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FINITE ELEMENT MODELLING OF FUNCTIONALLY GRADED SPHERICAL PRESSURE VESSELS

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Abstract: The main objective of this paper is to present an easly feasiblemethod for solving the steady-state thermoelastic problem of functionally graded components using commercial finite element codes. In our example a radially graded spherical body is considered which is subjected tospherically symmetric mechanical and thermal loads.The material properties of the functionally graded material depend on the radial coordinate and the temperature field, the used FEM software is the ABAQUS CAE.The multilayered approach is used to get the functions of the displacement and temperature field, the normal stresses and equivalent stresses.
Keywords: FGM part, thermoelastic, finite element modelling

1. INTRODUCTION

Pure metals are used less in engineering applications because of the demand of conflicting property requirement. For example, an application may require a material that is hard as well as ductile, there is no such material existing in nature. In order to solve this problem, metal are combined with other metals or non-metals in order to improve the properties of material. The concept of the functionally graded materials (FGMs) was first considered in Japan in 1984 during a hypersonic spaceplane project. The body of the spaceplane would be exposed to very high temperature environment (about 2000K), with a temperature gradient of approximately 1000K, between inside and outside of the spaceplane. At that time there was no uniform material able to endure such conditions. The researchers wanted to create a material by gradually changing (grading) the material composition in order to improve both the thermal resistance and the mechanical properties [1], [2]. The smooth transitions between the constituent materials(and their properties) eliminate the potential cracking and debonding what is one of the advantages over the other modern material group, the composites. In recent years this concept has become more popular in Europe.

From the point of view of fabrication methods the FGM structural components are divided into two groups: surface coating and bulk FGMs [1]. This paper deals with the modelling of the bulk functionally graded parts. Bulk FGMsare produced using powder metallurgy technique, centrifugal casting method, solid freeform (SFF) technology etc. [3]. To produce bulk functionally graded materials the laser based SFF methods are utilized mostly (laser cladding based method, Selective Laser Sintering, 3-D Printing and Selective Laser Melting). Solid freeform is an additive manufacturing process that offers lots of advantages for example higher speed of production, less energy intensive, maximum material utilization, ability to produce complex shapes and design freedom as parts are produced directly from CAD data [4]. By this method after the processing of the CAD files the component is built layer by layer (an example can be seen in Fig. 1). This means, that in many cases the multilayered approach can be accurate enough to deal with the mechanical analysis of such components.

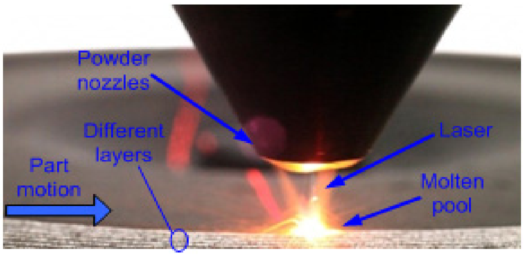

Figure 1. Fabrication possibility of FGMs (SFFmethod)

In the past few years significant amount of reviews dealt with the mechanics of the fuctionally graded materials from various aspects. The analytical solution for the thermomechanical problems of the FGMs is very complicated but we can find a few of them in some papers [5-11] in which the material parameters depend only on the spatial position. The derivation of closed form solutions for the displacement field and thermal stresses of functionally graded components with temperature- and spatial coordinate dependent properties is very hard even by some simple structural components (such as disks or spherical bodies), so the finite element simulation seems to be an effective way to deal with this kind of problems. Furthermore we want to look for a method which can be executed without any special software enviroment. This means that we want to avoid user subroutines (where special compilers are needed) and other special tools and only use the preprocessor of the chosen FE code: the ABAQUS CAE.

