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NUMERICAL STUDY ON WEHICLE QUARTER MODEL WITH PROGRESSIVE SPRINGS AND LINEAR DAMPING IN CASE OF WEAK HARMONIC EXCITATION

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Abstract: In this paper quarter model known from vehicle dynamics with nonlinear springs and linear dampers is studied. Model is solved numerically in case of weak harmonic excitation. Transmission function is calculated for special parameters. Stroboscopic mapping is used for showing out features of the motion.
Keywords: nonlinear system, vehicle suspension, quarter model, numerical study

1. INTRODUCTION

Vibrations play important role in many different field of everyday life and in engineering [1,2].
Quarter model is a widely spreaded tool of vehicle dynamics [3]. Basic idea of it is a technical view of a car assembled from several parts. One wheel and attended parts form a subassembly which may have in itself relevant features determining dynamical behaviour of the whole system. This is why quarter model is often studied separately. Imagine, one cuts a car into four pieces in a symmetric way so that each of them involves one wheel, and the remaining part is compressed into a single point mass. Suspension linking the wheel with the car and effect of tyre are represented a spring and a damper.

In most simple case spring is linear in extension and damper is linear in velocity, i.e. the first time derivative of extension. Solution and transmission function of such a system are described in textbooks. Main features of the transmission function are continuity and two maxima at two different frequency.

2. QUARTER MODEL WITH A SPECIAL TYPE OF NONLINEARITY

Real suspension system contains nonlinear springs and nonlinear dampers. In our study damping is linear, but springs are nonlinear, so-called progressive springs. Nonlinearity is featured by a parameter ε. Force law can be written as

$$F = -k\Delta\ell - \varepsilon(\Delta\ell)^3,$$

where $\Delta\ell$ denotes extension of the spring, k the linear elastic coefficient and ε the nonlinear elastic coefficient. Third power ensures symmetry of the force law for elongation and compression. Negative sign of the nonlinear member means that this spring puts up force larger than corresponding linear spring. Difference between nonlinear and corresponding linear spring grows with extension of the spring. This type of nonlinear springs are often applied in wehicle structures, and are called hardening or progressive springs.

Quarter model built up nonlinear springs is sketched on Figure 1. Mass *M* equals the quarter of whole vehicle mas, *m* stands for mass of wheel and parts fixed to it. Both *M* and *m* are point masses, and can move only in vertical direction x, so position of them can be given by x coordinate of them. In equations of motion we will use displacements form the original position instead of x coordinates, and for the sake of simplicity of notations x_1 and x_2 means displacements of point masses instead of their absolute coordinates.

Indices *s* and *t* refers to „suspension" and „tyre". Effect of the suspension system is modelled by a spring and a damping with parameters k_s, ε_s, β_s . Suspension acts force

$$F_s = -k_s x_1 - \varepsilon_s x_1^3 + \beta_s \dot{x}_1.$$

Similarly, tyre is modelled by a nonlinear spring and a linear damping, and force acted by the tyre can be written as

$$F_t = -k_t x_2 - \varepsilon_t x_2^3 + \beta_t \dot{x}_2.$$

When a car moves, height of grund surface may vary. This generates a vertical excitation. In the quarter model it is taken into account by a vertical movement of the ground level. In general case this is stochastic. Now we study only the case when excitation of ground is a pure harmonic function, and can be described by formula $\varphi(t) = \varphi_0 \sin(\omega_g t)$.

Quarter model has two degrees of freedom. equations of motion are the followings:

$$M\ddot{x}_1 = -k_s(x_1 - x_2) - \varepsilon_s(x_1 - x_2)^3 - \beta_s(\dot{x}_1 - \dot{x}_2)$$

$$m\ddot{x}_2 = k_s(x_1 - x_2) + \varepsilon_s(x_1 - x_2)^3 + \beta_s(\dot{x}_1 - \dot{x}_2)$$

$$-k_t\left(x_2 - \phi(t)\right) - \varepsilon_t\left(x_2 - \phi(t)\right)^3 - \beta_t(\dot{x}_2 - \dot{\phi}(t))$$

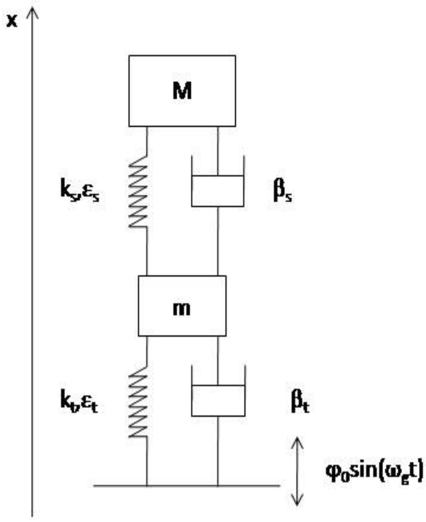

Figure 1. Quarter model of vehicles with nonlinear springs (notations are descibed int he text)

