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ANALYTICAL AND NUMERICAL STUDY OF OSCILLATORY MHD FLOW PAST A VERTICAL PLATE WITH VARIABLE TEMPERATURE AND CHEMICAL REACTION

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Abstract: An analytical study is performed to investigate thermal radiation and mass transfer effects on unsteady oscillatory MHD flow through porous medium past an infinite vertical plate with variable temperature and mass diffusion in the presence of heat source and chemical reaction. The fluid considered here is a gray, absorbing/emitting radiation but a non-scattering medium. At time t > 0, the plate temperature and concentration levels near the plate raised linearly with time t. The governing equations are solved by the Laplace transform technique and also numerically applying Crank-Nicolson difference scheme. The results of both the methods are found to be in good agreement. The effects of flow, heat transfer and mass transfer parameters on velocity, temperature and concentration distributions are shown with the help of graphs and tables. The velocity distribution exhibits 3-layer character depending upon (i) oscillatory (ii) constant (iii) no motion of the plate. Moreover, heating/cooling of the plate as well as positive/negative direction of motion of the plate contribute to symmetrical distribution about the direction of motion of the plate. Diffusion through aqueous medium is effective in reducing the skin friction, velocity of the plate subject to cooling and concentration distribution significantly in comparison with the air medium.
Keywords: MHD flow, thermal radiation, porous medium, chemical reaction, heat source

1. INTRODUCTION

Study of MHD flow with heat and mass transfer through porous medium plays an important role in various industrial problems such as power generation systems, geothermal sources investigation and nuclear fuel debris treatment etc. The effects of magnetic field and gravitational force on the flow characteristic of an electrically conducting fluid have been an active area of research in geophysics. Radiative heat and mass transfer play an important role in manufacturing industries for the design of reliable equipment. Nuclear power plants, gas turbine and various propulsion devices for air craft, missiles, satellite and space vehicles are examples of such engineering applications. If the temperature of surrounding fluid is high, radiation effects play an important role. In such cases one has to take into account the effect of thermal radiation and mass diffusion.

The problem of thermal radiation and mass diffusion effects on MHD flow with chemical reaction has been a subject of interest of several researchers. Radiation and mass transfer effects in two dimensional flows past an impulsively started vertical plate was studied by Prasad *et al* [1]. Makinde and Moitsheki [2] have applied non-perturbative technique for thermal radiation effect on natural convection past a vertical plate embedded in a saturated porous medium. The effect of oscillatory suction and heat source on heat and mass transfer in MHD flow along a vertical porous plate bounded by porous medium were studied by Sharma and Sharma [3]. Muthucumaraswamy [4] has considered the effect of heat and mass transfer on flow past on oscillatory vertical plate with variable temperature. Effect of magnetic field on flow through porous medium was studied by Geindreau and Auriault [5]. Das et al [6] have discussed MHD flow through a porous medium past a stretched vertical permeable surface in the presence of heat source/sink and a chemical reaction. Unsteady free convective flow and mass transfer in a rotating porous medium has been studied by Mahato [7]. Hassain and Thakhar [8] have discussed radiation effect on mixed convection along a vertical plate with uniform surface temperature. Barik et al [9] have studied the heat and mass transfer on MHD oscillatory flow through a porous medium over a stretching surface with heat source. Acharya et al. [10] have considered magnetic field effects on the free convection and mass transfer flow through porous medium with constant suction and constant heat flux. Kesavaiah et al. [11] have studied the effects of radiation and free convection currents on unsteady Couette flow between two vertical parallel plates with constant heat flux and heat source through porous medium. Rajesh and Varma [12] have considered heat source effects on MHD flow past an exponentially accelerated vertical plate with variable temperature through a porous medium. Radiation effects on MHD flow through porous media past an impulsively started vertical oscillating plate with variable mass diffusion has been discussed by Rajput and Kumar [13]. Kumar et al.[14] have worked on thermal diffusion and radiation effects on unsteady MHD flow, through porous medium with variable temperature and mass diffusion in the presence of heat source/sink. The governing equations are solved by Laplace transform technique. Makinde [15] has analyzed MHD mixed convection interaction with thermal radiation and nth order chemical reaction past a vertical porous plate embedded in a porous medium. The effect of heat and mass transfer on MHD free convection flow past an impulsively moving infinite vertical plate with ramped wall temperature have been studied by Pattnaik and Dash [16]. Sekhar and Reddy [17] have considered the effects of chemical reaction on MHD free convective oscillatory flow past a porous plate with viscous dissipation and heat sink. The thermal radiation and mass transfer effects on MHD flow past a vertical oscillating plate with variable temperature and variable mass diffusion has been studied by Kumar and Varma [18]. Thermal insulation of the radiating surface with porous material is essential in controlling heat transfer phenomena. Further, in the field of heat transfer, the concept of flow through porous media is of great consequence in the modern technology as the porous matrix acts as a good insulation to prevent energy loss. Thermal insulation of the radiating surface with porous material is essential in controlling heat transfer phenomena.

