ANNALS of Faculty Engineering Hunedoara – International Journal of Engineering

Tome XII [2014] – Fascicule 4 [November]
ISSN: 1584-2673 [CD-Rom, online]

a free-access multidisciplinary publication of the Faculty of Engineering Hunedoara

[1.] R. MUTHUCUMARASWAMY, [2.] P. SIVAKUMAR

THERMAL RADIATION AND FIRST ORDER CHEMICAL REACTION EFFECTS ON A PARABOLIC FLOW PAST AN INFINITE ISOTHERMAL VERTICAL PLATE WITH VARIABLE MASS DIFFUSION

[1.]Dep. Applied Mathematics, Sri Venkateswara College of Engineering, Irungattukottai, Sriperumbudur, INDIA
[2.] Dep. Mathematics, P.B. College of Engineering, Irungattukottai, Sriperumbudur, INDIA

Abstract: The paper studied the effects of thermal radiation and chemical reaction on a parabolic flow past an infinite isothermal vertical plate with variable mass diffusion, taking into account the homogeneous chemical reaction of first order. The plate temperature is raised uniformly and species concentration level near the plate is raised linearly with respect to time. The governing equations were solved using the Laplace transform method to obtain the analytical expressions of velocity, temperature and concentration. The effects of various parameters associated with flow like radiation parameter, magnetic parameter, chemical reaction parameter, thermal Grashof number, mass Grashof number, Schmidt number and time on the velocity, temperature, concentration are the effects of various parameters on flow variables are illustrated graphically, and the physical aspects of the problem are discussed. It is observed that the velocity increases with increasing values the thermal Grashof number or mass Grashof number.
Keywords: parabolic, radiation, isothermal, vertical plate, heat and mass transfer

1. INTRODUCTION

Combined heat and mass transfer problems with chemical reaction are of importance in many processes and have, therefore, received a considerable amount of attention in recent years. In processes such as drying, distribution of temperature and moisture over agricultural fields and graves of fruit trees, damage of crops due to freezing, evaporation at the surface of a water body, energy transfer in a wet cooling tower, and flow in a desert cooler, heat and mass transfer occur simultaneously. Possible applications of this type of flow can be found in many industries. For example, in the electric power industry, among the methods of generating electric power is one in which electrical energy is extracted directly from a moving conducting fluid. Chemical reaction can be codified as either homogeneous or heterogeneous processes. A homogeneous reaction is one that occurs uniformly through a given phase. In contrast, a heterogeneous reaction takes place in a restricted region or within the boundary of a phase. A reaction is said to be first order if the rate of reaction is directly proportional to the concentration itself which has many applications in different chemical engineering processes and other industrial applications such as polymer production, manufacturing of ceramics or glassware, and food processing

Das *et al.* [1] considered the effects of first order chemical reaction on the flow past an impulsively started infinite vertical plate with constant heat flux and mass transfer. Chambre and Young [2] have analyzed a first order chemical reaction in the neighborhood of a horizontal plate. Gupta et al. [3] studied free convection flow past a linearly accelerated vertical plate in the presence of viscous dissipative heat using perturbation method. Kafousias and Raptis [4] extended this problem to include mass transfer effects subjected to variable suction or injection. Mass transfer effects on flow past an accelerated vertical plate a uniformly accelerated vertical plate was studied by soundalgekar [5]. Mass transfer effects on flow past an accelerated vertical plate with uniform heat flux was analyzed by the Singh and Singh [6]. Basant kumar Jha *et al* [7] analyzed mass transfer effects on the flow past an accelerated infinite vertical plate with heat sources. Muthucumaraswamy et al. [8] studied mass transfer with a chemical reaction on unsteady flow past an accelerated isothermal vertical plate. Recently, Rajput and Sahu [9] investigated the effects of rotation and magnetic field on the flow past an exponentially accelerated vertical plate with constant temperature. Actually, many processes in new engineering areas occur at high temperature, and knowledge of radiation heat transfer becomes imperative for the design of the pertinent equipment. Nuclear power plants, gas turbines, and the various propulsion devices for aircraft, missiles, satellites, and space vehicles are examples of such engineering areas. Thermal radiation effect on flow past a vertical plate with mass transfer is examined by Muralidharan and Muthucumaraswamy [10]. The effects of radiation on free convection on the accelerated flow of a viscous incompressible fluid past an infinite vertical plate have many important technological applications in the astrophysical, geophysical, and engineering problem.

