on a surface have attracted many investigators during the past several decades because of its wide applications such as cooling of electronic devices by fans, cooling of nuclear reactors and many hydrodynamic processes. Hiemenz[1] has been investigated the two-dimensional stagnation flow over a plate and developed an exact solution to the Navier-Stokes equations. Raptis et al.[2] have presented the steady forced convection flow through a porous medium bounded by a semi-infinite plate when the fluid is viscous and the free stream velocity is not constant. The combined forced and free convection in stagnation flows becomes important as the buoyancy forces owing to the temperature differences between the surface and the free stream is large. The steady two-dimensional mixed convection of an incompressible fluid in a porous medium past a hot vertical impermeable plate is analyzed by Takhar et al.[3].

In many physical situations, the heat generation or absorption effects in the fluid are greatly dependent on temperature. Sparrow et al.[4] have discussed the temperature dependent heat sources or sinks in a stagnation point flow. Chamkha[5] has investigated steady two dimensional mixed convection flows of an electrically conducting and heat absorbing fluid near a stagnation point on a semi-infinite vertical permeable surface at arbitrary surface heat flux variations in the presence of a magnetic field. Gorla et al.[6] have discussed mixed convection in stagnation flows of micropolar fluids over vertical surfaces with non-uniform surface heat flux. Yih[7] has presented numerically the effect of heat source/sink on steady two- dimensional laminar MHD mixed convection owing to the stagnation flow against a vertical permeable flat plate with linear wall temperature in a fluid saturated porous medium. Wu et al.[8] have analyzed the stagnation point flow in porous media and presented non-linear exact and asymptotic solutions. Abdelkhalek[9] has discussed the skin friction in the MHD mixed convection stagnation point with mass transfer. Kumaran et al.[10] have employed a new implicit perturbation scheme to obtain approximate solution to stagnation point flow in porous media. Attia[11] has discussed the stagnation point flow and heat transfer of a micropolar fluid with uniform suction or blowing. Bachok et al.[12] have analyzed the MHD stagnation point flow of a micropolar fluid with prescribed wall heat flux. Singh et al.[13] have discussed the effects of volumetric heat generation/absorption on mixed convection stagnation point flow on an iso-thermal vertical plate in porous media. Dubey at el.[14] have studied the mixed convection of non-Newtonian fluids through porous medium along a heated vertical flat plate with magnetic. Recently, the mixed convection boundary layer flow past a vertical plate in porous medium with viscous dissipation and variable permeability has studied by Singh [15]. Fan et al.[16] have investigated the mixed convection heat transfer in horizontal channel filled with nanofluids.

The objective of the present study is to investigate the MHD mixed convection stagnation point flow and heat transfer in a porous medium in presence of a transverse uniform magnetic field. It is necessary to study the free convection effects on the flow through porous medium and to estimate its effect on the heat transfer which is one of the objectives of the present study. A few problems have been solved on buoyancy assisting flow with constant wall heat flux i.e. linear variation of wall temperature. The above cases referring Abdelkhalek [9] allowing the flow through a porous medium with an additional constraint has been considered.

MATHEMATICAL FORMULATION

Consider two-dimensional steady, laminar, hydromagnetic, mixed convection stagnation point flow impinging on a heated vertical semi-infinite permeable surface. The fluid is assumed Newtonian, viscous, electrically conducting and generates or absorbs heat at uniform rate. The y-axis is taken along the plate and the x-axis is normal to it. A uniform magnetic field is applied in the x-direction causing a flow resistive force in the y-direction. When the electrical conductivity is large i.e. for large magnetic Reynolds number, the diffusion of the magnetic field takes place in a narrow zone called the magnetic boundary layer and is of the same size as that of viscous and thermal boundary layers. In this case boundary layer equations for incompressible flow, for velocity and magnetic field must be solved simultaneously. When the electrical conductivity of the fluid is small i.e. for small magnetic Reynolds number, the thickness of the magnetic boundary layer is very large. In this case the flow direction component of the magnetic interaction of the corresponding Joule heating is only a function of the transverse magnetic field and the local velocity in the flow direction. Changes in the transverse magnetic field component and pressure across the boundary layer are negligible. The induced magnetic field is neglected in comparison with the applied magnetic field, which is taken in the transverse direction. Moreover, it is the transverse component of the magnetic field which affects the motion appreciably. The magnetic Reynolds number is assumed to be small, so that the induced magnetic field will be neglected. The free stream is moving with a uniform velocity U_∞ and is at a constant temperature T_∞. The permeable plate or surface is subjected to heat flux $q_w(x)$ and uniform suction or blowing. Under these conditions and taking into account the Boussinesq and the boundary layer approximation, the system of continuity, momentum and energy equations for a Darcyan viscous flow can be written as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0{}^2}{\rho}(u-U_\infty)-\frac{\upsilon}{K}(u-U_\infty)+zg\beta(T-T_\infty) \quad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+\frac{Q_0}{\rho C_p}(T-T_\infty) \quad (3)$$

It should be noted that in writing equations (1)-(3), the magnetic Reynolds number is assumed small so that the induced magnetic field is neglected. Also, the Hall effect of magnetohyrdodynamics, Joulean heating and the viscous dissipation are neglected. The heat generation or absorption term (last term of equation (3)) is assumed to vary linearly with the difference of the fluid temperature in the boundary layer and the ambient temperature.

