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[1.] Ľudovít KOLLÁTH, [2.] Ilja MARTIŠOVITŠ, [3.] Milada OMACHELOVÁ

# SOLVING THE PROBLEMS OF WORKSPACE IN PARALLEL KINEMATIC STRUCTURE

[1,3.] Slovak University of Technology, Faculty of Mechanical Engineering, Bratislava, SLOVAKIA
[2.] Microstep Spol. S.R.O, Bratislava, SLOVAKIA

**Abstract**: This paper deals with the problem of determining the size of the work area in a parallel kinematic structure. This problem was solved for Tricept device created at the Faculty of Mechanical Engineering, Slovak University of Technology. A study which takes into account the parameter of the tricept motion was elaborated. This study deals with the problem of ambiguity of the position of reference point of the movable platform in extreme position and delimitation of the "safe work area", i.e. the area where ambiguity in the position of the reference point is impossible.
**Keywords**: parallel kinematic structure, tricept motion, workspace

## 1. INTRODUCTION

Tricept device belongs to the wide range of parallel kinematic structures, whose main parts are a solid and movable platform, connected by parallel members, such as telescopic rods [1]. The Tricept device (Fig.1) designed at the faculty of Mechanical Engineering, STU within the project APVT, č. 68/2002 Riadiace systémy pre CNC rezacie centrá a č. 99-02-2604 Stroje pre extrémne rýchle delenie materiálov,, supported by the Grant Agency supported financially by VEGA 1/0584/12 is composed of a fixed platform in which primary joints, enabling the motion of telescopic rods, are placed. Telescopic rods are connected with a movable platform by means of secondary joints. The central rod is perpendicular to the movable platform and is firmly attached to it. The telescopic movement of the rods is secured by the screws controlled by electric motors.

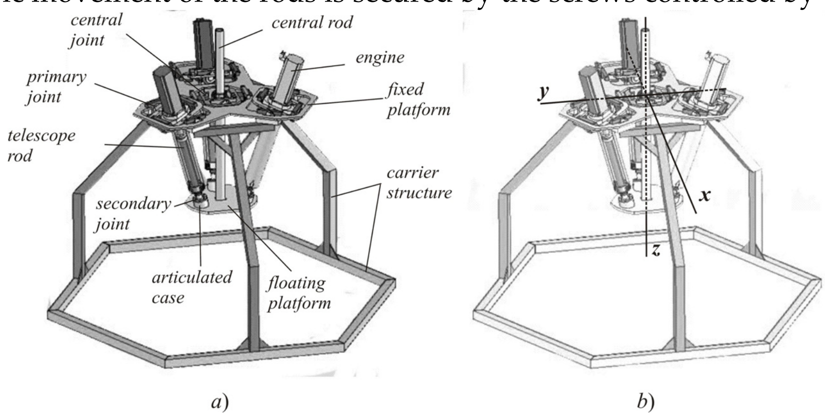


Figure 1. a) Basic parts of Tricept, b) Depicting of axes location of the Cartesian coordinate system

In this article we will analyze the shape of the Tricept work area. This space is created by moving the Q point (Fig. 2a) based on the equations derived in [2]. The shift from the starting point q with coordinates $\lfloor q_x, q_y, q_z \rfloor$ (when the rotation angles around x and y axes, as well as the shift along the axis equal zero) to the new point Q with the coordinates $\lfloor Q_x, Q_y, Q_z \rfloor$, with regard to the fixed platform – will be achieved as a transformation, which depends on the rotation around the axis y for the angle $\beta$, given by the matrix $O_y(\beta)$ and the rotation around the axis x for the angle $\alpha$, given by the matrix $O_x(\alpha)$, as well as the shift along the axis z, described as the vector $(\vec{q}+z\vec{e}_3)$. The matrix is as follows:

$$\vec{Q} = O_y(\beta).O_x(\alpha).(\vec{q}+z\vec{e}_3) \tag{1}$$

which means

$$\begin{pmatrix} Q_x \\ Q_y \\ Q_z \end{pmatrix} = \begin{pmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{pmatrix} . \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{pmatrix} . \begin{pmatrix} q_x \\ q_y \\ q_z + z \end{pmatrix} \tag{2}$$

The shift along the axis can be described as follows:

$$z = -q_z + S\sqrt{Q_x^2 + Q_y^2 + Q_z^2 - q_x^2 - q_y^2} \quad \text{where the variable } S = (\pm 1) \tag{3}$$