It is proposed to study the effects of on flow past an infinite isothermal vertical plate subjected to parabolic motion with variable mass diffusion, in the presence of thermal radiation. The dimensionless governing equations are solved using the Laplace-transform technique. The solutions are in terms of exponential and complementary error function.

2. MATHEMATICAL ANALYSIS

Here the unsteady flow of a viscous incompressible fluid past an infinite isothermal vertical plate with uniform diffusion, in the presence of thermal radiation has been considered. The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium. The x'-axis is taken along the plate in the vertically upward direction and the y -axis is taken normal to the plate. At time $t' \leq 0$, the plate and fluid are at the same temperature T_∞ and concentration C'_∞ . At time $t' > 0$, the plate is started with a velocity $u = u_0.t'^2$ in its own plane against gravitational field and the temperature from the plate is raised to Tw and the concentration level near the plate are also raised linearly with time. The plate is infinite in length all the terms in the governing equations will be independent of x'-and there is no flow along y-direction. Then under usual Boussinesq's approximation for unsteady parabolic starting motion is governed by the following equations:

$$\frac{\partial u}{\partial t'} = g\beta(T - T_\infty) + g\beta^*(C' - C'_\infty) + \nu\frac{\partial^2 u}{\partial y^2} \tag{1}$$

$$\rho C_p \frac{\partial T}{\partial t'} = k\frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y^2} - k_l(C' - C'_\infty) \tag{3}$$

With the following initial and boundary conditions:

$$u = 0, \quad T = T_\infty, \; C' = C'_\infty \text{ for all } y, t' \leq 0$$

$$t' > 0: u = u_0.t'^2, \; T=Tw, \; C' = C'_\infty + (C'_w - C'_\infty)At' \text{ at } y = 0, \text{ where } A = \left(\frac{u_0^2}{\nu}\right)^{\frac{1}{3}} \tag{4}$$

$$u \to 0 \quad T \to T_\infty, \quad C' \to C'_\infty \quad \text{as} \quad y \to \infty$$

The local radiant for the case of an optically thin gray gas is expressed by

$$\frac{\partial q_r}{\partial y} = -4a^* \sigma(T_\infty^4 - T^4) \tag{5}$$

It is assumed that the temperature differences within the flow are sufficiently small such that T^4 may be expressed as a linear function of the temperature. This is accomplished by expanding T^4 in a Taylor series about T_∞ and neglecting higher-order terms, thus

$$T^4 \cong 4T_\infty^3\, T - 3T_\infty^4 \tag{6}$$

By using equations (5) and (6), equation (2) reduces to

$$\rho\, C_p \frac{\partial T}{\partial t'} = k\, \frac{\partial^2 T}{\partial y^2} + 16a^* \sigma T_\infty^3\, (T_\infty - T) \tag{7}$$

On introducing the following non-dimensional quantities:

$$U = u\left(\frac{u_0}{\nu^2}\right)^{1/3}, \quad t = \left(\frac{u_0^{\,2}}{\nu}\right)^{1/3} t', \quad Y = y\left(\frac{u_0}{\nu^2}\right)^{1/3},$$

$$\theta = \frac{T - T_\infty}{T_w - T_\infty}, C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, \; Gr = \frac{g\beta(T - T_\infty)}{(\nu.u_0)^{1/3}}, \quad Gc = \frac{g\beta(C' - C'_\infty)}{(\nu.u_0)^{1/3}}, \tag{8}$$

$$R = \frac{16a^* \sigma.T_\infty^{\;3}}{k}\left(\frac{\nu^2}{u_0}\right)^{\frac{2}{3}}, \; \Pr = \frac{\mu C_p}{k}, \quad Sc = \frac{\nu}{D}$$

The equations (1), (3) and (7), reduces to the following dimensionless form

$$\frac{\partial U}{\partial t} = Gr\theta + GcC + \frac{\partial^2 U}{\partial Y^2} \tag{9}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial Y^2} - \frac{R}{\Pr}\theta \tag{10}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2} \tag{11}$$

