ANNALS OF FACULTY ENGINEERING HUNEDOARA
– International Journal of Engineering
Tome XI (Year 2013) – FASCICULE 4 (ISSN 1584 – 2673)

[1.] B.R. ROUT, [2.] H.B. PATTANAYAK

CHEMICAL REACTION AND RADIATION EFFECT ON MHD FLOW PAST AN EXPONENTIALLY ACCELERATED VERTICAL PLATE IN PRESENCE OF HEAT SOURCE WITH VARIABLE TEMPERATURE EMBEDDED IN A POROUS MEDIUM

[1.] DEPARTMENT OF MATHEMATICS, KRUPAJAL ENGINEERING COLLEGE, BHUBANESWAR-02, INDIA
[2.] DEPARTMENT OF COM. SC. AND APP., NIIS INSTITUTE OF BUSSINESS ADMINISTRATION, BHUBANESWAR, INDIA

ABSTRACT: In this paper we analyze the effect of chemical reaction and radiation on MHD fluid flow through a porous medium in a uniform magnetic field with variable temperature and mass diffusion with heat source past an exponentially accelerated vertical plate. The non-dimensional equations governing the above flow characteristics are solved by using Laplace Transformation and the effect of different physical parameters on the velocity profile, temperature profile and concentration profile are illustrated graphically.
KEYWORDS: Variable temperature, heat source, radiation, porous medium

INTRODUCTION

In recent years, MHD flow with heat and mass transfer in presence of chemical reaction are of importance in many practical processes such as distribution of temperature and moisture over agricultural field, energy transfer in a wet cooling tower, in method of generating and extracting power from a moving fluid. So many researchers have taken interest in study the above effects; Rajeswari, Jothiram and Nilson [1] studied chemical reaction, heat and mass transfer on nonlinear MHD boundary layer flow through a vertical porous surface in the presence of suction. The effect of chemical reaction and heat generation or absorption on double diffusive convection from vertical truncated cone in a porous media with variable viscosity is studied by Mahdy [2]. Pal and Talukdar [3] have studied perturbation analysis of unsteady MHD convective heat and mass transfer in boundary layer slip flow past a vertical permeable plate with thermal radiation and chemical reaction. Further the effect of thermal radiation and heat and mass transfer flow of a variable viscosity fluid past a vertical porous plate in presence of transverse magnetic field was investigated by Makinde and Ogulu [4]. Aziz [5] theoretically examined that a similarity solution for a laminar thermal boundary layer over a flat plate with a convective surface boundary condition. The effects of transversely applied magnetic field on the flow of an electrically conducting fluid past an impulsively started infinite isothermal vertical plate is studied by Soundalgekar et al.[6].

Radiation and chemical reaction of convective fluids in presence of heat source within a porous medium are of great practical importance in geophysics and energy related problem such as recovery of petroleum resources, cooling of underground electric cable, ground water pollution, fiber and granular insulation, chemical catalytic reactors and solidification of casting. The above applications attracts authors like; S.Bagai [7] studied the effect of variable viscosity on free convective over a non-isothermal axisymmetric body in a porous medium with internal heat generation, Malla et al. [8] presented natural convection flow along a vertical wavy surface with uniform surface temperature in presence of heat generation/absorption, Hady et al. [9] studied the heat generation or absorption effect on MHD free convection flow along a vertical wavy surface. Rout et al. [10] studied the influence of chemical reaction and radiation on MHD heat and mass transfer fluid flow over a moving vertical plate in presence of heat source with convective boundary condition. Singh and Naveen Kumar [11] established the free convection effects on flow past an exponentially accelerated vertical plate. Muthucumaraswamy et al.[12] presented the mass transfer effects on exponentially accelerated isothermal vertical plate. Effect of radiation on free convection flow past a moving plate is established by Raptis and Perdikis [13]. Rajput and Surendra Kumar [14] studied MHD flow past an impulsively vertical plate with variable temperature and mass diffusion.

In our knowledge, none of the researchers has taken interest in studying of MHD fluid flow through a porous medium in a uniform magnetic field with variable temperature and mass diffusion with heat source and radiation past an exponentially accelerated vertical plate.