The boundary conditions are

$$y=0:\ u(x)=0,\ v(y)=-v_0,\ \frac{\partial T(x)}{\partial y}=\frac{-q_w(x)}{K_f}=\frac{-a\,x^n}{K_f}\ ,\quad y\to\infty:\ \ u(x)\to U_\infty,\ \ T(x)\to T_\infty \quad (4)$$

Equations (2) and (3) are reduced to ordinary differential equations by the similarity transformation technique, which requires the introduction of the stream function ψ through the relations

$$u=\psi_y,\quad v=-\psi_x \quad (5)$$

In order to transform the partial differential equations into ordinary differential equations we introduced a new set of independent and dependent variables defined by

$$\psi=(\upsilon b)^{1/2}xf(\eta),\ \ \eta=\left(\frac{b}{\upsilon}\right)^{1/2}y,\ \ \gamma=Gr_x/(\mathrm{Re}_x)^{5/2},\ T=\left(\frac{q_w x}{K_f}\right)\mathrm{Re}_x{}^{-1/2}\,\theta+T_\infty \quad (6)$$

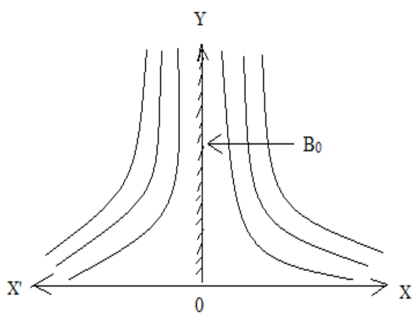

Diagram 1. Flow diagram

Equations (2) and (3) now reduced to the following coupled ordinary differential equations

$$f'''+ff''-(M+f'+\frac{1}{K_p})f'=-z\gamma\theta-(M+\frac{1}{K_p})m \quad (7)$$

$$\frac{1}{\Pr}\theta'' + f\theta' + \phi\theta = f'\theta \tag{8}$$

The appropriate flat plate, free convection boundary conditions are also transformed into the form

$$\eta = 0 \;:\quad f = f_w,\; f' = 0,\; \theta' = -1\;;\qquad \eta \to \infty:\; f' \to 0,\; \theta \to 0 \tag{9}$$

In the above equations, primes denote differentiation with respect to η, $\Pr = \upsilon/\alpha$ *is the Prandtl number,* $M = \sigma B_0^{\,2}/(b\rho)$ *is the magnetic parameter,* $K_p = bK/\upsilon$ *is the permeability parameter,* $\mathrm{Re}_x = bx^2/\upsilon$ *is the local Reynolds number,* $m = U_\infty/(bx)$, $Gr_x = g\beta q_w \mathrm{Re}_x^{\,2}/(b^2K_f)$ *is the local Grashoff number,* $\phi = Q_0/(b\rho C_p)$ *is the heat generation or absorption coefficient,* $\gamma = Gr_x/(\mathrm{Re}_x)^{5/2}$ *is the buoyancy parameter and* $f_w = v_0/\sqrt{\upsilon b}$ *is the suction parameter.*

The resulting differential equations contain arbitrary parameters, the Prandtl number Pr, the magnetic parameter M, the porosity parameter K_p *and the buoyancy parameter* γ. *Bhatanagar and Palekar[17] the solutions of the resulting semi-infinite domain, nonlinear equations are accomplished with a three part series method. The series for velocity and temperature are given below:*

$$f = k_0 + \varepsilon f_1 + \varepsilon^2 f_2 + \varepsilon^3 f_3 + \dots\dots\dots\dots\dots\dots\dots\dots \tag{10}$$

$$\theta = \varepsilon\theta_1 + \varepsilon^2\theta_2 + \varepsilon^3\theta_3 + \dots\dots\dots\dots\dots\dots\dots\dots\dots \tag{11}$$

The employed series, equation (10), contains term k_0 *that satisfies the boundary conditions and differential equations at infinity, the second term satisfies the boundary conditions at zero and is the solution to the initial homogeneous differential equation and the additional terms that are utilized to obtain increased numerical accuracy. This accuracy is limited by the number of terms that will not initiate divergence of the numerical results.*

The corresponding boundary conditions are

$$\begin{gathered}\eta = 0:\; f_1 = f_w,\; f_2 = 0, f_3 = 0\;\;,\; f_1' = 0, f_2' = 0,\; f_3' = 0\;\;\; \theta_1' = -1, \theta_2' = 0, \theta_3' = 0\\ \eta \to \infty:\;\; f_1' \to 0, f_2' \to 0, f_3' \to 0\;\;,\;\; \theta_1 \to 0, \theta_2 \to 0, \theta_3 \to 0\end{gathered} \tag{12}$$