The initial and boundary conditions in non-dimensional quantities are

$$\begin{aligned} &U = 0, \quad \theta = 0, \quad C = 0 \quad \text{for all } Y, t \le 0 \\ &t > 0: \quad U = t^2, \quad \theta = 1, \quad C = t \quad \text{at} \quad Y = 0 \\ &U \to 0, \quad \theta \to 0, \quad C \to 0 \quad \text{as} \quad Y \to \infty \end{aligned} \tag{12}$$

The dimensionless governing equations (6) to (8) and the corresponding initial and boundary conditions (9) are tackled using Laplace transform technique.

$$\theta = \frac{1}{2}\left[\exp\left(2\eta\sqrt{\Pr at}\right) erfc\left(\eta\sqrt{\Pr} + \sqrt{at}\right) + \exp\left(-2\eta\sqrt{\Pr at}\right) erfc\left(\eta\sqrt{\Pr} - \sqrt{at}\right)\right] \tag{13}$$

$$\begin{aligned} c = {} & \frac{t}{2}\left[\exp\left(2\eta\sqrt{Sc\,K\,t}\right) erfc\left(\eta\sqrt{Sc} + \sqrt{Kt}\right) + \exp\left(-2\eta\sqrt{Sc\,K\,t}\right) erfc\left(\eta\sqrt{Sc} - \sqrt{Kt}\right)\right] \\ & - \frac{\eta\sqrt{Sc}\sqrt{t}}{2\sqrt{K}}\left[\exp\left(-2\eta\sqrt{Sc\,K\,t}\right) erfc\left(\eta\sqrt{Sc} - \sqrt{Kt}\right) - \exp\left(2\eta\sqrt{Sc\,K\,t}\right) erfc\left(\eta\sqrt{Sc} + \sqrt{Kt}\right)\right] \end{aligned} \tag{14}$$

$$\begin{aligned} U = {} & 2\left(\frac{t^2}{6}\left[\left(3 + 12\eta^2 + 4\eta^4\right) erfc(\eta) - \frac{\eta}{\sqrt{\pi}}\left(10 + 4\eta^2\right)\exp(-\eta^2)\right]\right) \\ & + d\left(\begin{aligned} & erfc(\eta) - \frac{\exp(bt)}{2}\left[\exp\left(2\eta\sqrt{bt}\right) erfc\left(\eta + \sqrt{bt}\right) + \exp\left(-2\eta\sqrt{bt}\right) erfc\left(\eta - \sqrt{bt}\right)\right] \\ & - \frac{1}{2}\left[\exp\left(2\eta\sqrt{\Pr a t}\right) erfc\left(\eta\sqrt{\Pr} + \sqrt{at}\right) + \exp\left(-2\eta\sqrt{\Pr a t}\right) erfc\left(\eta\sqrt{\Pr} - \sqrt{at}\right)\right] \\ & + \frac{\exp(bt)}{2}\left[\exp\left(2\eta\sqrt{\Pr(a+b)t}\right) erfc\left(\eta\sqrt{\Pr} + \sqrt{(a+b)t}\right) + \exp\left(-2\eta\sqrt{\Pr(a+b)t}\right) erfc\left(\eta\sqrt{\Pr} - \sqrt{(a+b)t}\right)\right] \end{aligned}\right) \\ & + e\left(\begin{aligned} & erfc(\eta) - \frac{\exp(ct)}{2}\left[\exp\left(2\eta\sqrt{ct}\right) erfc\left(\eta + \sqrt{ct}\right) + \exp\left(-2\eta\sqrt{ct}\right) erfc\left(\eta - \sqrt{ct}\right)\right] \\ & - \frac{1}{2}\left[\exp\left(2\eta\sqrt{Sc\,K\,t}\right) erfc\left(\eta\sqrt{Sc} + \sqrt{Kt}\right) + \exp\left(-2\eta\sqrt{Sc\,K\,t}\right) erfc\left(\eta\sqrt{Sc} - \sqrt{Kt}\right)\right] \\ & + \frac{\exp(ct)}{2}\left[\exp\left(2\eta\sqrt{Sc(K+c)t}\right) erfc\left(\eta\sqrt{Sc} + \sqrt{(K+c)t}\right) + \exp\left(-2\eta\sqrt{Sc(K+c)t}\right) erfc\left(\eta\sqrt{Sc} - \sqrt{(K+c)t}\right)\right] \end{aligned}\right) \\ & + e(c)\left(\begin{aligned} & t\left[\left(1 + 2\eta^2\right) erfc(\eta) - \frac{2\eta}{\sqrt{\pi}}\exp(-\eta^2)\right] - \frac{t}{2}\left[\exp\left(2\eta\sqrt{Sc\,K\,t}\right) erfc\left(\eta\sqrt{Sc} + \sqrt{Kt}\right) + \exp\left(-2\eta\sqrt{Sc\,K\,t}\right) erfc\left(\eta\sqrt{Sc} - \sqrt{Kt}\right)\right] \\ & + \frac{\eta\sqrt{Sc}\sqrt{t}}{2\sqrt{K}}\left[\exp\left(-2\eta\sqrt{Sc\,K\,t}\right) erfc\left(\eta\sqrt{Sc} - \sqrt{Kt}\right) - \exp\left(2\eta\sqrt{Sc\,K\,t}\right) erfc\left(\eta\sqrt{Sc} + \sqrt{Kt}\right)\right] \end{aligned}\right) \end{aligned} \tag{15}$$