MATHEMATICAL FORMULATION

Consider an unsteady MHD flow of a viscous incompressible fluid through a porous medium past an infinite isothermal vertical plate with uniform mass diffusion in presence of a first order chemical reaction and radiation effect. The x'-axis is taken along the plate in the vertical upward direction and y'-axis is perpendicular to it. A uniform magnetic field is applied in the direction normal to x'-axis. The fluid is subjected to variable temperature such as at time $t' \le 0$ the plate and fluid are at same temperature T_∞ and at time $t' > 0$, the plate is exponentially accelerated with a velocity $u = u_0 \exp(a't')$. To neglect the induced magnetic field effect we choose the flow of small Reynolds number. Hull effect and Joule heating effect are neglected. To allow heat generation effect we place a heat source in the flow. The governing equations under the above physical conditions are given by

$$\frac{\partial u}{\partial t'} = \nu \frac{\partial^2 u}{\partial y^2} + g\beta\left(T - T_\infty\right) + g\beta^*\left(C' - C'_\infty\right) - \frac{\sigma B_0^2}{\rho} u - \frac{\nu}{K_r} u \qquad (1)$$

$$\rho c_p \frac{\partial T}{\partial t'} = k \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} - Q_0 (T - T_\infty) \qquad (2)$$

$$\frac{\partial C'}{\partial t} = D \frac{\partial^2 C'}{\partial y^2} - k_1 (C' - C'_\infty) \qquad (3)$$

Boundary condition

$$u = 0\,,\ T = T_\infty\,,\ C' = C'_\infty \quad at \quad y = 0$$

$$t' > 0: u = u_0 \exp(a't'),\ T = T_\infty + (T_w - T_\infty)At',\ C' = C'_\infty + (C'_w - C'_\infty)At' \quad at \quad y = 0\ .$$

$$u \to 0\,,\ T \to T_\infty\,,\ C' \to C'_\infty \quad as \quad y \to \infty \qquad (4)$$

where $A = \frac{u_0^2}{\nu}$

Introducing the following non-dimensional parameters

$$U = \frac{u}{u_0},\quad t = \frac{t' u_0^2}{\nu},\ a = \frac{a'\nu}{u_0^2},\ M = \frac{\sigma B_0^2 \nu}{\rho u_0^2},\ \theta = \frac{T - T_\infty}{T_w - T_\infty},$$

$$\Pr = \frac{\mu c_p}{k},\ K_0 = \frac{u_0^2 k_r}{\nu^2},\ R = \frac{16\sigma^* T_\infty^3}{3k_p k}$$

$$Sc = \frac{\upsilon}{D},\ Gr = \frac{\nu g\beta(T_w - T_\infty)}{u_o^3},\ Gc = \frac{\nu g\beta^*(C'_w - C'_\infty)}{v_o^3},$$

$$K = \frac{\upsilon}{u_0^2} k_1,\quad Q = \frac{\nu Q_0}{u_0^2 \rho C_p},\ Y = \frac{u_o y}{\nu},\quad C = \frac{C' - C'_\infty}{C'_w - C'_\infty} \qquad (5)$$

By using the Rosseland approximation q_r *takes the value*

$$q_r = -\frac{4\sigma^*}{3k_p} \frac{\partial T^4}{\partial y} \qquad (6)$$

Where σ^* *and* k_p *are the Stefan-Boltzman constant and the Rossenland mean absorption coefficient respectively. Assuming the temperature difference within the flow are sufficiently small, by Taylor series expansion neglecting the higher order terms we can express* T^4 *as a linear function of temperature of the form*

$$T^4 \approx 4T_\infty^3 - 3T_\infty^4 \qquad (7)$$

By using (6) and (7)

$$\frac{\partial q_r}{\partial y} = -\frac{16\sigma^* T_\infty^3}{3k_p} \frac{\partial^2 T}{\partial y^2} \qquad (8)$$

Using (5) and (8) in the system of Eq. (1) - (3), the reduced non-dimensional Eqs. are given by

$$\frac{\partial U}{\partial t} = Gr\theta + GcC + \frac{\partial^2 U}{\partial Y^2} - \left(M + \frac{1}{K_0} \right) U \tag{9}$$

$$\frac{\partial \theta}{\partial t} = \frac{1+R}{\Pr} \frac{\partial^2 \theta}{\partial Y^2} - Q\theta \tag{10}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc} \frac{\partial^2 C}{\partial Y^2} - KC \tag{11}$$

Corresponding boundary conditions are

$$\begin{aligned} &U = 0,\quad \theta = 0,\ C = 0 \quad \text{for all } Y, t \le 0 \\ &t > 0 : U = e^{at},\quad \theta = t,\ C = t \quad \text{at } Y = 0 \\ &U \to 0,\ \theta \to 0,\ C \to 0 \qquad \text{as } Y \to \infty \end{aligned} \tag{12}$$