Rotating by mens of primary joints can be described as

$$\sin\beta = \frac{-q_x.Q_z + K\sqrt{Q_x^2 + Q_z^2 - q_x^2}.Q_x}{Q_x^2 + Q_z^2} \tag{4}$$

$$\cos\beta = \frac{-q_x.Q_x + K\sqrt{Q_x^2 + Q_z^2 - q_x^2}.Q_z}{Q_x^2 + Q_z^2} \tag{5}$$

$$\sin\alpha = \frac{\sqrt{Q_x^2 + Q_y^2 + Q_z^2 - q_x^2 - q_y^2} - \sqrt{Q_x^2 + Q_z^2 - q_x^2} - q_y^2}{Q_x^2 + Q_y^2 + Q_z^2 - q_x^2} \quad \text{where the variable } K = (\pm 1) \tag{6}$$

$$\cos\alpha = \frac{K.S\sqrt{Q_x^2 + Q_z^2 - q_x^2} - \sqrt{Q_x^2 + Q_y^2 + Q_z^2 - q_x^2 - q_y^2} - Q_y.q_y}{Q_x^2 + Q_y^2 + Q_z^2 - q_x^2} \tag{7}$$

The length of extension of the particular rods (Fig. 2b) can be calculated as follows:

$$A_{-1} = \sqrt{R^2 + r^2 + z^2 - 2R\left(\frac{1}{4}r.\cos\beta + \cos\alpha\left(\frac{3}{4}r - \frac{1}{2}z.\sin\beta\right) + \frac{\sqrt{3}}{2}\sin\alpha\left(z + \frac{1}{2}r.\sin\beta\right)\right)} \tag{8}$$

$$A_0 = \sqrt{R^2 + r^2 + z^2 - 2R(r.\cos\beta + z.\sin\beta.\cos\alpha)} \tag{9}$$

$$A_{-1} = \sqrt{R^2 + r^2 + z^2 - 2R\left(\frac{1}{4}r.\cos\beta + \cos\alpha\left(\frac{3}{4}r - \frac{1}{2}z.\sin\beta\right) - \frac{\sqrt{3}}{2}\sin\alpha\left(z + \frac{1}{2}r.\sin\beta\right)\right)} \tag{10}$$

The magnitude of the working area is given by the parameters of the designed device. Its main limitations are the length of the telescopic rods and the max. angles to which the rods can be opened without is no collision with the fixed platform, or the frame of the carrier structure.

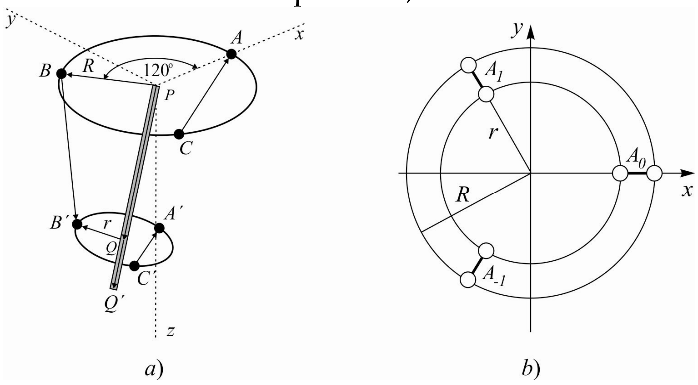


Figure 2. Scheme of location of telescopic rods and joints on the platform

To understand the abilities and restrictions of the presented type of the Tricept design we have made a thorough analysis which covered also the areas which can only be reached theoretically, or under special conditions. This analysis included also the so called“ambiguity areas”. By gradually increasing the rod length (the size of the rods being within the interval [589; 1189]) we created the work area in which 2 areas of ambiguity could be observed (Fig.3). These are the areas where at the same rod length we can get 2 different locations of the Q point. These areas did not appear at the rod length of 1090 mm. This length cannot be reached practically, as the real maximum rod length is 858 mm.

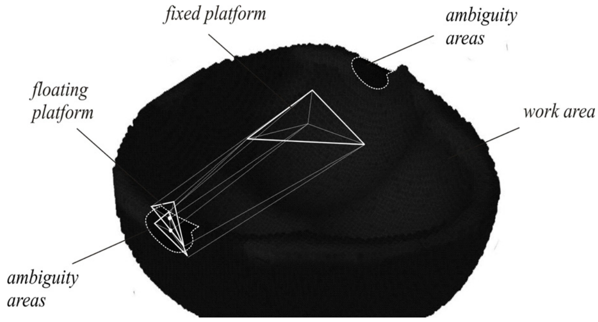


Figure 3. Theoretical work area with ambiguity areas

This analysis has revealed that the designed work area is symmetric only according to the plane xz, and not according to any platform passing through primary and central joints perpendicular on the fixed platform. One of the proofs is the ambiguity areas, their number and location. In case of symmetry, which would agree with the primary joints location, there should be three areas. The deviation from 120 ° symmetry is very small and at current work area angles almost unobservable (Fig. 4).