where $a = \frac{R}{\Pr}, b = \frac{R}{1 - \Pr}, c = \frac{KSc}{1 - Sc}, d = \frac{Gr}{b(1 - \Pr)}, e = \frac{Sc}{c^2(1 - Sc)}$ and $\eta = \frac{y}{2\sqrt{t}}$

ISSN: 1584-2673 [CD-Rom, online]

3. RESULTS AND DISCUSSION

For physical understanding of the problem numerical computations are carried out for different physical parameters K, R, Sc, Gr, Gc and t upon the nature of the flow and transport. The value of the Schmidt number is taken 0.6 to be which corresponds to water-vapor. Also, the value of Prandtl number are chosen such that they represent air (Pr = 0.71). The numerical values of the velocity are computed for different physical parameters like Radiation, chemical reaction parameter, Schmidt number, thermal Grashof number, mass Grashof number and time.

Figure 1 represents the effect of concentration profiles at time t = 0.2 for different Schmidt number (Sc = 0.16, 0.3, 0.6, 2.01). The effect of Schmidt number plays an important role in concentration field. The profiles have the common feature that the concentration decreases in a monotone fashion from the surface to a zero value far away in the free stream. It is observed that the wall concentration increases with decreasing values of the Schmidt number. The velocity profiles for different values of the chemical reaction parameter (K = 2, 5, 10) and t= 0.4 are shown in Figure 2. It is observed that the velocity increases with decreasing chemical reaction parameter.