The temperature representation i.e. equation (11) along with equation (10) and the associated boundary conditions (12) contain an undetermined parameter ε *which helps in the collection of terms for each set of the resulting differential equations. Substituting the series given in equations (10) and (11) in equations (7) and (8) respectively and equating the like powers of* ε, *we get the following equations.*

$$f_1''' + k_0 f_1'' - (M + \frac{1}{K_p}) f_1' = -z\gamma\theta_1 \tag{13}$$

$$\frac{1}{\Pr}\theta_1'' + k_0\theta_1' + \phi\theta_1 = 0 \tag{14}$$

$$f_2''' + k_0 f_2'' - (M + \frac{1}{K_p}) f_2' = -f_1 f_1'' + (f_1')^2 - z\gamma\theta_2 \tag{15}$$

$$\frac{1}{\Pr}\theta_2'' + k_0\theta_2' + \phi\theta_2 = f_1'\theta_1 - f_1\theta_1' \tag{16}$$

$$f_3''' + k_0 f_3'' - (M + \frac{1}{K_p}) f_3' = -f_1 f_2'' - f_1'' f_2 + 2 f_1' f_2' - z\gamma\theta_3 \tag{17}$$

$$\frac{1}{\Pr}\theta_3'' + k_0\theta_3' + \phi\theta_3 = -f_1\theta_2' - f_2\theta_1' + f_1'\theta_2 + f_2'\theta_1 \tag{18}$$

Solutions of the above equations are given by

$$\theta_1 = \frac{1}{a_1} e^{-a_1\eta} \tag{19}$$

$$f_1 = a_4 + a_5 e^{-a_3\eta} + a_2 e^{-a_1\eta} \tag{20}$$

$$\theta_2 = a_7 e^{-a_1\eta} + a_8 e^{-(a_1+a_3)\eta} + a_9\eta e^{-a_1\eta} \tag{21}$$

$$f_2 = a_{20} + a_{19} e^{-a_3\eta} + a_{18} e^{-a_1\eta} + a_{15}\eta e^{-a_3\eta} + a_{16} e^{-(a_1+a_3)\eta} \tag{22}$$

$$\theta_3 = (a_{28}\eta^2 + a_{29}\eta + a_{35}) e^{-a_1\eta} + (a_{30}\eta + a_{31}) e^{-(a_1+a_3)\eta} + a_{32} e^{-2a_1\eta} + a_{33} e^{-(a_1+2a_3)\eta} + a_{34} e^{-(a_3+2a_1)\eta} \tag{23}$$

$$\begin{gathered} f_3 = a_{55} + (a_{46}\eta^2 + a_{47}\eta + a_{53}) e^{-a_3\eta} + (a_{48}\eta^2 + a_{49}\eta + a_{50}) e^{-a_1\eta} + (a_{51}\eta + a_{52}) e^{-(a_1+a_3)\eta} \\ + a_{56} e^{-2a_1\eta} + a_{57} e^{-(a_1+2a_3)\eta} + a_{58} e^{-(a_3+2a_1)\eta}\end{gathered} \tag{24}$$

Substituting equations (19)-(24) into equations (10) and (11), we can get the required representation for f and θ. The constant k_0 is determined by satisfying the boundary condition f(0) and is a function of Pr and M. Getting the velocity and the temperature expression, we can calculate the skin friction and the rate of heat transfer in terms of Nusselt number.

RESULTS AND DISCUSSION

The steady two dimensional MHD mixed convection stagnation point flow past a heated vertical semi-infinite permeable surface embedded in a porous medium is carried out for different pertinent parameters. The main aim of the discussion is to bring out the effect of buoyancy assisting parameter (z) and linear non-uniform wall heat flux parameter (n) in the presence of uniform porous matrix and uniform magnetic field inducing a flow resistive force in the y-direction. Further, the effect of buoyancy parameter is also to be investigated.

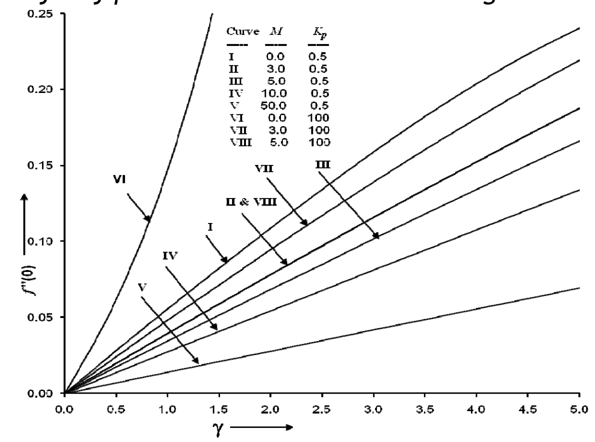

Fig-1 Effect of M & K_p on non-dimensional wall velocity gradient when Pr=0.7, f_w=0.0, k =1.3592 and η=0.0

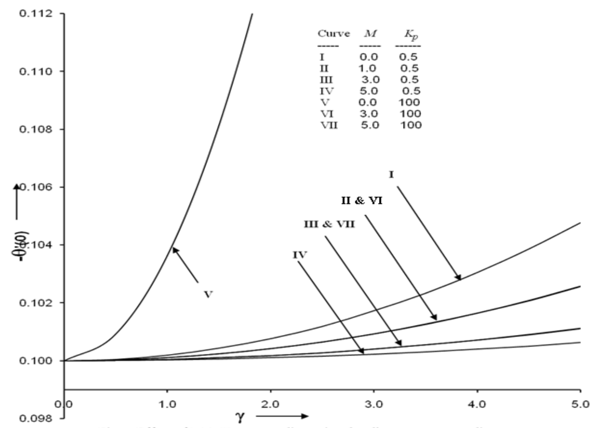

Fig-2 Effect of M & K_p on non-dimensional wall temperature gradient when Pr=0.7, f_w=0.0, k =1.3592, ϕ=0.0 and η=0.0

