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RADIATION AND CHEMICAL REACTION EFFECTS ON MHD FLOW PAST AN OSCILLATING VERTICAL POROUS PLATE WITH CONSTANT HEAT FLUX

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Abstract: This paper presents an analysis of combined heat and mass transfer MHD flow past an oscillating vertical porous plate under the action of radiation effects and chemical reaction when heat is supplied to the plate at constant rate. The governing equations are solved in closed analytical method. The results are obtained for velocity, temperature, concentration, skin friction, Nusselt number and Sherwood number. The effects of various parameters on flow variables are illustrated graphically, and the physical aspects of the problem are discussed qualitatively.
Keywords: Radiation, MHD, Heat and Mass transfer, Chemical reaction, Skin-friction

1. INTRODUCTION

The phenomenon of MHD flow with heat and mass transfer has been a subject of interest of many researchers because of its varied applications in science and technology. Such phenomena are observed in buoyancy induced motions in the atmosphere, in bodies of water, quasi-solid bodies such as earth, etc. Unsteady oscillatory free convective flows play an important role in chemical engineering, turbo- machinery and in aerospace technology. Such flows arise either due to unsteady motion of boundary or boundary temperature. Besides, unsteadiness may also be due to oscillatory free stream velocity or temperature. In natural processes and industrial applications many transport processes exist where transfer of heat and mass takes place simultaneously as a result of combined buoyancy effects of thermal diffusion and diffusion of chemical species. The phenomenon of heat and mass transfer is also very common in chemical process industries such as food processing and polymer production. Several researchers have analyzed the free convection and mass transfer flow of a viscous fluid through porous medium. In their studies, the permeability of the porous medium is assumed to be constant while the porosity of the medium may not necessarily be constant because the porous material containing the fluid is a non-homogeneous medium. Therefore, the permeability of the porous medium may not necessarily be a constant. In light of these facts, Soundalgekar and Gupta (1977) investigated the effect of free convection on oscillatory flow past an infinite vertical plate with variable suction and constant heat flux. Georgantopolous et al. (1981) have estimated the effect of mass transfer on free convective hydromagnetic oscillatory flow past an infinite vertical porous plate. Hayat et al. (1998) have reported the periodic unsteady flows of a non-Newtonian fluid. Kim (2000) studied the unsteady MHD convective heat transfer past a semi-infinite vertical porous moving plate with variable suction. Asghar et al. (2004) have reported the flow of a non-Newtonian fluid induced due to the oscillations of a porous plate.

The problem of three dimensional free convective flow and heat transfer through a porous medium with periodic permeability has been discussed by Singh and Sharma (2002). Bathul (2005) discussed the heat transfer in a three dimensional viscous flow over a porous plate moving with a harmonic disturbance. Singh et al (2005) have analyzed the heat and mass transfer in MHD flow of a viscous fluid past a vertical plate under oscillatory suction velocity. Ogulu and Prakash (2006) considered heat transfer to unsteady magneto hydrodynamic flow past an infinite vertical moving plate with variable suction. Singh and Gupta (2005) have investigated the MHD free convective flow of a viscous fluid through a porous medium bounded by an oscillatory porous plate in the slip-flow regime with mass transfer. Das et al. (2006) discussed the free convective and mass transfer flow of a viscous fluid past an infinite vertical porous plate through a porous medium in presence of source/sink with constant suction and heat flux. Reddy et al (2013) discussed the thermal radiation and magnetic field effects on unsteady mixed convection flow and mass transfer over a porous stretching surface with heat generation.

In the studies mentioned above, the oscillatory surface velocity in presence of time dependent viscosity along with the influence of uniform magnetic field with radiation parameter have not been discussed while such flows are very common in geophysical and astrophysical problems and also in soil sciences. Therefore, the objective of the present study is to analyze the effects of permeability variation and oscillatory surface with constant heat flux in MHD free convective heat and mass transfer flow of a viscous incompressible fluid past an infinite vertical porous plate through a porous medium when the plate is subjected to a time dependent suction velocity normal to the plate in the presence of a uniform transverse magnetic field with heat flux. The solutions for velocity field, temperature field and concentration distribution are obtained using perturbation technique. The results obtained are discussed for Grashof number, $Gr > 0$ corresponding to cooling of the plate. The effects of the flow parameters such as magnetic parameter M, Grashof number for heat and mass transfer Gr, Gm, permeability parameter K, Prandtl number $\Pr$, Schmidt number Sc radiation parameter R, chemical reaction parameter Kr on the velocity, temperature and concentration distribution of the flow field have been studied analytically and presented graphically. Further, the effects of the flow parameters on skin friction, Nusselt number and Sherwood number have been discussed.