SOLUTION PROCEDURE

The dimensionless equations (9) -(11) under the boundary conditions (12) are solved by Transform technique and solutions are given by Laplace Trasformation technique.

$$\begin{aligned} U = {} & \frac{e^{at}}{2}\left[\begin{array}{l} e^{2\eta\sqrt{(a+N)t}}\, erfc\left(\eta + \sqrt{(a+N)t}\right) \\ + e^{-2\eta\sqrt{(a+N)t}}\, erfc\left(\eta - \sqrt{(a+N)t}\right) \end{array}\right] + e\frac{e^{bt}}{2b^2}\left[\begin{array}{l} e^{2\eta\sqrt{(b+N)t}}\, erfc\left(\eta + \sqrt{(b+N)t}\right) \\ + e^{-2\eta\sqrt{(b+N)t}}\, erfc\left(\eta - \sqrt{(b+N)t}\right) \end{array}\right] \\ & - \frac{e}{2b^2}\left[\begin{array}{l} e^{2\eta\sqrt{Nt}}\, erfc\left(\eta + \sqrt{Nt}\right) \\ + e^{-2\eta\sqrt{Nt}}\, erfc\left(\eta - \sqrt{Nt}\right) \end{array}\right] - \frac{e}{2b}\left[\begin{array}{l} \left(t + \frac{\eta\sqrt{t}}{\sqrt{N}}\right) e^{2\eta\sqrt{Nt}}\, erfc\left(\eta + \sqrt{Nt}\right) \\ + \left(t - \frac{\eta\sqrt{t}}{\sqrt{N}}\right) e^{-2\eta\sqrt{Nt}}\, erfc\left(\eta - \sqrt{Nt}\right) \end{array}\right] \\ & + f\frac{e^{dt}}{2d^2}\left[\begin{array}{l} e^{2\eta\sqrt{(d+N)t}}\, erfc\left(\eta + \sqrt{(d+N)t}\right) \\ + e^{-2\eta\sqrt{(d+N)t}}\, erfc\left(\eta - \sqrt{(d+N)t}\right) \end{array}\right] - f\frac{1}{2d^2}\left[\begin{array}{l} e^{2\eta\sqrt{Nt}}\, erfc\left(\eta + \sqrt{Nt}\right) \\ + e^{-2\eta\sqrt{Nt}}\, erfc\left(\eta - \sqrt{Nt}\right) \end{array}\right] \\ & - f\frac{1}{2d}\left[\left(t + \frac{\eta\sqrt{t}}{\sqrt{N}}\right) e^{2\eta\sqrt{Nt}}\, erfc\left(\eta + \sqrt{Nt}\right) + \left(t - \frac{\eta\sqrt{t}}{\sqrt{N}}\right) e^{-2\eta\sqrt{Nt}}\, erfc\left(\eta - \sqrt{Nt}\right)\right] \\ & - e\frac{e^{bt}}{2b^2}\left[e^{2\eta\sqrt{A(b+Q)t}}\, erfc\left(\sqrt{A}\eta + \sqrt{(b+Q)t}\right) + e^{-2\eta\sqrt{A(b+Q)t}}\, erfc\left(\sqrt{A}\eta + \sqrt{(b+Q)t}\right)\right] \\ & + e\frac{1}{2b^2}\left[e^{2\eta\sqrt{AQt}}\, erfc\left(\sqrt{A}\eta + \sqrt{Qt}\right) + e^{-2\eta\sqrt{AQt}}\, erfc\left(\sqrt{A}\eta - \sqrt{Qt}\right)\right] \\ & + e\frac{1}{2b}\left[\left(t + \frac{\eta\sqrt{At}}{\sqrt{Q}}\right) e^{2\eta\sqrt{AQt}}\, erfc\left(\sqrt{A}\eta + \sqrt{Qt}\right) + \left(t - \frac{\eta\sqrt{At}}{\sqrt{Q}}\right) e^{-2\eta\sqrt{AQt}}\, erfc\left(\sqrt{A}\eta - \sqrt{Qt}\right)\right] \\ & - f\frac{e^{dt}}{2d^2}\left[e^{2\eta\sqrt{Sc(d+K)t}}\, erfc\left(\sqrt{Sc}\eta + \sqrt{(d+K)t}\right) + e^{-2\eta\sqrt{Sc(d+K)t}}\, erfc\left(\sqrt{Sc}\eta - \sqrt{(d+K)t}\right)\right] \\ & + f\frac{1}{2d^2}\left[e^{2\eta\sqrt{ScKt}}\, erfc\left(\sqrt{Sc}\eta + \sqrt{Kt}\right) + e^{-2\eta\sqrt{ScKt}}\, erfc\left(\sqrt{Sc}\eta - \sqrt{Kt}\right)\right] \\ & + f\frac{1}{2d}\left[\left(t + \frac{\eta\sqrt{Sc\,t}}{\sqrt{K}}\right) e^{2\eta\sqrt{ScKt}}\, erfc\left(\sqrt{Sc}\eta + \sqrt{Kt}\right) + \left(t - \frac{\eta\sqrt{Sct}}{\sqrt{K}}\right) e^{-2\eta\sqrt{ScKt}}\, erfc\left(\sqrt{Sc}\eta - \sqrt{Kt}\right)\right] \end{aligned} \tag{13}$$

