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AN EFFICIENT ITERATIVE METHOD FOR SOLVING DISCRETE KDV EQUATION

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Abstract: In this study, accurate analytical solution for KDV Equation is derived. This solution is called Reconstruction Variational Iteration Method (RVIM). Numerical results show the efficiency of the proposed algorithm. By using RVIM, only a few iterations lead us to high accuracy of the solutions and it is valid for whole solution domain.
Keywords: Reconstruction Variational Iteration Method (RVIM), KDV Equation

1. INTRODUCTION

Differential equations are widely used to describe physical problems. In most cases, the exact solution of these problems may not be available. In addition, it is much easier computing and analyzing these solutions by means of the numerical methods without wasting time or spending money for experimenting problems. Alternatively, the numerical methods can provide approximate solutions rather than the exact solutions. But most of these methods have low accuracy and are highly time consuming. Reaching to a high accurate approximation for linear and nonlinear equations has always been important while it challenges tasks in science and engineering. Therefore several numbers of approximate methods have been established like Homotopy perturbation Method (HPM), Variational Iteration Method (VIM) and so on each of which has advantages and disadvantages. We introduce a new analytical method of nonlinear problems called the reconstruction of variational iteration method, which in the case of comparing with VIM [1-4] and HPM [5-8], not uses Lagrange multiplier as variational methods do and not requires small parameter in equations as the perturbation techniques. RVIM has been shown to solve a large class of nonlinear problems with approximations converging to solutions rapidly, effectively, easily, and accurately. The method used gives rapidly convergent successive approximations. As stated before, we aim to achieve analytic solutions to problems. We also aim to approve that the reconstruction of variational iteration method is powerful, efficient, and promising in handling scientific and engineering problems.

2. Basic Idea of RVIM

To clarify the basic ideas of our proposed method in [9], we consider the following differential equation same as VIM based on Lagrange multiplier [10]

$$Lu(x_1,\cdots,x_k)+Nu(x_1,\cdots,x_k)=f(x_1,\cdots,x_k) \quad (1)$$

By suppose that

$$Lu(x_1,\cdots,x_k)=\sum_{i=0}^{k} L_{xi}u(x_i) \quad (2)$$

where L is a linear operator, N a nonlinear operator and $f(x_1, \ldots x_k)$ an inhomogeneous term.

We can rewrite equation (1) down a correction functional as follows:

$$L_{xj}u(x_j) = \underbrace{f(x_1,\cdots,x_k) - Nu(x_1,\cdots,x_k) - \sum_{\substack{i=0\\ i\neq j}}^{k} L_{xi}u(x_i)}_{h((x_1,\cdots,x_k),u(x_1,\cdots,x_k))} \tag{3}$$

therefore

$$L_{xj}u(x_j) = h((x_1,\cdots,x_k),u(x_1,\cdots,x_k)) \tag{4}$$

With artificial initial conditions being zero regarding the independent variable xj .

By taking Laplace transform of both sides of the equation (4) in the usual way and using the artificial initial conditions, we obtain the result as follows

$$P(s).U(x_1,\cdots,x_{i-1},s,x_{i+1},x_k) = H((x_1,\cdots,x_{i-1},s,x_{i+1},x_k),u) \tag{5}$$

Where P(s) is a polynomial with the degree of the highest derivative in equation (5), (the same as the highest order of the linear operator L_{xj}). The following relations are possible;

$$\ell[\mathrm{h}] = \mathrm{H} \tag{6-a}$$

$$\mathrm{B(s)} = \frac{1}{\mathrm{P(s)}} \tag{6-b}$$

$$\ell[\mathrm{b(x_i)}] = \mathrm{B(s)} \tag{6-c}$$

Which that in equation (6-a) the function $H((x_1,\cdots,x_{i-1},s,x_{i+1},x_k),u)$ and $h((x_1,\cdots,x_{i-1},x_i,x_{i+1},x_k),u)$ have been abbreviated as *H, h* respectively.

Hence, rewrite the equation (5) as;

$$U(x_1,\cdots,x_{i-1},s,x_{i+1},x_k) = H((x_1,\cdots,x_{i-1},s,x_{i+1},x_k),u).B(s) \tag{7}$$

Now, by applying the inverse Laplace Transform on both sides of equation (7) and by using the (6-a) - (6-c), we have;

$$\mathrm{u}(\mathrm{x}_1,\cdots,\mathrm{x}_{i-1},\mathrm{x}_i,\mathrm{x}_{i+1},\mathrm{x}_k) = \int_0^{\mathrm{x}_i} \mathrm{h}((x_1,\cdots,x_{i-1},\tau,x_{i+1},x_k),u).b(x_i-\tau)d\tau \tag{8}$$

Now, we must impose the actual initial conditions to obtain the solution of the equation (1). Thus, we have the following iteration formulation:

$$\begin{aligned}\mathrm{u}_{n+1}(\mathrm{x}_1,\cdots,\mathrm{x}_{i-1},\mathrm{x}_i,\mathrm{x}_{i+1},\mathrm{x}_k) &= \mathrm{u}_0(\mathrm{x}_1,\cdots,\mathrm{x}_{i-1},\mathrm{x}_i,\mathrm{x}_{i+1},\mathrm{x}_k)\\ &\quad + \int_0^{\mathrm{x}_i} \mathrm{h}((x_1,\cdots,x_{i-1},\tau,x_{i+1},x_k),u).b(x_i-\tau)d\tau\end{aligned} \tag{9}$$

where u_0 is initial solution with or without unknown parameters. Assuming u_0 is the solution of *Lu*, with initial/boundary conditions of the main problem, In case of no unknown parameters, u_0 should satisfy initial/ boundary conditions. When some unknown parameters are involved in u_0, the unknown parameters can be identified by initial/boundary conditions after few iterations, this technology is very effective in dealing with boundary problems. It is worth mentioning that, in fact, the Lagrange multiplier in the He's variational iteration method is $\lambda(\tau) = b(x_i-\tau)$, as shown in [9].