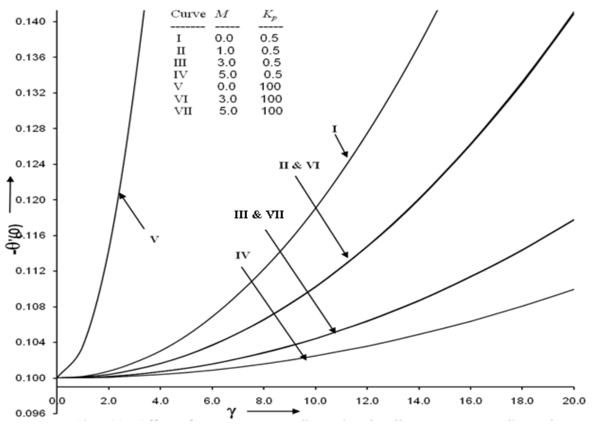

Fig-2(a) Effect of M & K_p on non-dimensional wall temperature gradient when Pr=0.7, f_w=0.0, k =1.3592, ϕ=0.0 and η=0.0

Figs.1 and 2 depict the effects of buoyancy parameter (γ) and magnetic parameter (M) on the non-dimensional surface velocity gradient $f''(0)$ and non-dimensional wall temperature gradient $-\theta'(0)$ respectively with and without porous matrix. As magnetic parameter increases, the surface velocity gradient as well as the wall temperature gradient decreases. In absence of magnetic field and porous matrix (M=0.0 and K_p=100), a sudden rise in the surface velocity gradient and the wall temperature gradient is marked (Curve-VI of fig.1 and Curve-V of fig.2). In absence of porous matrix, an increase in magnetic field leads to decrease the velocity gradient as well as the wall temperature gradient which is in good agreement with Abdelkhalek [9](Table-1). It is concluded that the imposed magnetic field reduces the velocity field as well as temperature field. Due to onset of free convection current and an increasing value of buoyancy parameter for buoyancy assisting flow, the surface velocity gradient and wall temperature gradient increases which means velocity and temperature distribution decreases. This may be attributed to the cooling of the plate. From the figure-2 it is clear that an increase in γ (increase in Gr_x), leads to an increase in the temperature gradient($-\theta'(0)$) at the plate.

Fig.2(a) shows the effect of buoyancy parameter (γ) and magnetic parameter (M) on the non-dimensional wall temperature gradient $-\theta'(0)$ with and without porous matrix. It is marked that for larger value of the buoyancy parameter, the change in wall temperature gradient follows the same trend as that of smaller value in fig.2.

Table-1

	Present results		Abdelkhalek[13]	
M	f''(0)	-θ'(0)	f''(0)	-θ'(0)
0	0.05548	0.10019	0.14707	0.10362
3	0.03946	0.10004	0.04799	0.10010
5	0.03438	0.10002	0.03943	0.10004

Table-2

	Present results		Abdelkhalek[13]	
P_r	f''(0)	-θ'(0)	f''(0)	-θ'(0)
0.70	0.03946	0.10004	0.04799	0.10010
100.00	0.00001	0.10000	0.00001	0.10000

Figs.3 and 4 illustrates the various values of the Prandtl number (Pr) on the non-dimensional wall velocity gradient and non-dimensional wall temperature gradient profiles respectively in presence and absence of porous matrix. An increase in Prandtl number reduces the thermal boundary layer thickness along the plate. This gives a reduction in the fluid temperature. This trend is due to the fluid with higher viscosity and this causes a reduction in shear stress. Increase in the buoyancy parameter (γ) increases the skin-friction and Nusselt number for buoyancy assisting flows (z =1). In absence of porous matrix (K_p=100), increase in Prandtl number decreases both surface velocity gradient and wall temperature gradient which is good agreement with the result obtained by Abdelkhalek [9](Table-2). It is interesting to note from the curves VI and VII of fig.3 that presence of porous matrix is found to be ineffective to modify the velocity gradient in case of high Prandtl number fluid i.e. the fluid with low thermal diffusivity.

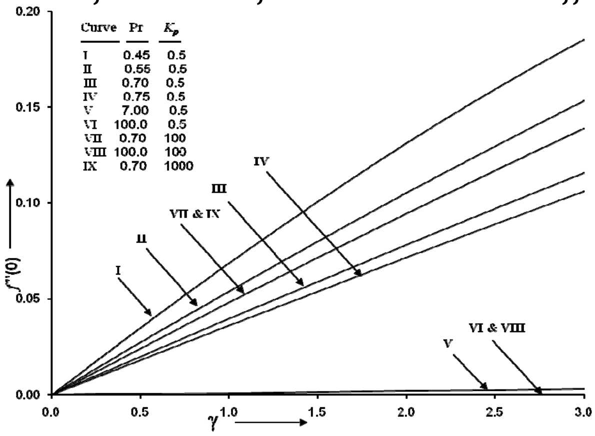

Fig-3 Effect of Pr & K_p on non-dimensional wall velocity gradient when M=3.0, f_w=0.0, k =1.3592, ϕ=3.0 and η=0.0