2. MATHEMATICAL ANALYSIS

We consider a two-dimensional magnetohydrodynamic flow of an incompressible and electrically conducting viscous incompressible fluid along an infinite vertical porous plate. The x' - axis is taken on the infinite plate and parallel to the free stream velocity and y'- axis normal to it. Initially, the plate and the fluid are at same temperature T'_∞ with concentration level C'_∞ at all points. At time $t' > 0$, the plate concentration is changed to C'_w with heat supplied at a constant rate to the plate and it starts oscillating with a velocity $U_R \cos \omega t$ in its own plane. It is assumed that there exists a homogeneous chemical reaction of first order with constant rate k_r between the diffusing species and the fluid. Since the plate is infinite in extent therefore the flow variables are the functions of y' and t' only. The fluid is considered to be gray absorbing-emitting radiation but non scattering medium. The radiative heat flux in the x' direction is considered negligible in comparison with of y' direction. It is assumed that there is no applied voltage of which implies the absence of an electric field. The transversely applied magnetic field and magnetic Reynolds number are very small and hence the induced magnetic field is negligible. The MHD term is derived from an order-or-magnitude analysis of the full Navier-Stokes equation. The governing equation for this investigation is based on the balances of mass, linear momentum, energy, and concentration species. Taking into consideration the assumptions made above, these equations can be written in Cartesian frame of reference, as follows

$$\frac{\partial u'}{\partial t'} = \nu \frac{\partial^2 u'}{\partial y'^2} + g\beta\left(T' - T'_\infty\right) + g\beta^*\left(C' - C'_\infty\right) - \frac{\sigma B_0^2}{\rho} u' - \frac{\nu}{k'} u' \quad (1)$$

$$\frac{\partial T'}{\partial t'}=\frac{k}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2}-\frac{1}{\rho c_p}\frac{\partial q_r'}{\partial y'} \tag{2}$$

$$\frac{\partial C'}{\partial t'}=D\frac{\partial^2 C'}{\partial y'^2}-k_r C' \tag{3}$$

with following initial and boundary conditions:

$$\begin{aligned} &u'=0,\ T'=T_\infty',\ C'=C_\infty' \quad \forall y',\ t'\le 0,\\ &u'=U_R\cos\omega' t',\ \frac{\partial T'}{\partial y'}=-\frac{q}{\kappa},\quad C'=C_w' \qquad \text{at} \quad y'=0,\ t'>0\\ &u'\to 0,\ T'\to T_\infty',\quad C'\to C_\infty' \qquad \text{as} \quad y'\to\infty,\ t'>0 \end{aligned} \tag{4}$$

By using the Rosseland's approximation, the radiative heat flux vector q_r can be written as:

$$q_r'=-\frac{4\sigma^*}{3\kappa^*}\frac{\partial T'^4}{\partial y'} \tag{5}$$

where, σ^* and κ^* are respectively the Stefan-Boltzmann constant and the mean absorption coefficient. We assume that the temperature difference within the flow is sufficiently small such that T'^4 may be expressed as a linear function of the temperature. This is accomplished by expanding in a Taylor series about the free stream temperature T_∞' and neglecting higher order terms, thus

$$T'^4\cong 4T_\infty'^3T'-3T_\infty'^4 \tag{6}$$

By using (5) and (6), (2) gives

$$\rho C_p\frac{\partial T'}{\partial t'}=\kappa\frac{\partial^2 T'}{\partial y'^2}+\frac{16\sigma^* T_\infty'^3}{3\kappa^*}\frac{\partial^2 T'}{\partial y'^2} \tag{7}$$

In order to write the governing equations and the boundary conditions in dimensionless form, the following non-dimensional quantities are introduced.