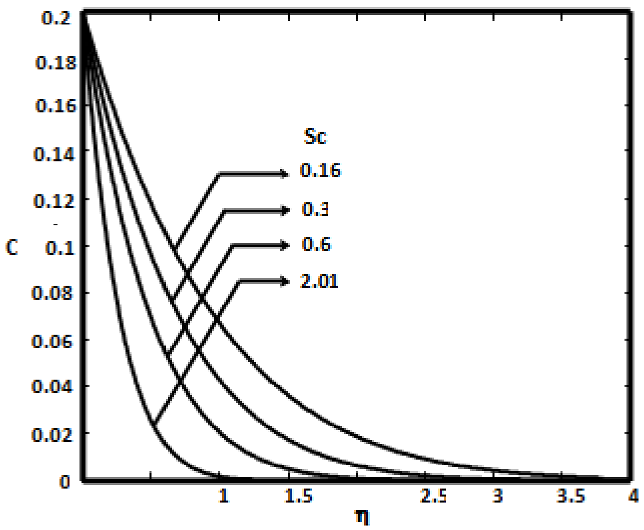

Figure.1 Concentration profile for different values Sc

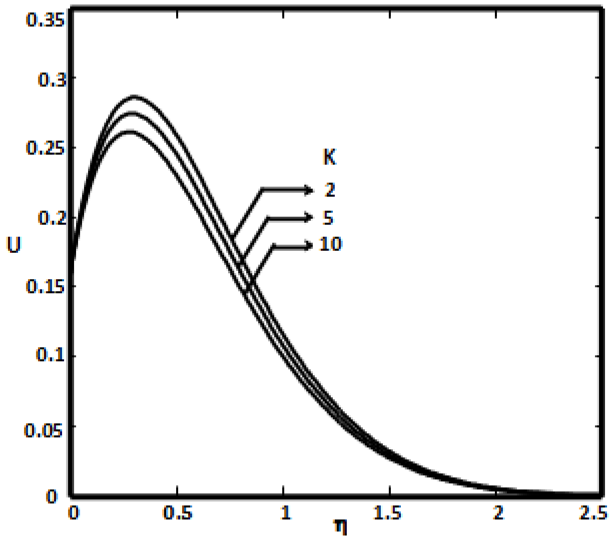

Figure 2. Velocity profiles for different K

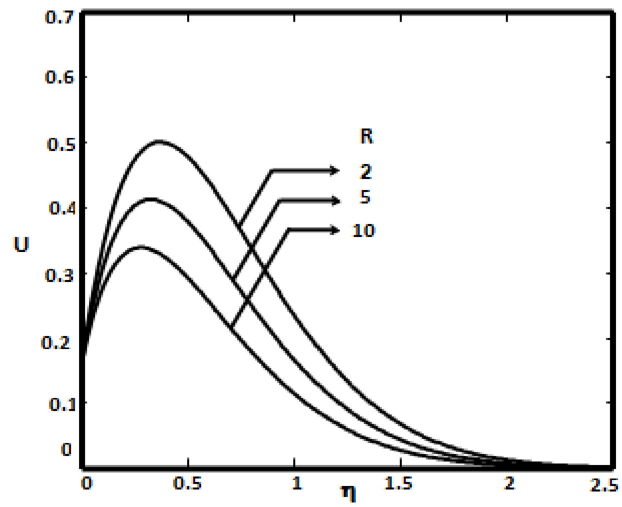

Figure 3. Velocity profiles for different R

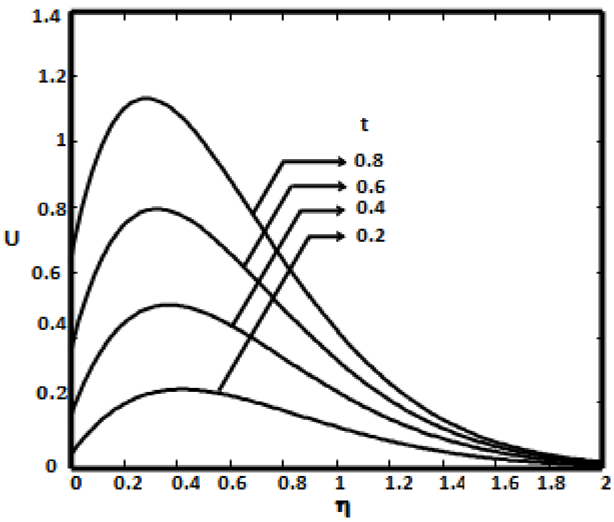

Figure 4. Velocity profiles for different t

The velocity profiles are calculated for different values of the thermal radiation parameter (R= 2, 5, 10) at time t = 0.4 are shown in Figure 3. It is observed that the temperature increases with the decreasing radiation parameter. The trend shows that there is a fall in plate temperature due to higher thermal radiation. Figure 4 demonstrates the velocity profiles for different values of time (t = 0.2, 0.4, 0.6, 0.8), K= 2, Gr = 2, Gc = 5, R = 2 are studied and presented in. It is observed that velocity increases with increasing values of time.

Figure 5 illustrates the effect of the velocity profiles for different the Schmidt number (Sc =0.16, 0.3, 0.6), K=2 and t=0.4. The profiles have the common values of feature that the concentration decreases in a monotone fashion from the surface to a zero value far away in the free stream. It is observed that the concentration increases with the decreasing Schmidt number. Figure 6 demonstrates the effect velocity fields for different thermal Grashof number (Gr = 2, 5, 5), mass Grashof number (Gc = 2, 2, 5) and t = 0.4. It is observed that the velocity increases with increasing values of the thermal Grashof number or mass Grashof number.