We will consider aspherical pressure vesselwithspherically symmetric thermal and mechanical load. There are first kind thermal boundary conditions prescribed on the inner (t_{inner}) and on the outer (t_{outer}) boundary surfaces, the constant pressure exerted on the inner and outer surfaces are denoted by p_{inner} and p_{outer} respectively. The material parameters are depend on the radial coordinate (r) and temperature field ($T(r)$). The functionally graded spherical vessel is modelled as a multilayered body with constant (from the point of view of the spatial- dependency), temperature-dependent material properties. The number of the layers is denoted by n. Our aim is to determine the displacement field ($u(r)$), radial stress (σ_r) and the tangential stresses ($\sigma_\phi=\sigma_\vartheta$).

2. THE MATERIAL PROPERTIES

Since functionally graded components are most commonly used in high temperature environment where significant changes in mechanical properties of the constituent materials are to be expected, it is essential to take into consideration this temperature-dependency for accurate prediction of the mechanical response. Thus, the effective Young's modulus E_f, Poisson's ratio ν_f, thermal expansion coefficient $\alpha_f$and thermal conductivity $\lambda_f$are assumed to be temperature dependent.Several micromechanics models have been developed over the years to infer the effective properties of FGMs. The Mori–Tanaka scheme [12], [2] for estimating the effective moduli is applicable to regions of the graded microstructure which have a well-defined continuous matrix and a discontinuous particulate phase. It takes into account the interaction of the elastic fields among neighboring inclusions. Another method is the self-consistent method [2] which assumes that each reinforcement inclusion is embedded in a continuum material and does not distinguish between matrix and reinforcement phases. But in many cases the material parameters can be expressed as a nonlinear functions of the temperature field [13], [2]:

$$M_{eff}(T)=P_0(P_{-1}T^{-1}+1+P_1T+P_2T^2+P_3T^3)\,. \tag{1}$$

In Eq. (1) $M_{eff}(T)$ denotes the function of the considered effective material property (E_f, ν_f, α_f or λ_f), P_0, P_{-1}, P_1, $P_2$and P_3 are material dependent coefficients of temperature (T [K]). Using this result the functions of the temperature- and position-dependent functionally graded material properties:

$$M_{eff}(r,T)=\left[M_{eff1}(T)-M_{eff2}(T)\right]\left[C(r)\right]^N+M_{eff2}(T)\,, \tag{2}$$

where the $C(r)$ function is $C^{Sphere}(r)=\dfrac{r-a}{b-a}$, by spherical bodies (this will be used for the further calculations) and $C^{Plate}(r)=\dfrac{2z-h}{2h}$, by plates, the 1 and 2 indexes denote the metal and ceramic components, h is the thickness of the considered structural component (for example the thickness of a plate), z is the thickness coordinate,a and b denote the inner and outer radii of the sphere, and N is the volume fraction exponent (Fig. 2) of the material.

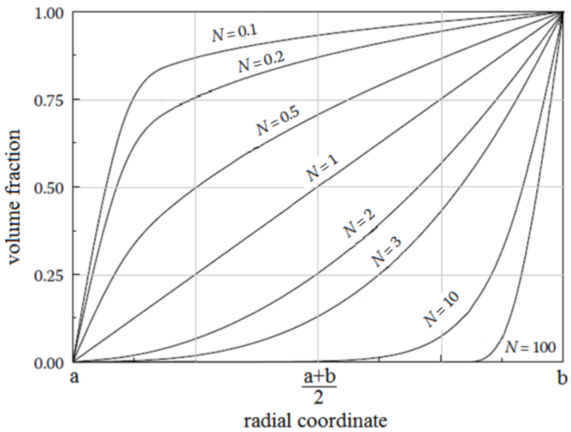

Figure 2. The volume fraction of material 1 along the radial coordinate

In our example a stainless steel - silicon nitride FGM will be considered. The material parameters of this FGM can be seen in Table 1.