The initial values are usually used for selecting the zero[th] approximation u_0. With u_0 determined, then several approximations u_0, $n > 0$, follow immediately. Consequently, the exact solution may be obtained by using

$$\mathrm{u}(\mathrm{x}_1,\cdots,\mathrm{x}_{i-1},\mathrm{x}_i,\mathrm{x}_{i+1},\mathrm{x}_k) = \lim_{n\to\infty}\mathrm{u}_n(\mathrm{x}_1,\cdots,\mathrm{x}_{i-1},\mathrm{x}_i,\mathrm{x}_{i+1},\mathrm{x}_k). \tag{10}$$

3. Application of RVIM to Discrete KDV Equation

In this section, we apply He's variational iteration method (VIM) for solving the governing equation of a physical problem which is a discrete KdV and is given by [10]:

$$\frac{du_n}{dt} = u_n^2(u_{n+1} - u_{n-1}). \tag{11}$$

with initial conditions

$$u_n(0) = 1 - \frac{1}{n^2} \tag{12}$$

The exact solution of the above problem is given by

$$u_n(t) = 1 - \frac{1}{(n+2t)^2} \tag{13}$$

Here, auxiliary linear operator is selected as $L_t u(x,t) = u_t$. By using the Eq. (11) we have the following operator form equation:

$$L_t u(t) = u_{n_t} = \overbrace{u_n^2 (u_{n+1} - u_{n-1})}^{w(u(t))} \tag{14}$$

Now Laplace transform is implemented with respect to independent variable x on both sides of eq. (14) and by using the new artificial initial condition (which all of them are zero) we have

$$s \cup (t) = \ell\{h(t,u)\} \tag{15}$$

$$\cup (t) = \frac{\ell\{h(t,u)\}}{s} \tag{16}$$

And whereas Laplace inverse transform of 1/s is as follows

$$\ell^{-1}[\frac{1}{s}] = 1 \tag{17}$$

Therefore by using the Laplace inverse transform and convolution theorem it is concluded that

$$u(t) = \int_0^t h(\varepsilon, u) d\varepsilon \tag{18}$$

Hence, we arrive the following iterative formula for the approximate solution of subject to the initial condition (13).

So, in exchange with applying recursive algorithm, following relations are achieved

$$u_{n,m+1}(t) = u_{n,0}(t) + \int_0^t u_{n,m+1}{}^2 (u_{n+1,m+1} - u_{n-1,m+1}) d\tau \tag{19}$$

Therefore we begin with $u_n(0) = 1 - \frac{1}{n^2}$, accordingly by the equation (19) one can get the higher order approximation of the exact solution as the following relations;

$$u_{n,1}(t) = 1 - \frac{1}{n^2} + \frac{4}{n^3} t$$

$$u_{n,2}(t) = 1 - \frac{1}{n^2} + \frac{4}{n^3} t - \frac{12}{n^4} t^2$$

$$u_{n,3}(t) = 1 - \frac{1}{n^2} + \frac{4}{n^3} t - \frac{12}{n^4} t^2 + \frac{32}{n^5} t^3$$

$$\vdots$$

The series solution is given by

$$u_n(t) = 1 - \frac{1}{n^2} + \frac{4}{n^3} t - \frac{12}{n^4} t^2 + \frac{32}{n^5} t^3 - \ldots$$

and the closed form solution is given as

$$u_n(t) = 1 - \frac{1}{(n+2t)^2}$$

4. CONCLUSION

In this paper, the RVIM method has been successfully applied to find the solution of the physical problem related to a discrete KdV equation. It gives rapidly convergent successive approximations through using the RVIM's iteration relation without any restrictive assumptions or transformation that may change the physical behavior of the problems. Moreover, RVIM reduces the size of calculations by not requiring the tedious Adomian polynomials, and hence the iteration is direct and straightforward.

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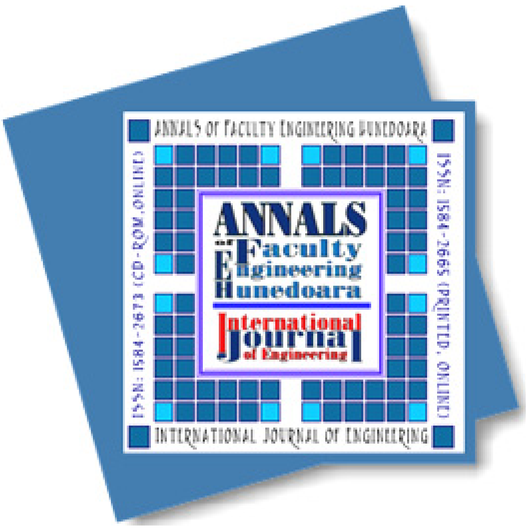

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