It is convenient to introduce following notations:

$$\mu = \frac{M}{m}, \lambda_s^2 = \frac{k_s}{M}, \alpha_s = \frac{\beta_s}{M}, \lambda_t^2 = \frac{k_t}{M}, \alpha_t = \frac{\beta_t}{M}, \alpha_s = \frac{\varepsilon_s}{M}, \alpha_t = \frac{\varepsilon_t}{M}, \alpha = \frac{\varepsilon_t}{\varepsilon_s} = \frac{\alpha_t}{\alpha_s}.$$

For writing dimensionless equation we scale time with factor $T = T_g = \frac{2\pi}{\omega_g}$, and length with factor $L = \frac{\lambda_s}{\sqrt{\alpha_s}}$. In the followings x_1 and x_2 and their derivatives will denote dimensionless quantities. Form of equations of motion is resulted in:

$$\ddot{x}_1 = -\lambda_s^2 T_g^2((x_1 - x_2) + (x_1 - x_2)^3) - \alpha_s T_g(\dot{x}_1 - \dot{x}_2)$$

$$\frac{1}{\mu}\ddot{x}_2 = \lambda_s^2 T_g^2((x_1 - x_2) + (x_1 - x_2)^3) - \alpha_s T_g(\dot{x}_1 - \dot{x}_2)$$

$$-\lambda_t^2 T_g^2 (x_2 - I\sin(2\pi t)) - \lambda_s^2 T_g^2 \varepsilon (x_2 - I\sin(2\pi t))^3 - \alpha_t T_g(\dot{x}_2 - I\frac{2\pi}{T_g}\cos(2\pi t))$$

This is a fourth-order nonautomous system, which could be rewritten to a fifth-order autonomous system.

Instead of then physical parameters, seven independent parameters are involved in this equation system:

$$\lambda_s^2 T_g^2,\ \alpha_s T_g,\ \mu,\ \lambda_t^2 T_g^2,\ \alpha_t T_g,\ \varepsilon,\ I = \frac{\varphi_0\sqrt{\varepsilon_s}}{\lambda_s}.$$

3. TRANSMISSION FUNCTION IN CASE OF WEAK EXCITATION

Solution of this system of differential equation can be approximated by numerical methods. We applied fourth order Runge-Kutta method. In a case study we set following physical parameters:

masses:	M=400kg,	m=10 kg,	
suspension parameters:	k_s=5000 N/m,	β_s=200 Ns/m,	ε_s=100 N/m^3,
tyre parameters:	k_t=50000 N/m,	β_t=10 Ns/m,	ε_t=1000 N/m^3,
ground excitation amplitude:		ϕ_0=0,05m.	

For the amplitude of excitation is small, this study investigates behaviour of the system in case of weak excitation. This means that parameter *I* is small.

The length scale factor is $L = \frac{\lambda_s}{\sqrt{\alpha_s}} = \sqrt{50}m$. Time scale factor $T = \frac{2\pi}{\omega_g}$ changes with frequency.

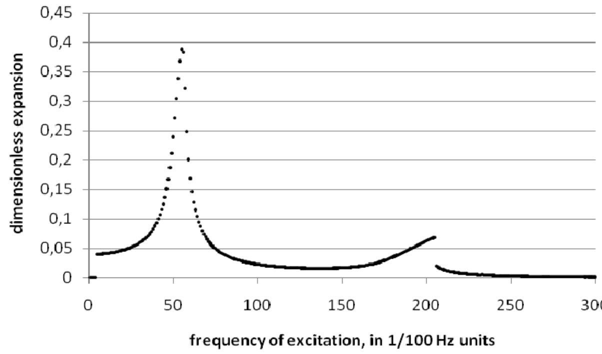

Figure 2. Transmission function of M, i.e. amplitude of $x_1(t)$ as function of frequency of excitation

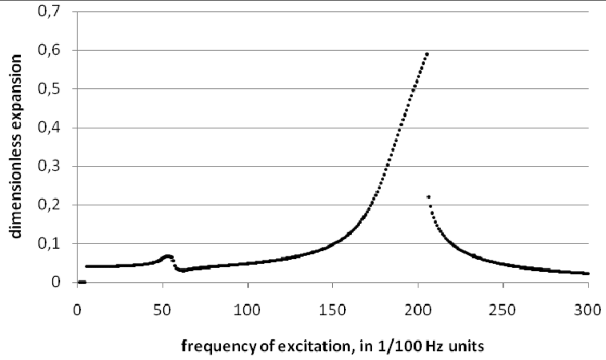

Figure 3. Transmission function of m, i.e. amplitude of $x_2(t)$ as function of frequency of excitation

Transmission functions on Figure 2 and Figure 3 shows general features of transmission function of quarter models with special nonlinearities in the spring (described above). These functions are similar to transmission function of linear quarter model in the sense they have two peaks. However a significant difference is obvious: peak at higher frequency has discontinuity, which is a tipical feature of nonlinear systems. This discontimuity appears in resonance function of one-dimensional Duffing-oscillator too [4].