Table 1. The material properties of the considered FGM

Material property (M_{eff})	stainless steel (1)				silicon nitride (2)			
	P_{m0}	$P_{m1}(10^{-3})$	$P_{m2}(10^{-7})$	$P_{m3}(10^{-10})$	P_{c0}	$P_{c1}(10^{-3})$	$P_{c2}(10^{-7})$	$P_{c3}(10^{-11})$
λ(W/mK)	15.39	-1.264	20.92	-7.223	12.723	-1.032	5.466	-7.876
α (1/K)	$12.33\cdot10^{6}$	0.8086	0	0	$3.873\cdot10^{-6}$	0.9095	0	0
E (Pa)	$2.01\cdot10^{11}$	0.3079	-6.534	0	$3.484\cdot10^{11}$	-0.307	2.16	-8.946
ν (-)	0.3262	-0.1	3.797	0	0.24	0	0	0

The steps of the modelling method are:

- The first step of the modelling is the creation ofthe segmentedgeometry (as the partitioning of the functionally graded body). The material properties within the segments (for example by FGM plates with one dimensional grading: the segments are layers) are temperature dependent, so the direction of the grading designate the position of the segments, while the volume fraction exponent and theparameters (especially their change) of the constituent materials and the desired accuracy of the simulationspecify the geometry (thickness) of the different partitions.
- The second step is the calculation of the material parameters for each segment. In this step first we determine the temperature-dependent functions of the material properties for each segments as $M(T)=M(r=r_{median},T)$, then create these functions in discrete points. The points are generated by appropriate stepts in the radial coordinate.
- The third step is the creation of the assembled geometric model. Within the coupled temperature-displacement step the boundary conditions and loads can be adjusted. Among the constrains the osculant surfaces of the adjacent segmentsmust be tied together.
- In the last step we have to create the mesh and choose the most appropriate element group (from the element types of the coupled temperature-displacement analysis).

3. MODELLING OF THE PROBLEM

A thick-walled (r/h=5.95) spherical pressure vessel is considered with d_{inner}=1m inner diameter and h=84mm wallthickness, in addition a=0.5m, b=0.584m, t_{inner}=698K, t_{outer}=303K, p_{inner}=100MPa, p_{outer}=0MPa, N=3 and three cases with three different layer numbers were investigated(n_1=7, n_2=14, n_3=28).

The problem is axisymmetric, so a quarter of the spherical vessel is modeled. The functionally graded sphere is modelled as a multilayered body. Due to the radial grading, the layers

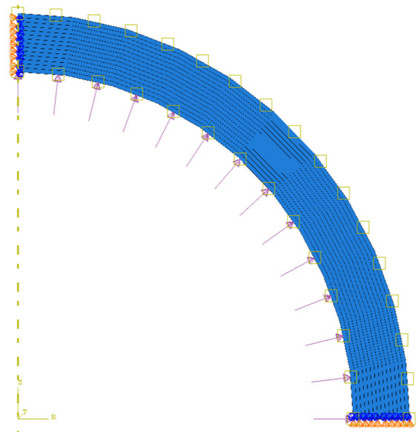

Figure 3. The modell of the multilayered (n=14) spherical vessel with the mechanical load (arrows: p_{inner}), the first-kind thermal boundary condition (squares) and the boundary conditions (symmetry conditions)

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should beconcentrical hollow spheres with h/n wall thickness.Thelayers have temperature-dependent material parameters and thematerial properties (M: E, α, λ and ν) for the different layers can be calculated by:

$$R_{mi} = \frac{R_i + R_{i+1}}{2}, \quad M(T) = M(r = R_{mi}, T), \qquad i = 1 \ldots n \tag{3}$$

The next step is the input of the material parameter-temperature functions for every different layers using the simpliest method of the preprocessor: tabular input. *Maple 15* mathematical software was used to calculate these couples of values from 25^{0}C to 425^{0}C by the step of 25^{0}C then it was copied into the material creation module. In the assembly module the previously created layers can be assemebled by position constrains. The problem can be solved using the coupled temperature-displacement stepof the ABAQUS CAE. In this case we choose the steady-state option but there is a transient option too. In the boundary condition module we allow the movement of the nodal points on the horizontal edge only in X direction, on the vertical edge only in the Ydirection, furthermore the temperature of the inner and outer boundary surfaces (first-kind thermal boundary conditions) were entered (Figure 3). In the next step we tie the osculant surfaces of the adjacent layers (Constraints/Tie module). For the meshing of the model an 8-node, coupled temperature-displacement, quadrilateral element group was used (CAX8T). The number of elements and nodes for the three different meshes (Figure 4) are: (350; 1771), (700; 3542) and (2800; 14084).