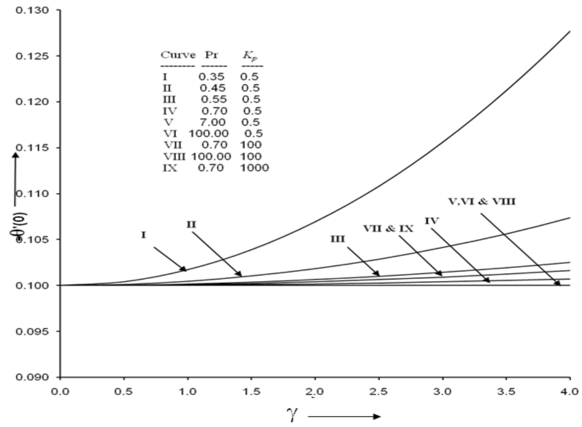

Fig-4 Effect of Pr & K_p on non-dimensional wall temperature gradient when M= 3.0, f_w=0.0, k =1.3592, ϕ=0.0 and η=0.0

Curves for K_p =100 and K_p =1000 and more coincide and hence we treat high value of K_p as absence of porous matrix (Curves VII & IX).

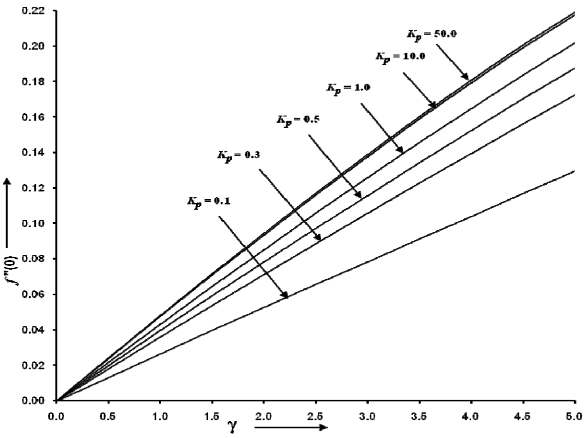

Fig-5 Effect of K_p on non-dimensional wall velocity gradient when Pr = 0.7, M= 3.0, f_w=0.0, k =1.3592 and η=0.0

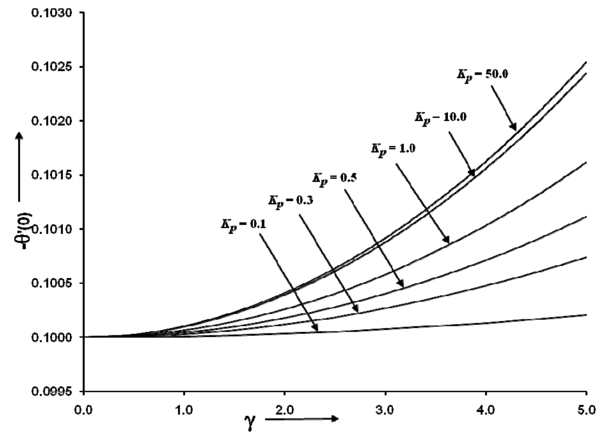

Fig-6 Effect of K_p on non-dimensional wall temperature gradient when Pr = 0.7, M= 3.0, f_w=0.0, k =1.3592, ϕ=0.0 and η=0.0

Figs.5 and 6 show the effect of permeability parameter (K_p) on the dimensionless surface velocity gradient and dimensionless wall temperature gradient respectively. It is observed that in absence of porous matrix, both surface velocity gradient and wall temperature gradient increase. Which indicates greater shearing stress is experienced on the surface of the wall.

Figs.7 and 8 depict the effect of suction or blowing parameter (f_w) on the dimensionless surface velocity gradient and dimensionless wall temperature gradient respectively in presence and in absence of porous matrix. It should be noted here that positive value of f_w indicates fluid suction at the surface while negative values of f_w correspond to fluid blowing or injection at the surface. The effect of suction is to make the velocity and temperature distribution more uniform within the boundary layer. Imposition of fluid suction at the surface has a tendency to reduce both the hydrodynamic and thermal thickness of the boundary layer where viscous effects dominate. This results in decreasing the surface velocity gradient and increasing surface temperature gradient in presence of porous matrix. Also the same effect has been occurred for fluid blowing at the surface in *presence of porous matrix. This result of ours is in good agreement with the device of boundary layer control by applying suction on the moving surface. So in the present problem presence of porous matrix suggest a controlling device for reducing boundary-layer thickness.*

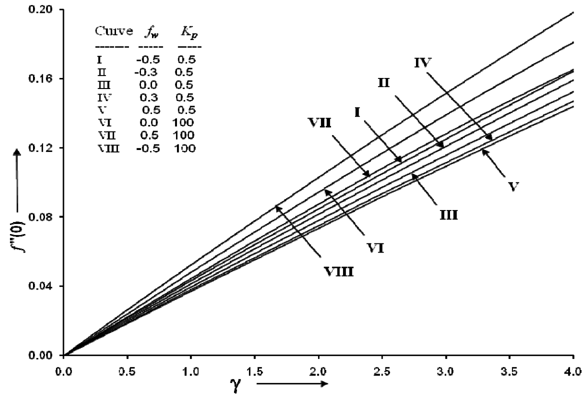

Fig-7 Effect of f_w & K_p on non-dimensional wall velocity gradient when Pr = 0.7, M= 3.0, f_w=0.0, k =1.3592 and η=0.0