$$\begin{aligned} &t=\frac{t'}{t_R},\ y=\frac{y'}{L_R},u=\frac{u'}{U_R},\omega=\omega' t_R,M=\frac{\sigma B_0^2\nu}{\rho U_R^2},K=\frac{K'U_R^2}{\nu^2},K_r=\frac{K_r'U_R^2}{\nu^2}\\ &\theta=\frac{T'-T_\infty'}{q\nu/\kappa U_R},C=\frac{C'-C_\infty'}{C_w'-C_\infty'},\Pr=\frac{\nu\rho C_p}{\kappa},Sc=\frac{\nu}{D},Gr=\frac{g\beta q\nu^2}{\kappa U_R^2},\\ &Gm=\frac{\nu\beta^* g\left(C_w'-C_\infty'\right)}{U_R^2},R=\frac{\kappa^*\kappa}{4\sigma^* T_\infty'^3},\Delta T=\left(T_w'-T_\infty'\right),\\ &U_R=\left(\nu g\beta\Delta t\right)^{1/3},L_R=\left(\frac{g\beta\Delta T}{\nu^2}\right)^{-1/3},\mathrm{t_R}=\left(g\beta\Delta T\right)^{-2/3}\nu^{1/3}, \end{aligned} \tag{8}$$

In view of Equations (1) to (3) reduce to the following nondimensional form:

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial y^2}+G_r\theta+G_m C-\left(M+\frac{1}{K}\right)u \tag{9}$$

$$\Pr\frac{\partial\theta}{\partial t}=\frac{1}{\Pr}\left(1+\frac{4}{3R}\right)\frac{\partial^2\theta}{\partial y^2} \tag{10}$$

$$\frac{\partial C}{\partial t}=\frac{1}{Sc}\frac{\partial^2 C}{\partial y^2}-K_r C \tag{11}$$

The corresponding dimensionless boundary conditions are

$$\begin{aligned} &u=0,\ \theta=0,\ C=0 \quad \forall\ y\ ,\ t\le 0\\ &t>0,\ \begin{cases} u=\cos\omega t, & \dfrac{\partial\theta}{\partial y}=-1,\quad C=1 \quad \text{at} \quad y=0\\ u\to 0, & \theta\to 0,\quad C\to 0 \quad \text{as} \quad y\to\infty, \end{cases} \end{aligned} \tag{12}$$

3. SOLUTION OF PROBLEM

In order to reduce the above system of partial differential equations to a system of ordinary equations in dimensionless form, we may represent the velocity, temperature and concentration as

$$u(y,t) = u_0(y)e^{nt} \tag{13}$$

$$\theta(y,t) = \theta_0(y)e^{nt} \tag{14}$$

$$C(y,t) = C_0(y)e^{nt} \tag{15}$$

The corresponding boundary conditions can be written as

$$\begin{aligned} u_0 &= \cos\omega t, \quad \frac{\partial\theta_0}{\partial y} = -1, \; C_0 = 1, \qquad \text{at} \qquad y = 0 \\ u_0 &= 0, \quad \theta_0 \to 0, \; C_0 \to 0, \qquad \text{as} \qquad y \to \infty \end{aligned} \tag{16}$$

Substituting (13), (14) and (15) in Equations (9) - (11) and equating harmonic and non-harmonic terms, and neglecting the higher order terms of O(ε^2), we obtain the solutions of velocity, temperature and concentration are given by

$$u(y,t) = k_7 e^{-k_4 y} + k_5 e^{-k_3 y} + k_6 e^{-k_1 y}$$

$$\theta(y,t) = \frac{1}{k_3} e^{-k_3 y}$$

$$C(y,t) = e^{-k_1 y}$$

Here constants are not given due shake of brevity.

4. RESULTS AND DISCUSSION

For different values of the magnetic field parameter *M*, the velocity profiles are plotted in Fig.1. It is obvious that the effect of increasing values of *M* parameter results in decreasing velocity distribution across the boundary layer because of the application of transfer magnetic field will result a restrictive type force (Lorentz force) similar to drag force which tends to resist the fluid flow and thus reducing its velocity.