$$\begin{aligned} \theta = {} & \frac{t}{2}\left[e^{2\eta\sqrt{AQt}}\, erfc\left(\sqrt{A}\eta + \sqrt{Qt}\right) + e^{-2\eta\sqrt{AQt}}\, erfc\left(\sqrt{A}\eta - \sqrt{Qt}\right)\right] \\ & - \frac{\eta\sqrt{At}}{2\sqrt{Q}}\left[e^{-2\eta\sqrt{AQt}}\, erfc\left(\sqrt{A}\eta - \sqrt{Qt}\right) - e^{2\eta\sqrt{AQt}}\, erfc\left(\sqrt{A}\eta + \sqrt{Qt}\right)\right] \end{aligned} \tag{14}$$

$$\begin{aligned} C = {} & \frac{t}{2}\left[e^{2\eta\sqrt{ScKt}}\, erfc\left(\sqrt{Sc}\eta + \sqrt{Kt}\right) + e^{-2\eta\sqrt{ScKt}}\, erfc\left(\sqrt{Sc}\eta - \sqrt{Kt}\right)\right] \\ & - \frac{\eta\sqrt{Sct}}{2\sqrt{K}}\left[e^{-2\eta\sqrt{ScKt}}\, erfc\left(\sqrt{Sc}\eta - \sqrt{Kt}\right) - e^{2\eta\sqrt{ScKt}}\, erfc\left(\sqrt{Sc}\eta + \sqrt{Kt}\right)\right] \end{aligned} \tag{15}$$

where $A = \frac{\Pr}{1+R},\ b = \frac{AQ-N}{1-A},\ d = \frac{ScK-N}{1-Sc},\ e = \frac{Gr}{A-1},$

$f = \frac{Gc}{Sc-1},\ N = M + \frac{1}{K_0},\ \eta = \frac{Y}{2\sqrt{t}}$

RESULT AND DISCUSSION

An analysis of MHD flow past an exponentially accelerated vertical plate embedded in a porous medium with variable temperature and mass diffusion in presence of heat source and chemical reaction of first order with radiation effect is carried out. To illustrate the behavior of the velocity, temperature and concentration field, the solution is obtained by Laplace Transform and the parameters those describes the flow characters and the results are displayed in form of graphs. In the present study the following parameter values are taken for computation $K_0 = 1$*, a=1, Pr=7, Sc=0.6, Gr = Gc =5, Q = 0.1, K = 5, R= 0.1, t= 0.2, M =2.*

The variation of velocity profile on permeability parameter K_0 *is demonstrated in figure-1. It is observed that velocity profile increases with increase in* K_0*. From the figure-2 it is observed that permeability parameter gives less variation in fluid velocity. In figure-2, the curvature depicts that the fluid velocity increases with respect to the time.*

The effect of magnetic field parameter on velocity profile is shown in figure-3 and it is observed that velocity profile decreases with increase in magnetic field parameter because the presence of magnetic field yields a drag force called Lorentz force which retards the fluid velocity. Velocity profile increases with increase in the value of velocity expansion parameter a, which is shown in figure-4. It is evident from the figure-5 that the velocity profile increases with increase in modified Grashof number Gc due to increase of mass buoyancy effect which leads to more velocity. There is no variation of velocity profile with respect to the Grashof number Gr, so graph of that variation is not specified.