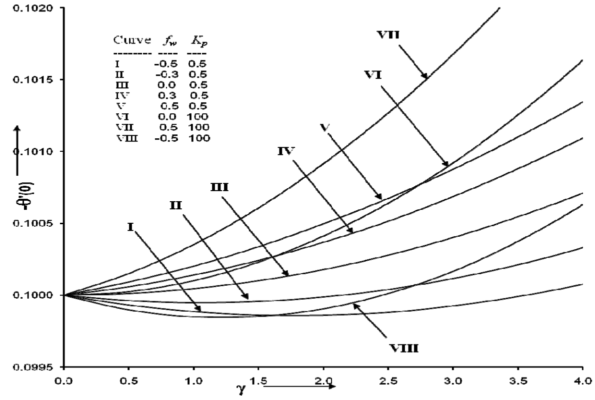

Fig-8 Effect of f_w & K_p on non-dimensional wall temperature gradient when Pr = 0.7, M= 3.0, f_w=0.0, k =1.3592, ϕ=0.0 and η=0.0

Further in case of blowing the same effect is observed in the presence of porous matrix suggesting that boundary-layer effect can be controlled by embedding the moving surface in a porous matrix. But in absence of porous matrix (K_p=100), the fluid suction and the fluid blowing enhance the surface velocity gradient. On the other hand, in absence of porous matrix (K_p=100), the fluid suction enhances the surface temperature gradient at all points (Curve-VII of fig.8) where as fluid blowing reduces the surface temperature gradient up to certain range of buoyancy parameter and then increases(Curve-VIII of fig.8). On careful observation, the curve VIII exhibits the initial fall and rise in temperature gradient due to the presence of suction and high value of permeability. Further, it is to note that presence of injection with high permeability accelerates the wall temperature gradient. The above facts reveal that suction with low buoyancy effect (i.e. small value of is γ) is found to be counterproductive for enhancing the wall temperature gradient. The high value of buoyancy effect is beneficial for higher temperature gradient. Hence, buoyancy parameter has a dual role in modifying the temperature gradient in the presence of suction.

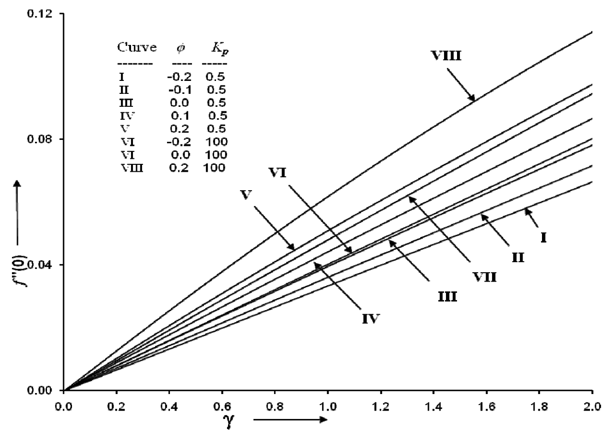

Fig-9 Effect of ϕ & K_p on non-dimensional wall velocity gradient when Pr = 0.7, M= 3.0, f_w=0.0, k =1.3592 and η=0.0

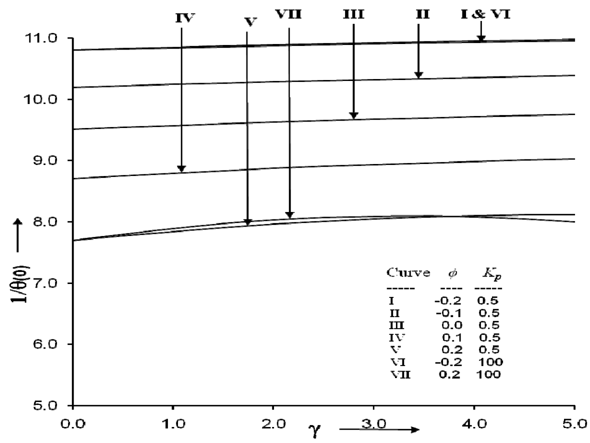

Fig-10 Effect of ϕ & K_p on Nusselt number when Pr = 0.7, M= 3.0, f_w=0.0, k =1.3592 and η=0.0

Figs.9 and 10 illustrate the effect of heat generation or absorption coefficient (ϕ) on both non-dimensional surface velocity gradient and Nusselt number respectively with or without porous matrix. It should be noted that positive value of ϕ means heat generation (source) and negative value of ϕ means heat absorption (sink). In presence of porous matrix decrease in heat generation or absorption coefficient decreases the non-dimensional surface velocity gradient and increases the Nusselt number for buoyancy assisting flow. Also, in absence of porous matrix, decrease in heat generation or absorption coefficient decreases the non-dimensional surface velocity gradient and increases the Nusselt number which is an excellent agreement with the

Table-3

	Present results		Abdelkhalek[13]	
Φ	f"(0)	1/θ(0)	f"(0)	1/θ(0)
-0.2	0.03333	10.84475	0.04025	10.85369
0.2	0.05085	7.84438	0.06175	7.89531

result obtained by Abdelkhalek [9](Table-3). The change in γ produces change(both decrease/increase) in buoyancy effect in the presence of heat source causing the rise/fall in velocity gradient whereas change in ϕ fails to affect rate of heat transfer γ wise variation but addition or deletion of heat energy varies with the strength of heat source/sink.