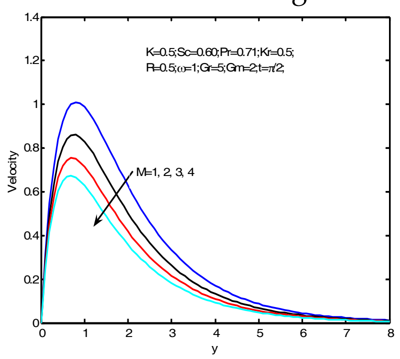

Fig-1. Velocity profiles for different values of magnetic parameter

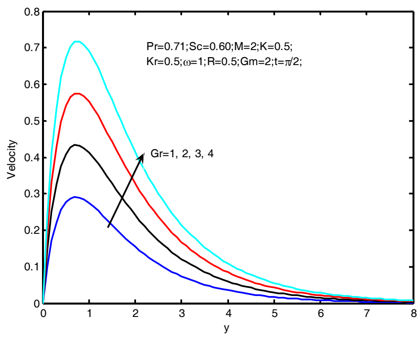

Fig-2. Velocity profiles for different values of Grashof number

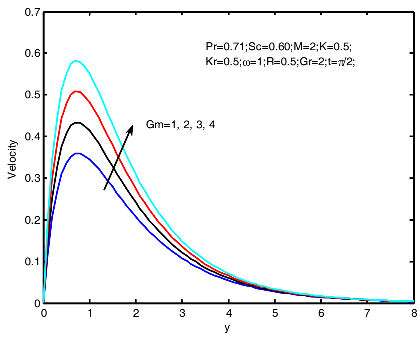

Fig-3. Velocity profiles for different values of modified Grashof number

The velocity profiles for different values of Grashof number Gr are described in Fig.2. It is observed that an increasing in Gr leads to a rise in the values of velocity. Here the Grashof number represents the effects of the free convection currents. Physically, $Gr > 0$ means heating of the fluid of cooling of the boundary surface $Gr < 0$ means cooling of the fluid of heating of the boundary surface and $Gr = 0$ corresponds to the absence of free convection current. In addition, the curves show that the peak value of velocity increases rapidly near the wall of the porous plate as Grashof number increases, and then decays to the relevant free stream velocity. The velocity profiles for different values of modified Grashof number Gm is described in Fig.3. It is observed that an increasing in Gm leads to a rise in the values of velocity. In addition, the curves show that the peak value of velocity increases rapidly near the wall of the porous plate as modified Grashof number increases.

For various values of Grashof number (Gr) and modified Grashof number (Gm), the velocity profiles are plotted in Figs. (2) and (3). The Grashof number Gr signifies the relative effect of the thermal buoyancy force to the viscous hydrodynamic force in the boundary layer. It is observed that there is a rise in the velocity due to the enhancement of thermal buoyancy force. The modified Grashof number (Gm) defines the ratio of the species buoyancy force to the viscous hydrodynamic force. As expected, the fluid velocity increases and the peak value is more distinctive due to increase in the species buoyancy force. The velocity distribution attains a distinctive maximum value in the vicinity of the surface. It is noticed that the velocity increases with an increasing values of the modified Grashof number. Fig-4 illustrates the variation of velocity distribution across the boundary layer for various values of the permeability parameter K. The velocity increases with increases in permeability parameter K.