4. STUDY OF MOTION IN THE PHASE SPACE

Motion is studied in details with physical parameters given in the previous chapter and $\omega_g = 2\pi Hz.$ Length scale factor is identical with value int he previous section. Time scale factor $T = \frac{2\pi}{\omega_g} = 1$, so in this special case, time in the dimensionless system is the same as magnitude of real time.

Time function of x_1 and x_2 are shown on Figures 4. and 5. After a transient part, a periodic motion follows. Instead of time functions stroboscopic mapping is a more effective tool for analysing the motion. Extended phase space of forced quarter model is five dimensional with coordinates $x_1, \dot{x}_1, x_2, \dot{x}_2, t.$ Stroboscopic mapping can show if there is an attractor in the system, and if yes, features of that can be studied. In phase space $x_1, \dot{x}_1, x_2, \dot{x}_2$ stroboscopic picture of a steady state motion can be one point of finite number of points. A four-dimensiomal space can not visualized on a paper sheet. That's why image points of stroboscopic mapping are projected on planes, namely $(x_1, \dot{x}_1)$ and $(x_2, \dot{x}_2)$ planes. Figures 6-9 show such projections. Motion was studied through 5000 periods of excitation, so 5000 image points of stroboscopic mapping were calculated. On Figure 6. first 34 image points are omitted, because they are strongly transient. remaining points comes more and more closer to a small area of the $(x_1, \dot{x}_1)$ plane. This small area is shown enlarged on Figure 7, and first 200 image point omitted. There is 4800 point on Figure 7, but they coincide with 4 points. This means, steady state motion has a 4T time period, not T, however four parts of the period are highly similar to each other. Figures 8 and 9 show same diagrams for mass m, i.e. the wheel.

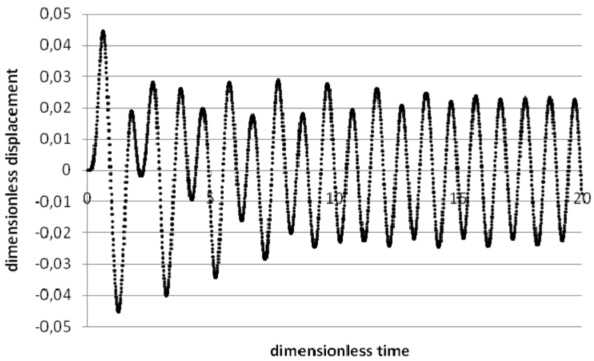

Figure 4. Time function of x_1, namely the displacement of M (the quarter body of the car)

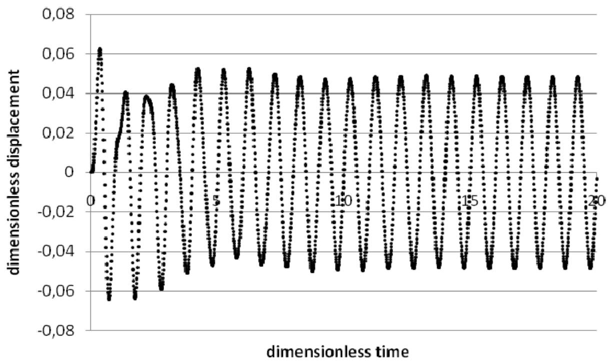

Figure 5. Time function of x_2, namely the displacement of m (the wheel)

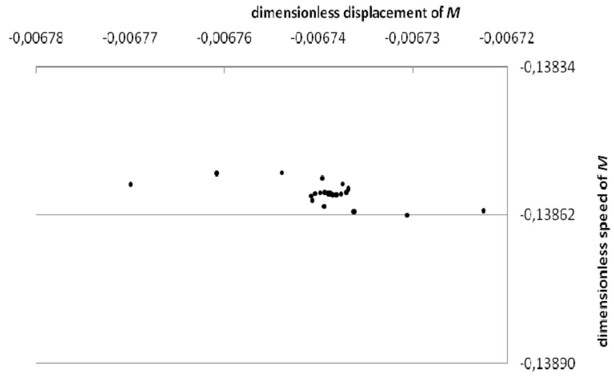

Figure 6. Stroboscopic mapping of motion of M projected on plane $(x_1, \dot{x}_1)$, first 34 transient point are left out, 4966 image points are shown