CONCLUSIONS

A theoretical study of steady two dimensional laminar MHD mixed convection stagnation point flow and heat transfer against a heated vertical semi-infinite permeable surface in a porous medium has been presented. Some of the important findings of the study are given below.

- *In presence of porous matrix the imposed magnetic field reduces the velocity field as well as the temperature field.*
- *An increase in Prandtl number reduces the thermal boundary layer along the plate.*
- *Due to the presence of porous matrix, both surface velocity gradient and wall temperature gradient decrease.*
- *The effect of fluid suction and fluid blowing is to reduce the surface velocity gradient and to enhance the wall temperature gradient respectively in presence of porous matrix.*
- *In presence of porous matrix decrease in heat generation or absorption coefficient decreases the non-dimensional surface velocity gradient and increases the Nusselt number.*

Nomenclature

x, y Coordinate axes
u, v Velocity components in the x and y directions
n, a Constants
b Characteristic time or time rate constant
B_0 The magnetic induction
C_p The specific heat at constant pressure
v_0 The suction or injection velocity
f_w The wall suction parameter
g The acceleration and z is a parameter such that z =1 denotes buoyancy assisting $T_w > T_\infty$ and z =-1 corresponds to buoyancy opposing $T_w < T_\infty$
Gr_x The local Grashoff number
K_f The fluid thermal conductivity
M The Hartmann number
Pr The Prandtl number
Q_0 The dimensional heat generation or absorption coefficient
Re_x The local Reynolds number
T Temperature of the fluid
T_∞ Temperature of the fluid far away from the plate
T_w Wall temperature
U Non-dimensional velocity component

Greek letters

α The thermal diffusivity
β The coefficient of thermal expansion
υ The kinematic viscosity
ρ The fluid density
ϕ The dimensionless heat generation or absorption coefficient
σ The fluid electrical conductivity

Subscripts

w Condition on the wall
∞ Free-stream condition

Appendix

$$\lambda_1=\frac{1}{2}\left(-\Pr k_0+\sqrt{(\Pr k_0)^2-4\phi\Pr}\right),\quad a_1=\frac{1}{2}\left(\Pr k_0+\sqrt{(\Pr k_0)^2-4\phi\Pr}\right),\quad \theta_1=\frac{1}{a_1}e^{-a_1\eta}\quad \alpha_1=\frac{1}{2}\left(-k_0+\sqrt{k_0{}^2+4(M+\frac{1}{K_p})}\right),$$

$$a_2=\frac{z\gamma}{a_1{}^2\left(a_1{}^2-k_0a_1-(M+\frac{1}{K_p})\right)},\ a_3=\frac{1}{2}\left(k_0+\sqrt{k_0{}^2+4(M+\frac{1}{K_p})}\right),\quad a_4=f_w+\frac{a_1a_2}{a_3}-a_2,\ a_5=-\frac{a_1a_2}{a_3},$$

$$a_6=a_5-\frac{a_3a_5}{a_1},\ a_7=\frac{\Pr .a_4}{(\Pr k-2a_1)a_1}-\frac{a_6(a_1+a_3)}{a_1a_3(a_1+a_3+\lambda_1)},\ a_8=\frac{a_6}{a_3(a_1+a_3+\lambda_1)},\ a_9=\frac{\Pr .a_4}{\Pr k_0-2a_1},$$

$$a_{10}=-a_4a_1{}^2a_2-z\gamma a_7,\ a_{11}=-a_4a_3{}^2a_5,\ a_{12}=2a_1a_2a_3a_5-a_1{}^2a_2a_5-z\gamma a_8,\ a_{13}=-z\gamma a_9$$

$$a_{14}=-\frac{a_{10}}{a_1(a_1+\alpha_1)(a_3-a_1)},\ a_{15}=\frac{a_{11}}{3a_3{}^2-2k_0a_3-(M+\frac{1}{K_p})},\ a_{16}=-\frac{a_{12}}{(a_1+a_3)(a_1+a_3+\alpha_1)a_1}$$

$$a_{17}=-\frac{\left((2a_1+\alpha_1)(a_1-a_3)+a_1(a_1+\alpha_1)\right)a_{13}}{\left(a_1(a_1+\alpha_1)(a_1-a_3)\right)^2},\quad a_{18}=a_{14}+a_{15},\ a_{19}=\frac{a_{15}-a_1a_{18}-a_{16}(a_1+a_3)}{a_3},$$

$$a_{20}=-(a_{19}+a_{18}+a_{16}),\ a_{21}=a_1a_4a_9,\ a_{22}=a_1a_4a_9-a_4a_9+a_{20},\ a_{23}=a_{15}-a_3a_5a_9-\frac{a_3a_{15}}{a_1},$$

$$a_{24}=a_4a_8(a_1+a_3)+a_{19}-a_3a_5a_7-\frac{a_3a_{19}}{a_1}+\frac{a_{15}}{a_1},\quad a_{25}=-a_2a_9,\ a_{26}=a_5a_8(a_1+a_3)-a_3a_5a_8,$$