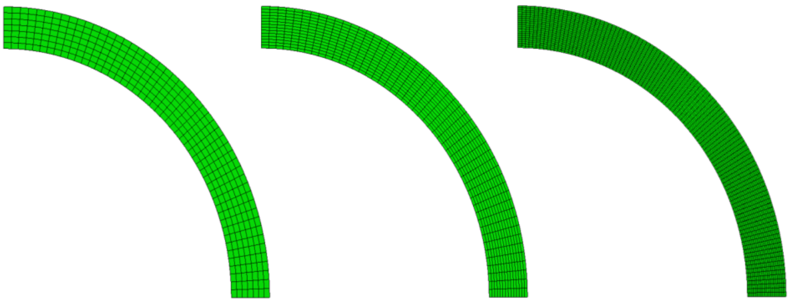

Figure 4. The investigated meshes(n=7,14 and 28).

The results and their convergence can be seen in Figures 5-7. The result for the temperature field were accurate enough even by n=7 layers.

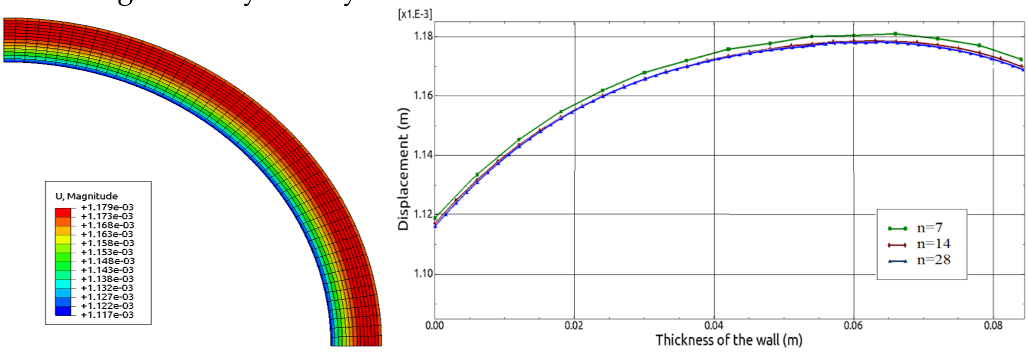

Figure 5. The displacement fields of the functionally graded vessel.

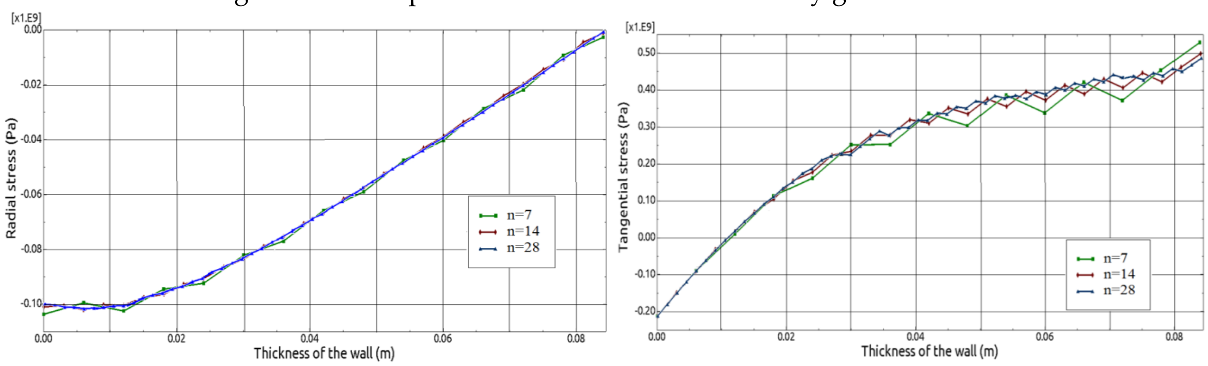

Figure 6. The normal stressesof the investigated cases.