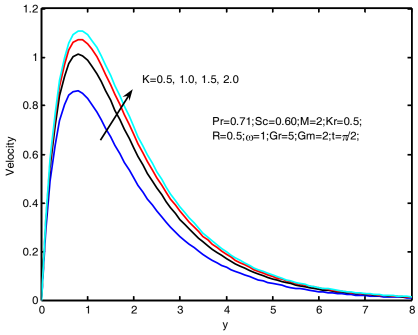

Fig-4. Velocity profiles for different values of permeability parameter

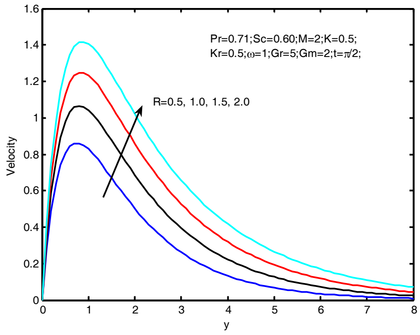

Fig-5. Velocity profiles for different values of radiation parameter

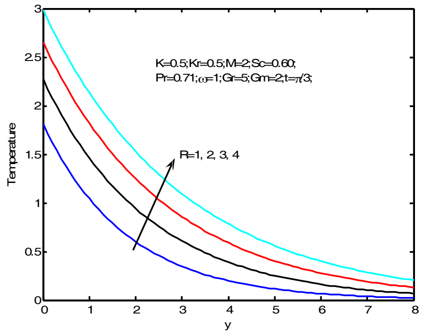

Fig-6. Temperature profiles for different values of radiation parameter

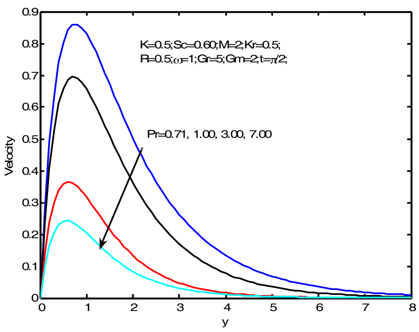

Fig-7. Velocity profiles for different values of Prandtl number

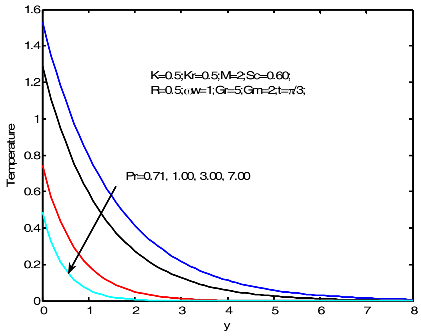

Fig-8. Temperature profiles for different values of Prandtl parameter

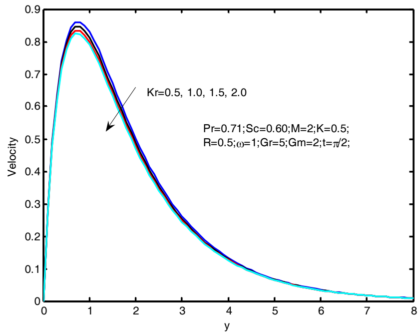

Fig-9. Velocity profiles for different values of chemical reaction parameter

For different values of the radiation parameter *R*, the velocity and temperature profiles are shown in Figs (5) and (6). It is noticed that an increase in the radiation parameter results an increase in the velocity and temperature within the boundary layer, also it increases the thickness of the velocity and temperature boundary layers.

Figs (7) and (8) are illustrates the velocity and temperature profiles for different values of Prandtl number. The numerical results show that the effect of increasing values of Prandtl number results in a decreasing velocity. Also, it is shown that increases in the Prandtl number results tend to a decreasing of the thermal boundary layer and in general it lowers the average temperature through the boundary layer. The reason is that, the smaller values of Pr are equivalent to increase in the thermal conductivity of the fluid and therefore heat is able to diffuse away from the heated surface more rapidly for higher values of Pr. Hence in the case of smaller Prandtl numbers, the thermal boundary layer is thicker and the rate of heat transfer is reduced.

For different values of chemical reaction parameter, the velocity and concentration profiles are plotted figs (9) and (10) respectively. As the chemical reaction parameter increases, the velocity and concentration profiles are decreases. Figs (11) and (12) are display the effects of Schmidt number on the velocity and concentration profiles, respectively. As the Schmidt number increases, the velocity and concentration decreases, because the smaller values of Sc are equivalent to increasing the chemical molecular diffusivity.

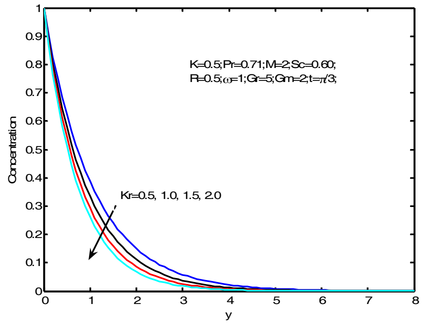

Fig-10. Concentration profiles for different values of chemical reaction parameter