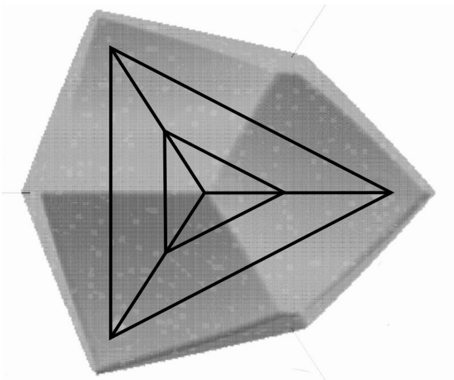

Figure 4. Representation of real work area (top view) with schematic drawing of triangles connecting the primary and secondary joints

To get a clear image of a real work area we have drawn it using contour (Fig.5) and we have projected its distribution into the cut (cross section) created by xz the plane (Fig.6). We also investigated what shape will the work area be, if its limitation will only be the geometrical shapes of the telescopic rods (Fig. 5a). We compared its size with the work area which shall be limited by the maximum deviation of each telescopic rod, before the motor or telescopic rod hits the fixed platform (Fig.5b). The restriction limit is the angle $\alpha = \pm 25^{\circ}$ a uhol $\beta = \pm 45^{\circ}$.

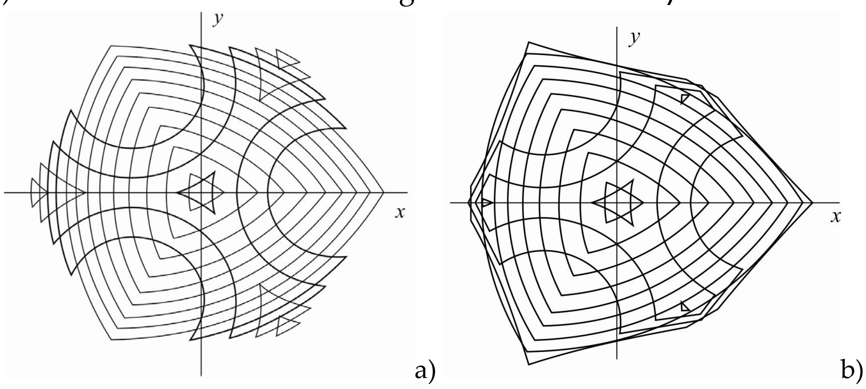


Figure 5. Comparing the contour lines of the top view of the Tricept work area a) without restricting the deviation angles of telescopic rods

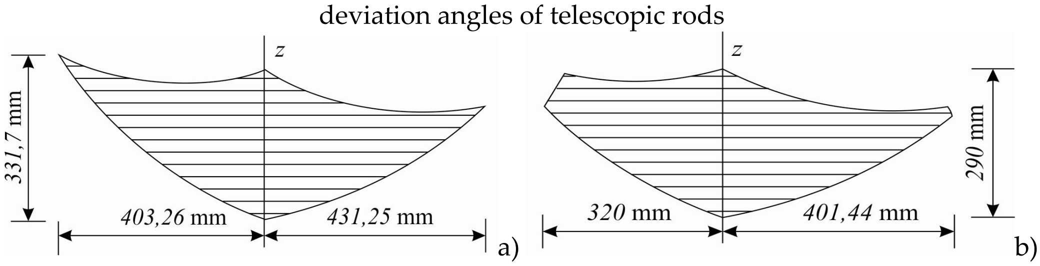


Figure 6. Comparing of cross sections of Tricept work areas with depicted contour lines a)without limiting angle deviation b) with limiting angle deviation of telescopic rods

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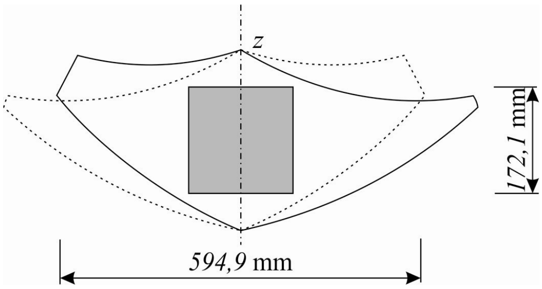

Figure 7. Clear work area with the cube

## CONCLUSION

All reflections and analyses concerning the work area are directed towards determining the dimensions of the largest cube, which can be placed into the clear work area as a whole. The clear work area is created by rotating the cross section (Fig.6) around z axis (Fig.7). The cube will be used for measuring the real Tricept motion.

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