The variation of concentration with Schimdt number, chemical reaction and with time is established in figure- 6 to 8. In figure-6 it is observed that the concentration profile decrease with increase in Sc, that happens as Sc increases there is decrease in molecular diffusivity which leads to less concentration of the fluid. As the fluid flow is subjected to a first order chemical reaction it reduces the fluid concentration i.e. concentration profile decreases with increase in chemical reaction parameter, which is displayed in figure-7. Concentration of the fluid with respect to time is specified in figure-8 and it is observed that the concentration profile increases with increase in time.

The variation of temperature profile for different values of the parameters like Prandtl number, heat source parameter and radiation parameter are analysed. In figure-9 the curvature indicated the temperature profile increases with decrease in Pr. It is due to that smaller value of Pr means increasing of thermal conductivities which enables diffusion of more heat.

Radiation parameter verses temperature profile graph is given in figure-10 and nature of its curve shows that increasing in radiation parameter there is increase in temperature profile as enhancement of the thermal radiation causes increase in the convection moment in the boundary layer and also increase the heat transfer coefficient of the medium. In figure 11 it is noticed that as increase in heat source parameter that reduces the temperature profiles.

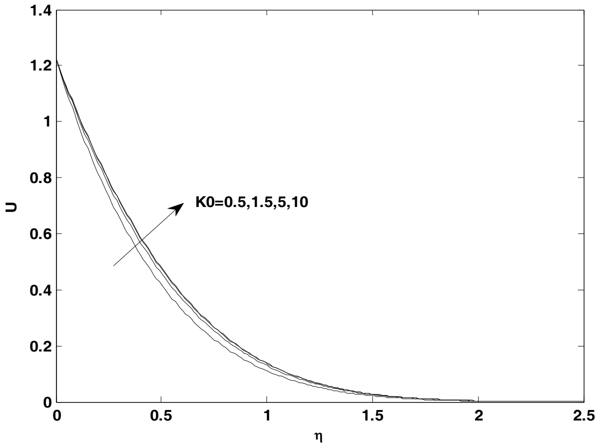


Fig. 1: Velocity profile for variation of K_0*.*

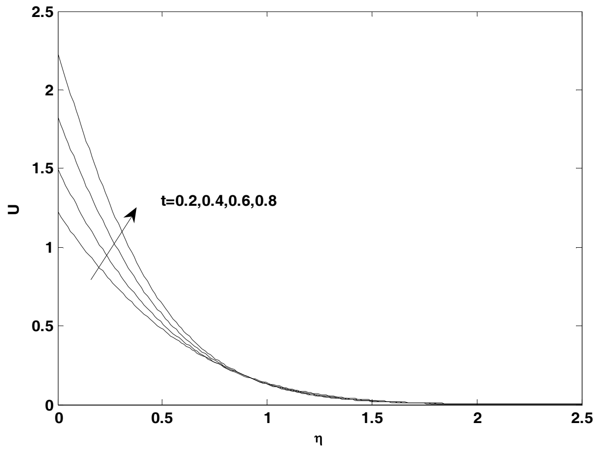


Fig. 2: Velocity profile for variation of t.

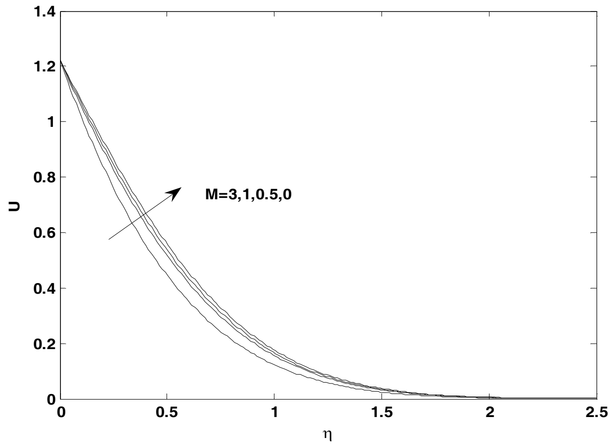


Fig. 3: Velocity profile for variation of M.