Recently Ahmed et al.[19] have studied MHD radiating flow over an infinite vertical surface bounded by a porous medium in presence of chemical reaction. They have not considered the effect of heat source/sink in the domain of their discussion which very often occurs in the reality. Moreover, Barik [20] has studied mass transfer and radiation effect on MHD flow past an impulsively started exponentially accelerated inclined porous plate with variable temperature in the presence of heat source and chemical reaction. Though he has considered heat source in his study, the motion is engendered exponentially with an inclined surface. Moreover, Barik [20] has only studied analytically. Therefore the present study is more general comparing with Ahmed [19] and the Barik's case can not be compared even if the boundary surface is made vertical. Therefore, the necessity of the present study arises in view of mathematical method and physical configuration.

The novelty of the present study is the embedding the oscillatory and radiative surface in a porous medium with uniform porosity. An humble attempt has been made to use a simple Darcian model to account for the flow through porous medium surrounding the vertical surface in the presence of a heat source. Another important aspect of the present study is to consider the reacting chemical species which has been taken care of by incorporating the first order chemical reaction.

Moreover, the heat generation is a common phenomenon in engineering applications. The oscillation of the velocity, temperature and concentration on the solid surface about which flow occurs is a common occurrence in practice. Therefore, we have considered oscillation of the plate and a heat-source term in heat-equation which is quite relevant in the present context. Hence, the main objective of the study is to bring out the effects of distributed characteristics, permeability of the medium, heat-source parameter and chemical reaction parameter over and above the effects of the other parameters considered by the previous authors referred above. The governing equations are solved analytically with the help of Laplace transform technique and numerically applying unconditionally stable Crank-Nicolson difference scheme. The analytical expressions for fluid velocity, temperature and concentration distribution in the flow domain have been obtained and skin friction, Nusselt number and Sherwood number have also been calculated for various values of pertinent flow parameters.

2. MATHEMATICAL FORMULATION

We consider an unsteady free convective oscillatory MHD flow of a viscous impressible electrically conducting fluid past an infinite vertical plate with variable temperature and mass diffusion through porous medium in the presence of heat source and chemical reaction. The x′-axis is taken along the plate in the vertical upward direction and y′-axis is taken normal to the plate. Initially it is assumed that the plate and fluid are at the same temperature T'_∞ and concentration level C'_∞ in the stationary condition for all the points. At time t′ > 0, the plate is given an oscillatory motion in its own plane with velocity u_0 cos (ω′t′). At the same time, the plate temperature is raised linearly with the time t and also mass is diffused from the plate linearly with time. A transverse magnetic field of uniform strength B_0 is assumed to be applied normal to the plate. The induced magnetic field is assumed to be negligible as the magnetic Reynolds number of the flow is taken to be very small. The fluid considered here is gray, absorbing/emitting radiation but a non-scattering medium.

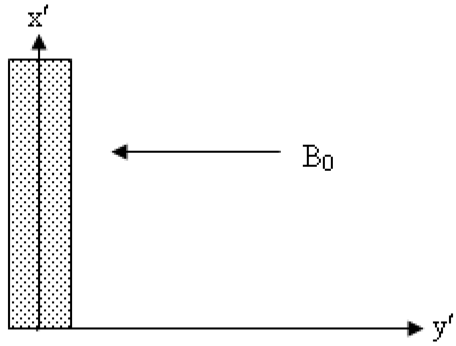

Figure 1. Schematic flow diagram

By taking into account the Boussinesq approximation and neglecting the Soret-Dufour (thermal diffusion and diffusion-thermo) effects because of low concentration level, the governing equations of motion, energy and diffusion are given by:

$$\frac{\partial u'}{\partial t'} = g\beta(T'-T'_\infty) + g\beta'(C'-C'_\infty) + \upsilon\frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^2 u'}{\rho} - \frac{\upsilon u'}{k'_p} \quad (1)$$

$$\rho C_p\frac{\partial T'}{\partial t'} = k\frac{\partial^2 T'}{\partial y'^2} - \frac{\partial q_r}{\partial y'} + Q'(T'_\infty - T') \quad (2)$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y'^2} - K'_c(C'-C'_\infty) \quad (3)$$

where g, β, β', k'_p, Q', K'_c, ρ, σ, υ, k, C_p, D, q_r, C', T' and u' are acceleration due to gravity, coefficient of volume expansion, coefficient of mass expression, permeability parameter, heat source parameter, chemical reaction parameter, density, electric conductivity, kinematic viscosity, thermal conductivity, specific heat at constant pressure, concentration diffusivity, radiative heat flux, concentration of fluid, temperature of fluid and velocity of fluid in x′-direction respectively.

With the following initial and boundary conditions:

$t' \le 0 : u' = 0,\ T' = T'_\infty,\ C' = C'_\infty$ for all y′

$t' > 0 : u' = u_0 \cos(\omega' t'),\ T' = T'_\infty + (T'_w - T'_\infty)\,At'$

$C' = C'_\infty + (C'_w - C'_\infty)At'$ at $y' = 0$

and $u' \to 0,\ T' \to T'_\infty,\ C' \to C'_\infty$ as $y' \to \infty$ (4)

where $A = \frac{u_0^2}{\upsilon}$. The local radiant for the case of an optically thin grey gas is expressed as

$$\frac{\partial q_r}{\partial y'} = -4a'\sigma\left(T'^4_\infty - T'^4\right) \quad (5)$$

and $T'^4 \cong 4T'^3_\infty T' - 3T'^4_\infty$ (6)

provided the temperature difference within the flow is small, where σ and a′ are the Stefan-Boltzmann constant and Mean absorption coefficients respectively.

Using equation (5) and (6) in equation (2) we get:

$$\rho C_p \frac{\partial T'}{\partial t'} = k\frac{\partial^2 T'}{\partial y'^2} + 16a'\sigma T_\infty'^3\left(T_\infty' - T'\right) + Q'\left(T_\infty' - T'\right) \tag{7}$$

Now, introducing the following non-dimensional quantities

$$y = \frac{u_0 y'}{\upsilon}, u = \frac{u'}{u_0}, t = \frac{t' u_0^2}{\upsilon}, \theta = \frac{T' - T_\infty'}{T_w' - T_\infty'}, C = \frac{C' - C_\infty'}{C_w' - C_\infty'}, S_c = \frac{\upsilon}{D}, P_r = \frac{\rho \upsilon C_p}{k},$$

$$G_r = \frac{\upsilon g\beta\left(T_w' - T_\infty'\right)}{u_0^3}, G_m = \frac{\upsilon g\beta'\left(C_w' - C_\infty'\right)}{u_0^3},\ M = \frac{\sigma B_0^2 \upsilon}{\rho u_0^2}, R = \frac{16a'\upsilon^2\sigma\ T_\infty'^3}{k u_0^2},$$

$$\omega = \frac{\omega'\upsilon}{u_0^2},\quad K_c = \frac{\upsilon K_c'}{u_0^2},\quad H = \frac{Q'\upsilon^2}{k u_0^2},\quad k_p = \frac{k_p' u_0^2}{\upsilon^2} \tag{8}$$

the equations (1), (3) and (7) reduce to

$$\frac{\partial u}{\partial t} = G_r\theta + G_m C + \frac{\partial^2 u}{\partial y^2} - Mu - \frac{u}{k_p} \tag{9}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{P_r}\frac{\partial^2 \theta}{\partial y^2} - \frac{1}{p_r}(R+H)\theta \tag{10}$$

$$\frac{\partial C}{\partial t} = \frac{1}{S_c}\frac{\partial^2 C}{\partial y^2} - K_c C \tag{11}$$