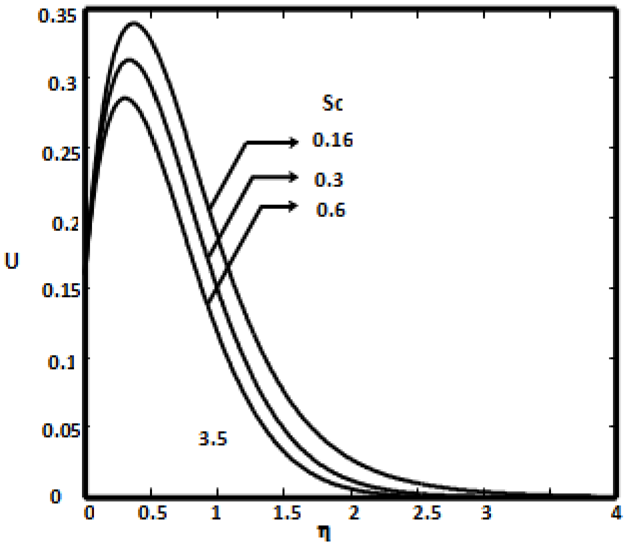

Figure 5. Velocity profiles for different Sc

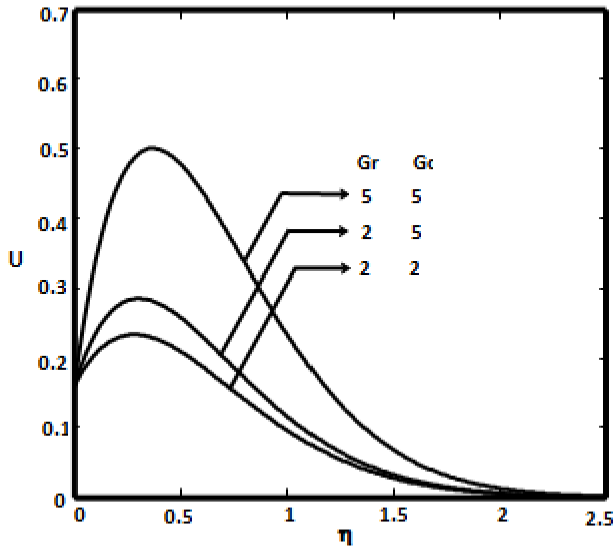

Figure 6. Velocity profiles for different Gr & Gc

4. CONCLUSION

The theoretical solution of flow past a parabolic starting motion of the infinite isothermal vertical plate in the presence of variable mass diffusion has been studied. The dimensionless governing equations are solved by the usual Laplace transform technique. The effect of different physical parameters like thermal radiation parameter, thermal Grashof number and mass Grashof number are studied graphically. It is observed that the velocity increases with increasing thermal Grashof number or mass Grashof number, but the trend is just reversed with respect to the thermal radiation parameter. The temperature of the plate increases with decreasing values of the thermal radiation parameter. The plate concentration increases with decreasing values of the Schmidt number.

» The velocity increases with increasing thermal Grashof number or mass Grashof number, but the trend is just reversed with respect to the chemical reaction parameter.
» The temperature of the plate increases with decreasing values of the Prandtl number.
» The plate concentration increases with decreasing values of the chemical reaction parameter

NOMENCLATURE

A-Constants
C′-species concentration in the fluid, kgm^{-3}
C-dimensionless concentration
Cp-specific heat at constant pressure, Jkg^{-1}*k*
D-mass diffusion coefficient, m^{2}*.s*$^{-1}$
Gc-mass Grashof number
Gr-thermal Grashof number
g- acceleration due to gravity $m.s^{-2}$
k-thermal conductivity, Wm^{-1}*K*$^{-1}$
Pr-Prandtl number
Sc-Schmidt number
T-temperature of the fluid near the plate, K
t′-time, s
u-velocity of the fluid in the x′-direction, ms^{-1}
u$_0$*-velocity of the plate, ms*$^{-1}$
u-dimensionless velocity

Greek symbols

β-volumetric coefficient of thermal expansion, K^{-1}
β-volumetric coefficient of expansion with concentration, K*$^{-1}$
μ-coefficient of viscosity, Rs s
υ-kinematic viscosity, m^{2} *s*$^{-1}$
φ-density of the fluid, kg m^{-3}
τ-dimensionless skin-friction, kg m^{-1} *s*^{2}
θ-dimensionless temperature
η-similarity parameter
erfc-complementary error function

Subscripts

ω-conditions at the wall
∞-free stream conditions

ISSN: 1584-2673 [CD-Rom, online]

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