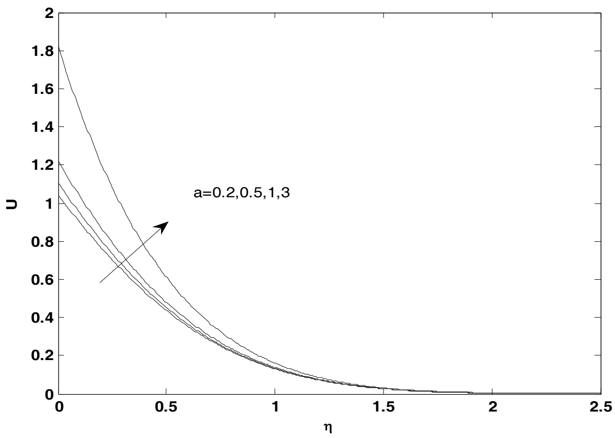


Fig.4: Velocity profile for variation of a

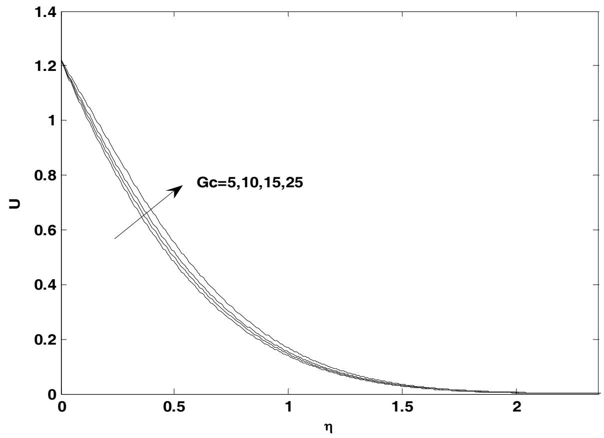


Fig. 5: Velocity profile for variation of Gc

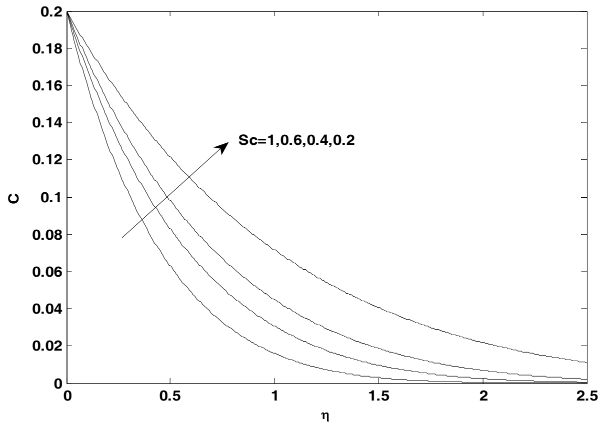


Fig. 6: Concentration profile for variation of Sc

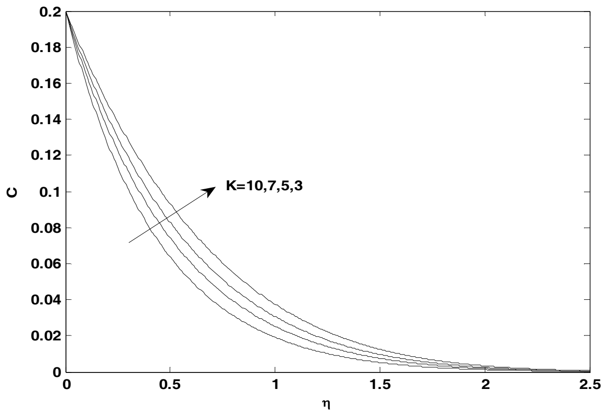


Fig. 7: Concentration profile for variation of Kr

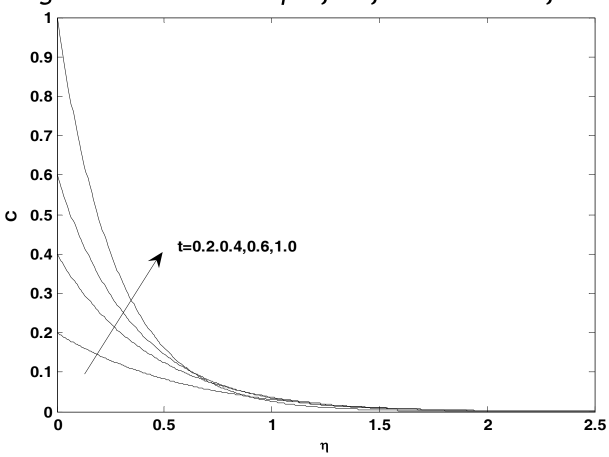


Fig. 8: Concentration profile for variation of t

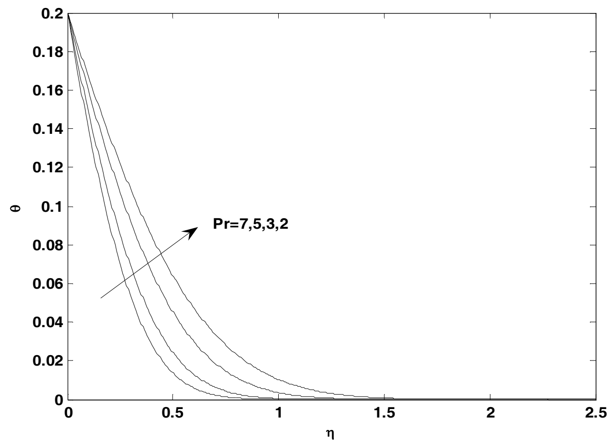


Fig. 9: Temperature profile for variation of Pr

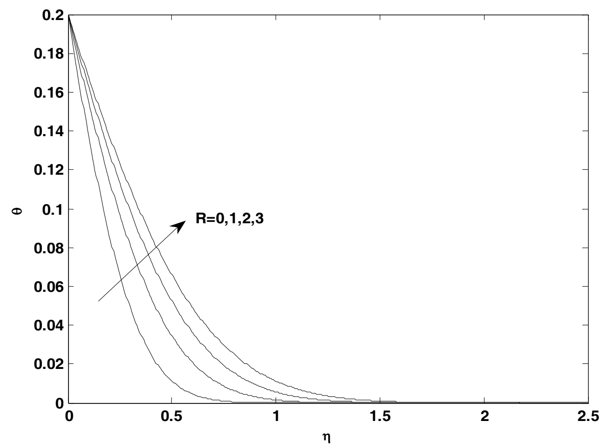


Fig. 10: Temperature profile for variation of R

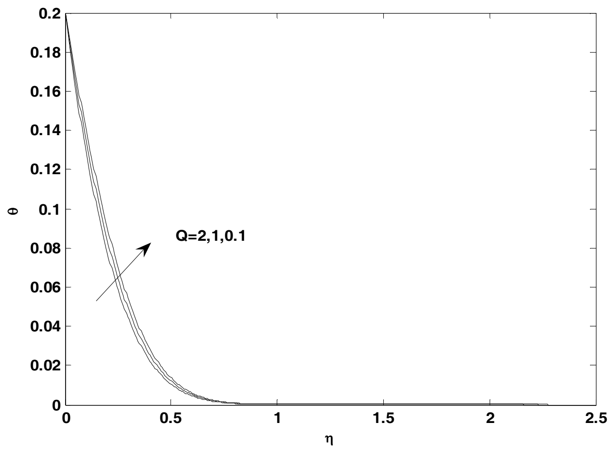


Fig. 11: Temperature profile for variation of Q

CONCLUSIONS

The present analysis has been carried out for MHD flow past an exponentially accelerated vertical plate in presence of heat source and chemical reaction with variable temperature embedded in a porous medium. The solutions for the model have been solved by Laplace Transformation technique. The conclusions of this study are as follows:

- *The velocity profiles increases with increasing for porosity parameter, modified Grashof number, velocity expansion parameter and time.*
- *The velocity profiles decreases with increasing magnetic field parameter.*
- *The increase in the strength of chemical reacting substances as well as Schimdt number causes to decrease the concentration.*