and the boundary conditions of (4) become

$$\begin{aligned} &t \le 0: u = 0,\ \theta = 0,\ C = 0\ \forall\ y \\ &t > 0: u = \cos\omega t,\ \theta = t,\ C = t \text{ for } y \to 0 \\ &\text{and } u \to 0,\ \theta \to 0 \text{ and } C \to 0 \text{ as } y \to \infty \end{aligned} \tag{12}$$

where G_r, the Grashof number for heat transfer, G_m, the Grashof number for mass transfer, P_r, the Prandtl number, S_c, the Schmidt number, M, the magnetic parameter, R, the Radiation parameter, K_c, the chemical reaction parameter, H, the heat source parameter and k_p, the permeability parameter.

3. METHOD OF SOLUTION

The unsteady, linear coupled partial differential equations (9) to (11) with their boundary conditions (12) are solved analytically using Laplace transform technique and the solutions for temperature, concentration and velocity field are given by:

Temperature:

$$\theta(y,t) = \left(\frac{t}{2} - \frac{yP_r}{4\sqrt{S}}\right)e^{-y\sqrt{S}} erfc\left(\frac{y}{2}\sqrt{\frac{P_r}{t}} - \sqrt{\frac{St}{P_r}}\right) + \left(\frac{t}{2} + \frac{yP_r}{4\sqrt{S}}\right)e^{y\sqrt{S}} erfc\left(\frac{y}{2}\sqrt{\frac{P_r}{t}} + \sqrt{\frac{St}{P_r}}\right) \tag{13}$$

Concentration:

$$C(y,t) = \left(\frac{t}{2} - \frac{y}{4}\sqrt{\frac{S_c}{K_c}}\right)e^{-y\sqrt{K_c S_c}}.erfc\left(\frac{y}{2}\sqrt{\frac{S_c}{t}} - \sqrt{K_c t}\right) + \left(\frac{t}{2} + \frac{y}{4}\sqrt{\frac{S_c}{K_c}}\right)e^{y\sqrt{K_c S_c}}.erfc\left(\frac{y}{2}\sqrt{\frac{S_c}{t}} + \sqrt{K_c t}\right) \tag{14}$$

Velocity:

$$\begin{aligned} u(y,t) = {} & \frac{1}{4}e^{-i\omega t}\left[e^{-y\sqrt{\lambda - i\omega}}.erfc\left(\frac{y}{2\sqrt{t}} - \sqrt{(\lambda - i\omega)t}\right) + e^{y\sqrt{\lambda - i\omega}}.erfc\left(\frac{y}{2\sqrt{t}} + \sqrt{(\lambda - i\omega)t}\right)\right] \\ & + \frac{1}{4}e^{i\omega t}\left[e^{-y\sqrt{\lambda + i\omega}}.erfc\left(\frac{y}{2\sqrt{t}} - \sqrt{(\lambda + i\omega)t}\right) + e^{y\sqrt{\lambda + i\omega}}.erfc\left(\frac{y}{2\sqrt{t}} + \sqrt{(\lambda + i\omega)t}\right)\right] \\ & -\left(A_4 + A_6\right)\left[e^{-y\sqrt{\lambda}}.erfc\left(\frac{y}{2\sqrt{t}} - \sqrt{\lambda t}\right) + e^{y\sqrt{\lambda}}.erfc\left(\frac{y}{2\sqrt{t}} + \sqrt{\lambda t}\right)\right] \\ & -A_7\left[\left(\frac{t}{2} - \frac{y}{4\sqrt{\lambda}}\right)e^{-y\sqrt{\lambda}}.erfc\left(\frac{y}{2\sqrt{t}} - \sqrt{\lambda t}\right) + \left(\frac{t}{2} + \frac{y}{4\sqrt{\lambda}}\right)e^{y\sqrt{\lambda}}.erfc\left(\frac{y}{2\sqrt{t}} + \sqrt{\lambda t}\right)\right] \\ & +A_4 e^{\alpha_1 t}\left[.e^{-y\sqrt{\lambda + \alpha_1}}.erfc\left(\frac{y}{2\sqrt{t}} - \sqrt{\left(\lambda + \alpha_1\right)t}\right) + e^{y\sqrt{\lambda + \alpha_1}}.erfc\left(\frac{y}{2\sqrt{t}} + \sqrt{\left(\lambda + \alpha_1\right)t}\right)\right] \\ & +A_6 e^{\alpha_2 t}\left[e^{-y\sqrt{\lambda + \alpha_2}}.erfc\left(\frac{y}{2\sqrt{t}} - \sqrt{\left(\lambda + \alpha_2\right)t}\right) + e^{y\sqrt{\lambda + \alpha_2}}.erfc\left(\frac{y}{2\sqrt{t}} + \sqrt{\left(\lambda + \alpha_2\right)t}\right)\right] \end{aligned}$$