$$a_{27}=a_{16}-a_1a_2a_8-\frac{a_{16}(a_1+a_3)}{a_1},\ a_{28}=-\frac{a_{21}}{2(a_1+\lambda_1)},\ a_{29}=-\frac{a_{22}(a_1+\lambda_1)+a_{21}}{(a_1+\lambda_1)^2},\ a_{30}=\frac{a_{23}}{a_3(a_1+a_3+\lambda_1)}$$

$$a_{31}=\frac{a_{23}(a_1+2a_3+\lambda_1)}{a_3^{\ 2}(a_1+a_3+\lambda_1)^2}+\frac{a_{24}}{a_3(a_1+a_3+\lambda_1)},\ a_{32}=\frac{a_{25}}{a_1(2a_1+\lambda_1)},\ a_{33}=\frac{a_{26}}{2a_3(a_1+2a_3+\lambda_1)},$$

$$a_{34}=\frac{a_{27}}{(2a_1+a_3+\lambda_1)(a_1+a_3)},\quad a_{35}=\frac{1}{a_1}\{a_{29}+a_{30}-a_{31}(a_1+a_3)-2a_1a_{32}-(a_1+2a_3)a_{33}-(a_3+2a_1)a_{34}\}$$

$$a_{36}=2a_3a_4a_{15}-a_4a_3^{\ 2}a_{19}-a_3^{\ 2}a_5a_{20},\quad a_{37}=-a_4a_3^{\ 2}a_{15},\quad a_{38}=-z\gamma a_{28},\quad a_{39}=-z\gamma a_{29},$$

$$a_{40}=-a_4a_1^{\ 2}a_{18}-a_1^{\ 2}a_2a_{20}-z\gamma a_{30},$$

$$a_{42}=-a_4(a_1+a_3)^2a_{16}-a_1^{\ 2}a_5a_{18}-a_2a_3^{\ 2}a_{19}-a_1^{\ 2}a_2a_{19}-a_3^{\ 2}a_5a_{18}+2a_1a_3a_5a_{18}+a_1a_2a_3a_{19}-a_1a_2a_{15}-z\gamma a_{31}$$

$$a_{43}=-z\gamma a_{32},\quad a_{44}=-a_5(a_1+a_3)^2a_{16}-a_3^{\ 2}a_5a_{16}+2a_3a_5(a_1+a_3)a_{16}-z\gamma a_{33},$$

$$a_{45}=-a_2(a_1+a_3)^2a_{16}-a_1^{\ 2}a_2a_{16}+2a_1a_2(a_1+a_3)a_{16}-z\gamma a_{34},\quad a_{46}=\frac{a_{36}}{2a_3(a_3+\alpha_1)},\quad a_{48}=\frac{a_{38}}{a_1(a_1+\alpha_1)(a_3-a_1)},$$

$$a_{47}=\frac{a_{37}}{a_3(a_3+\alpha_1)}+\frac{a_{36}(2a_3+\alpha_1)}{a_3^{\ 2}(a_3+\alpha_1)^2},\quad a_{49}=\frac{a_{39}}{a_1(a_1+\alpha_1)(a_3-a_1)}-\frac{(a_3-a_1)(2a_1+\alpha_1)2a_{38}}{[a_1(a_1+\alpha_1)(a_3-a_1)]^2},$$

$$a_{50}=\frac{a_{40}}{a_1(a_1+\alpha_1)(a_3-a_1)}-\frac{(a_3-3a_1-\alpha_1)a_{38}+(a_3-a_1)(2a_1+\alpha_1)a_{39}}{[a_1(a_1+\alpha_1)(a_3-a_1)]^2},\quad a_{51}=\frac{-a_{41}}{a_1(a_1+a_3)(a_1+a_3+\alpha_1)},$$

$$a_{52}=\frac{-a_{42}}{a_1(a_1+a_3)(a_1+a_3+\alpha_1)}-\frac{(2a_1+a_3)(a_1+a_3+\alpha_1)+a_1(a_1+a_3)a_{41}}{[a_1(a_1+a_3)(a_1+a_3+\alpha_1)]^2},\ a_{55}=-a_{53}-a_{54}$$

$$a_{53}=\frac{1}{a_3}\left[a_{47}+a_{49}-a_1a_{50}+a_{51}-(a_1+a_3)a_{52}-\frac{2a_1a_{43}}{2a_1(2a_1+\alpha_1)(a_3-2a_1)}+\frac{a_{44}}{(a_1+2a_3+\alpha_1)(a_1+a_3)}+\frac{a_{45}}{(a_3+2a_1+\alpha_1)2a_1}\right]$$

$$a_{54}=a_{50}+a_{52}+\frac{a_{43}}{2a_1(2a_1+\alpha_1)(a_3-2a_1)}-\frac{a_{44}}{(a_1+2a_3)(a_1+2a_3+\alpha_1)(a_1+a_3)}-\frac{a_{45}}{2a_1(a_3+2a_1+\alpha_1)(a_3+2a_1)},$$

$$a_{56}=\frac{a_{43}}{2a_1(2a_1+\alpha_1)(a_3-2a_1)},\quad a_{57}=\frac{-a_{44}}{(a_1+a_3)(a_1+2a_3)(a_1+2a_3+\alpha_1)},\quad a_{58}=\frac{-a_{45}}{2a_1(2a_1+a_3)(2a_1+a_3+\alpha_1)}$$

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