- *Temperature of the fluid flow increases as increase in radiation parameter.*
- *An interesting observation is noticed that heat source parameter and Prandtl number reduce the temperature profile.*

NOMENCLATURE

u velocity components along	U : nondimensional velocity component
x axis	B_0 : magnetic field strength
g : acceleration due to gravity	ν: kinematic viscosity
Pr : Prandtl number	Sc: Schimdt number
Q: heat source parameter	D : mass diffusivity
T : temperature	C : concentration
T_w : fluid temperature near wall	T_∞ :fluid temperature away from the wall
θ : non-dimensional temperature	C_w : fluid concentration near wall
C_∞ :fluid concentration away from the wall	Gr : thermal Grashof number
C: non-dimensional concentration	Gc : mass Grashof number
K : chemical reaction parameter	K_0 : non-dimensional permeability
R: radiation parameter Parameter	q_r :radiative heat flux
C_p : specific heat at constant pressure	K_r : permeability of the porous medium
β :coefficient of volume expansion temperature	ρ: fluid density
β^* : coefficient of volume expansion with concentration	σ : fluid electrical conductivity

REFERENCES

[1.] Rajeswari, R., Jothiram, B. and Nelson, V.K. (2009); Chemical reaction. Heat and Mass transfer of nonlinear MHDboundary layre flow through a vertical porous surface in the presence of suction. Appl. Math. Sci. 3(50), pp.2469-2480.