$$+A_3\left[\left(\frac{t}{2}-\frac{yP_r}{4\sqrt{S}}\right)e^{-y\sqrt{S}}.erfc\left(\frac{y}{2}\sqrt{\frac{P_r}{t}}-\sqrt{\frac{St}{P_r}}\right)+\left(\frac{t}{2}+\frac{yP_r}{4\sqrt{S}}\right)e^{y\sqrt{S}}.erfc\left(\frac{y}{2}\sqrt{\frac{P_r}{t}}+\sqrt{\frac{St}{P_r}}\right)\right]$$

$$+A_4\left[e^{-y\sqrt{S}}.erfc\left(\frac{y}{2}\sqrt{\frac{P_r}{t}}-\sqrt{\frac{St}{P_r}}\right)+e^{y\sqrt{S}}.erfc\left(\frac{y}{2}\sqrt{\frac{P_r}{t}}+\sqrt{\frac{St}{P_r}}\right)\right]$$

$$-A_4e^{\alpha_1 t}\left[e^{-y\sqrt{S+\alpha_1P_r}}.erfc\left(\frac{y\sqrt{P_r}}{2\sqrt{t}}-\sqrt{\left(\frac{S}{P_r}+\alpha_1\right)t}\right)+e^{y\sqrt{S+\alpha_1P_r}}.erfc\left(\frac{y\sqrt{P_r}}{2\sqrt{t}}+\sqrt{\left(\frac{S}{P_r}+\alpha_1\right)t}\right)\right]$$

$$+A_5\left[\left(\frac{t}{2}-\frac{y}{4}\sqrt{\frac{S_c}{K_c}}\right)e^{-y\sqrt{S_cK_c}}.erfc\left(\frac{y}{2}\sqrt{\frac{S_c}{t}}-\sqrt{K_ct}\right)\right]+\left[\left(\frac{t}{2}+\frac{y}{4}\sqrt{\frac{S_c}{K_c}}\right)e^{y\sqrt{S_cK_c}}.erfc\left(\frac{y}{2}\sqrt{\frac{S_c}{t}}+\sqrt{K_ct}\right)\right]$$

$$+A_6\left[e^{-y\sqrt{S_cK_c}}.erfc\left(\frac{y}{2}\sqrt{\frac{S_c}{t}}-\sqrt{K_ct}\right)+e^{y\sqrt{S_cK_c}}.erfc\left(\frac{y}{2}\sqrt{\frac{S_c}{t}}+\sqrt{K_ct}\right)\right]$$

$$-A_6e^{\alpha_2 t}\left[e^{-y\sqrt{S_c(K_c+\alpha_2)}}.erfc\left(\frac{y}{2}\sqrt{\frac{S_c}{t}}-\sqrt{(\alpha_2+K_c)t}\right)+e^{y\sqrt{S_c(K_c+\alpha_2)}}.erfc\left(\frac{y}{2}\sqrt{\frac{S_c}{t}}+\sqrt{(\alpha_2+K_c)t}\right)\right] \tag{15}$$

where $\lambda=M+\frac{1}{k_p}$, $A_1=\frac{G_r}{P_r-1}$, $A_2=\frac{G_m}{S_c-1}$, $S=R+H$, $\alpha_1=\frac{\lambda-S}{P_r-1}$, $\alpha_2=\frac{\lambda-S_cK_c}{S_c-1}$,

$$A_3=\frac{A_1}{\alpha_1}, A_4=\frac{A_1}{2\alpha_1^2}, A_5=\frac{A_2}{\alpha_2}, A_6=\frac{A_2}{2\alpha_2^2}, A_7=\frac{A_1}{\alpha_1}+\frac{A_2}{\alpha_2}$$