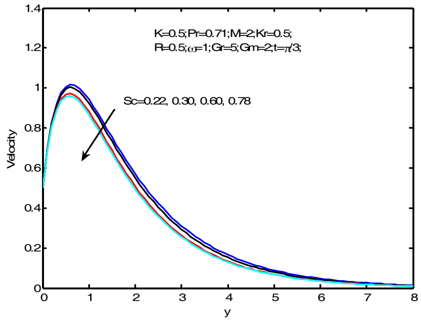

Fig-11. Velocity profiles for different values of Schmidt number

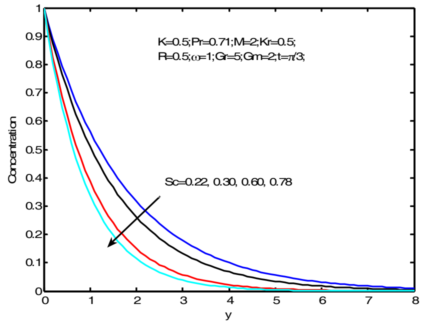

Fig-12. Concentration profiles for different values of Schmidt number

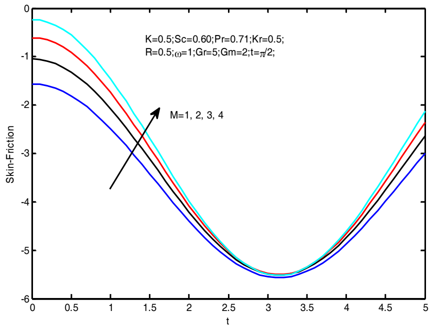

Fig-13. Skin-friction profiles for different values of magnetic parameter

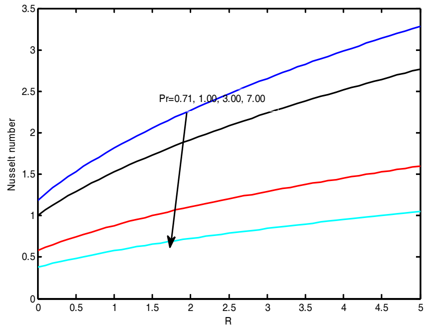

Fig-14. Nusselt number profiles for different values of Prandtl number

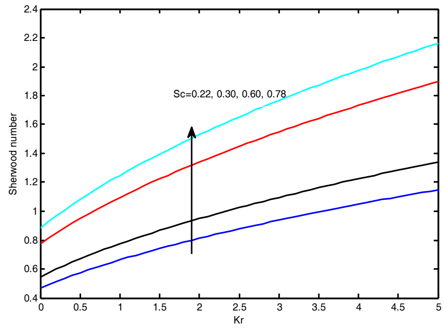

Fig-15. Sherwood number profiles for different values of Schmidt number

The effect of magnetic parameter (M) on the skin-friction is shown in Fig.13. As the magnetic parameter increases, the skin-friction is also increases. Fig.14 illustrates the effect of the Prandtl number (Pr) on the Nusselt number of the fluid under consideration. As the Prandtl number increases, the Nusselt number decreases. Fig.15 illustrates the effect of Schmidt number (Sc) on Sherwood number of the fluid under consideration. As the Schmidt number increases the Nusselt number is found to be increasing.

5. CONCLUSIONS

A mathematical model has been presented for analytically studies the radiation and chemical reaction effects on unsteady MHD double diffusive convective flow past an oscillating porous surface with constant heat flux. The fluid considered here is a gray, absorbing-emitting but non-scattering medium and the Rosseland approximation is used to describe the radiative heat flux in the energy equation. The dimensionless governing equations are solved using two term perturbation technique. The resulting velocity, temperature and concentration profiles are shown graphically for different values of physical parameters involved.

Numerical evaluations of closed form solutions were performed, and some graphical results were obtained to illustrate the details of flow, heat and mass transfer characteristics, and their dependence on some physical parameters. The physical aspects of the problem are also discussed. The result of the flow problem indicates the following:

(1) Fluid velocity decreases with an increase in magnetic parameter, Schmidt number, Prandtl number and chemical reaction parameter but increases with an increase in Grashof number, modified Grashof number, permeability parameter and radiation parameter.

(2) Temperature profile decreases with an increase in radiation parameter and Prandtl number increases the temperature profile decreases.

(3) Concentration profile decreases with an increase in chemical reaction parameter and Schmidt number.

(4) There is a rise in skin friction with increasing magnetic parameter and other parameters are fixed constants. The rate of heat transfer (Nusselt number) increases with an increase in the value of radiation parameter and Prandtl number. Sherwood number also increases with an increase in chemical reaction parameter and Schmidt number.

Nomenclature

U_R: Reference velocity
g: Gravitational acceleration
Cp: Specific heat at constant pressure
D: Mass diffusivity
β: Thermal expansion coefficient
β_C: Concentration expansion coefficient
ρ: Density
k: Thermal conductivity of fluid
k*: Mean absorption coefficient
σ: Electrical conductivity of fluid
ν: Kinematic viscosity
q_r: Radiative heat flux
σ: Stefan-Boltzmann constant
L_R: Reference length
t_R: Reference time
M: Magnetic parameter
K: Permeability parameter
Gm: Modified Grashof number
Pr: Prandtl number
Sc: Schmidt number
ω: Frequency of oscillation
u: Dimensionless velocity component
θ: Dimensionless temperature
C: Dimensionless concentration
μ: Viscosity of fluid
t: Time in dimensionless coordinate
R: Radiation parameter
Kr: Chemical reaction parameter

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