[2.] Mahdy, A (2010); Effect of chemical reaction and heat generation or absorption on double-diffusive convection from a vertical truncated cone in a porous media with variable viscosity. International communications in Heat and Mass Transfer.37.pp.548-554.

[3.] Pal,A Talukdar, B.(2010); Perturbation analysis of unsteady Magnetohydrodynamic convective heat and mass transfer in a boundary layer slip flow past a vertical permeable plate with a thermal radiation and chemical reaction, Commun Nonlinear Sci. Numerical Simulation. 15, pp. 1813-1830.

[4.] Makinde, O.D. and Ogulu, A.(2008) The effect of thermal radiation on the heat and mass transfer flow of a variable viscosity fluid past a vertical porous plate permeated by a transverse magnetic field, Chemical Eng,Communication.195(12).pp.1575-1584.

[5.] Aziz, A (2009); A Similarity solution for laminar thermal boundary layer over a fiat plate with a convective surface boundary condition. Communication Nonlinear science Numerical Simulation.14.pp.1064-1068.

[6.] Soundalgekar, V.M. Gupta, S.K. and Birajdar, N.S.(1979) Effects of mass transfer and free convection currents on MHD Stokes problem for a vertical plate, Nuclear Engineering Design.53.pp.339-346.

[7.] Bagai, S (2004) Effect of variable viscosity on free convection over a non-isothermal axisymmetric body in a porous medium with internal heat generation, Acta Mechanica.169 (1-4), pp187-194.

[8.] Molla M M, Hossain, M A and Yao, L S (2004) Natural convection flow along a vertical wavy surface with uniform surface temperature in presence of heat generation/absorption. International Journal of Thermal Sciences, 43(2).pp.157-163.

[9.] Hady, F M, Mohamed, A and Mahdy, A(2006) MHD free convection flow along a vertical wavy surface with heat generation or absorption effect. International communication in Heat and Mass Transfer, 33(10).pp.1253-1263.

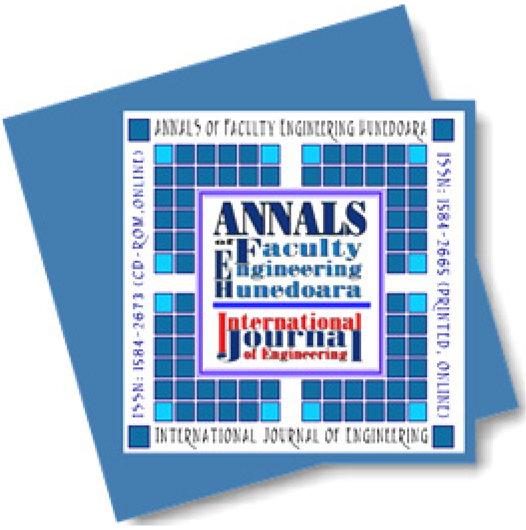
[10.] Rout, B R, Parida, S K and Panda, S (2013) MHD Heat and Mass Transfer of Chemical Reaction Fluid Flow over a Moving Vertical Plate in Presence of Heat Source with Convective Surface Boundary Condition. International Journal of Chemical Engineering, Volume 2013 (2013), ArticleID 296834, 10pages, http://dx.doi.org/10.1155/2013/296834.

[11.] Singh, A K and Naveen Kumar (1984) Free convection flow past an exponentially accelerated vertical plate, Astrophysics and Space,98.pp.245-258.

[12.] Muthucumaraswamy, R. Sathappan, K.E. and Natarajan, R(2008) Mass transfer effects on exponentially accelerated isothermal vertical plate.International Journal of Applied Mathematics and Mechanics,4(6),pp.19-25.

[13.] Raptis, A. and Perdikis, C. (1999) Radiation and free convection flow past a moving plate. International Journal of Applied Mechanics and Engineering, 4, pp.817-821.

[14.] Rajput, U.S. and Surendra Kumar (2011) MHD flow past an impulsively vertical plate with variable temperature and mass diffusion. Applied Mathematical Sciences, 5(3), pp.149-157.



ANNALS of Faculty Engineering Hunedoara

– International Journal of Engineering