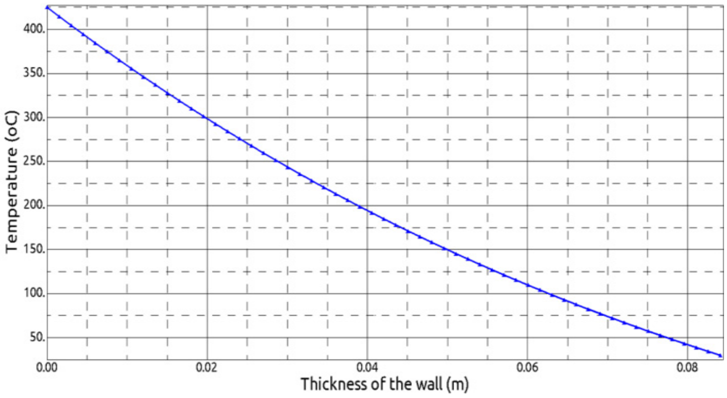

Figure 7. The temperature field (n=28).

In Figure 6 we can see that the function of the tangential stresses contains significant oscillations and the values of the first and last layers have greater error than in the other layers. In order to reduce these errors it is recommended to copy the results (the stress values, except the last value of the tangential stresses) of the simulation and fit a curve to these calculated values. For example we can use the Maple 15 and its curve fitting method (via least squares method) to execute this step. In the last step we can calculate the equivalent stress and analyze the yield streng of the component, for example we can use the Mises criteria:

$$\sigma_{eq-Mises} = \left|\sigma_{\varphi} - \sigma_r\right| . \tag{3}$$

In our case there is 9.8% relative error between the maximum values of the equivalent stress values of the cases (Figure 8):n=28 and n=7, and 3.2% between the results of the cases:n=28 and n=14.

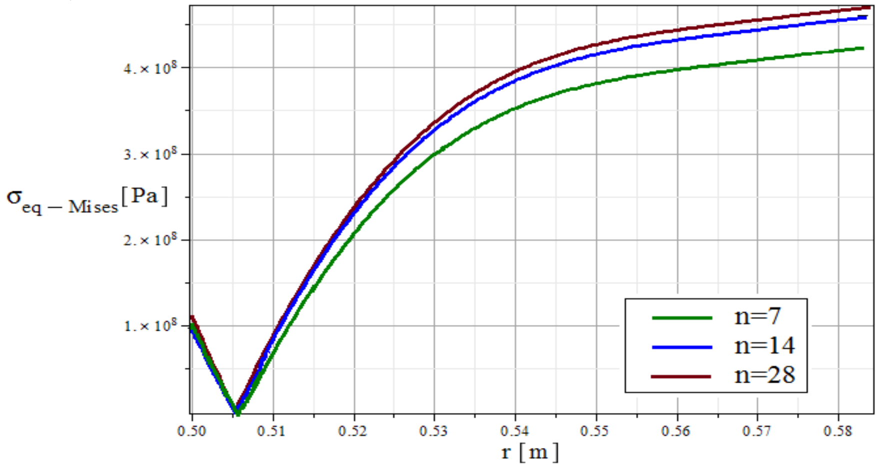

Figure 8. Building the equivalent stress functions from the approximated curves

4. CONCLUSIONS

This paper presents a finite element modelling technique for functionally graded components via the example of asteady-state spherically symmetric thermoelastic problem of a functionally graded spherical pressure vessel which is subjected to mechanical and thermal loads. The body of the component is modeled as a segmented body with different discretized material properties within each segments. Three cases were invesigated with three different layer numbers and the equivalent stresses are presented. In order to solve this problem, the ABAQUS CAE FE software and Maple 15 mathematical softwarewere used.

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