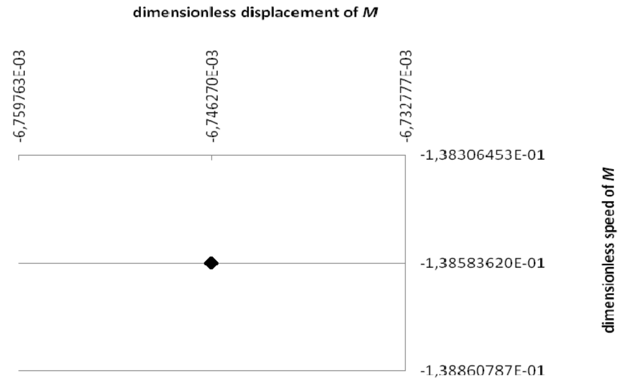

Figure 7. Stroboscopic mapping of motion of M projected on plane $(x_1, \dot{x}_1)$, first 200 transient point are left out, so they are 4800 image points

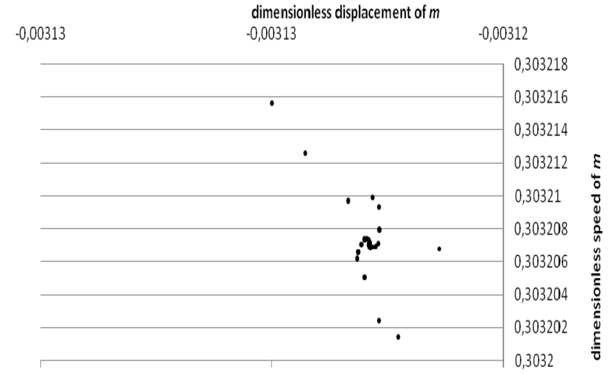

Figure 8. Stroboscopic mapping of motion of m projected on plane $(x_2, \dot{x}_2)$, first 34 transient point are left out, 4966 image points are shown

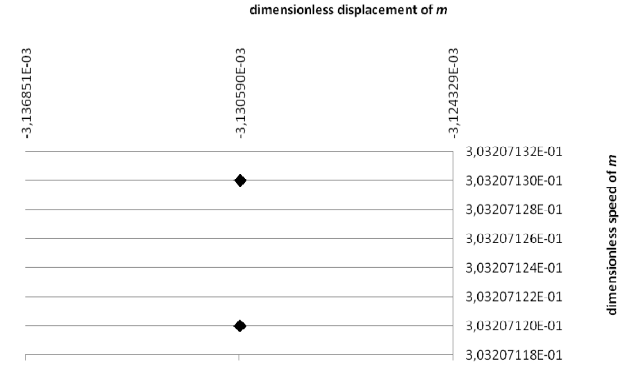

Figure 9. Stroboscopic mapping of motion of m projected on plane $(x_2, \dot{x}_2)$, first 200 transient point are left out, so they are 4800 image points

5. CONCLUSIONS

Vehicle quarter model with hardening springs and linear damping in case of weak excitation. Equation system of motion was solved numerically by Runge-Kutta method. Transmission function has two peaks, one of them shows jump phenomenon, what is typical in nonlinear systems. Stroboscopic mapping shows invariant sets of points in pilot calculations.

In the future research work is planned to extend for strong excitations.

References

[1.] Isak KARABEGOVIĆ, Bekir NOVKINIĆ, Ermin HUSAK, Safet ISIĆ: Influence of self-exited vibrations ont he surface roughness of workpieces obtained by longitudinal turning, International Journal of Engineering, XII. (2014) 1 p163-166 ISSN:1584-2673

[2.] Ioan SEBEŞAN, Mădălina DUMITRIU: Validation of the theoretical model for the study of dynamic behaviour on vertical direction of railway vehicles, International Journal of Engineering, XII. (2014) 2 p153-160 ISSN:1584-2673

[3.] DUKKIPATI, V. R. et al. (2008): Road Vehicle Dynamics. SAE International, ISBN 978-0-7680-1643-7

[4.] Tamás Kalmár-Nagy, Balakumar Balachandran: Forced harmonic vibration of a Duffing oscillator with linear viscous damping, in: I. Kovacic and M. J. Brennan (eds.): The Duffing equation: Nonlinear oscillators and their behaviour, First Edition, 2011 John Willey and Sons Ltd. ISBN 978-0-470-71549-9

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