Skin friction (τ_w):

$$-\frac{\partial u}{\partial y}\bigg]_{y=0}=\frac{1}{2}e^{-i\omega t}\left[\sqrt{\lambda-i\omega}.erf\left(\sqrt{(\lambda-i\omega)t}\right)+\frac{1}{\sqrt{\pi t}}e^{-(\lambda-i\omega)t}\right]$$

$$+\frac{1}{2}e^{i\omega t}\left[\sqrt{\lambda+i\omega}.erf\left(\sqrt{(\lambda+i\omega)t}\right)+\frac{1}{\sqrt{\pi t}}e^{-(\lambda+i\omega)t}\right]-\left(A_4+A_6\right)\left[2\sqrt{\lambda}.erf\left(\sqrt{\lambda t}\right)+\frac{2}{\sqrt{\pi t}}e^{-\lambda t}\right]$$

$$-A_7\left[\frac{1}{2\sqrt{\lambda}}erf\left(\sqrt{\lambda t}\right)+t\sqrt{\lambda}.erf\left(\sqrt{\lambda t}\right)+\frac{t}{\sqrt{\pi t}}e^{-\lambda t}\right]+A_4e^{\alpha_1 t}\left[2\sqrt{\lambda+\alpha_1}.erf\left(\sqrt{(\lambda+\alpha_1)t}\right)+\frac{2}{\sqrt{\pi t}}e^{-(\lambda+\alpha_1)t}\right]$$

$$+A_6e^{\alpha_2 t}\left[2\sqrt{\lambda+\alpha_2}.erf\left(\sqrt{(\lambda+\alpha_2)t}\right)+\frac{2}{\sqrt{\pi t}}e^{-(\lambda+\alpha_2)t}\right]$$

$$+A_3\left[\frac{\mathrm{P_r}}{2\sqrt{S}}erf\left(\sqrt{\frac{St}{P_r}}\right)+t\sqrt{S}.erf\left(\sqrt{\frac{St}{P_r}}\right)+\frac{\sqrt{P_rt}}{\sqrt{\pi}}e^{-\left(\frac{St}{P_r}\right)}\right]+A_4\left[2\sqrt{S}.erf\left(\sqrt{\frac{St}{P_r}}\right)+\frac{2\sqrt{P_r}}{\sqrt{\pi t}}e^{-\left(\frac{St}{P_r}\right)}\right]$$

$$-A_4.e^{\alpha_1 t}\left[2\sqrt{S+\alpha_1P_r}.erf\left(\sqrt{\left(\frac{S}{P_r}+\alpha_1\right)t}\right)+\frac{2\sqrt{P_r}}{\sqrt{\pi t}}e^{-\left(\frac{S}{P_r}+\alpha_1\right)t}\right]$$

$$+A_5\left[\frac{1}{2}\sqrt{\frac{S_c}{K_c}}erf\sqrt{K_ct}+t\sqrt{S_cK_c}erf\sqrt{K_ct}+\left(\sqrt{\frac{S_ct}{\pi}}\right)e^{-(K_ct)}\right]+A_6\left[2\sqrt{K_cS_c}erf\left(\sqrt{K_ct}\right)+2\sqrt{\frac{S_c}{\pi t}}e^{-K_ct}\right]$$

$$-A_6.e^{\alpha_2 t}\left[2\sqrt{S_c\left(K_c+\alpha_2\right)}.erf\left(\sqrt{(\alpha_2+K_c)t}\right)+\frac{2\sqrt{S_c}}{\sqrt{\pi t}}e^{-(K_c+\alpha_2)t}\right] \tag{16}$$

Nusselt Number (N_u):

$$-\frac{\partial\theta}{\partial y}\bigg]_{y=0}=\frac{P_r}{2\sqrt{S}}.erf\left(\sqrt{\frac{St}{P_r}}\right)+t\sqrt{S}.erf\left(\sqrt{\frac{St}{P_r}}\right)+\frac{t\sqrt{P_r}}{\sqrt{\pi t}}e^{-\left(\frac{St}{P_r}\right)} \tag{17}$$

Sherwood Number (S_h):

$$-\frac{\partial c}{\partial y}\bigg]_{y=0}=\frac{1}{2}\sqrt{\frac{S_c}{K_c}}.erf\left(\sqrt{K_ct}\right)+t\sqrt{S_cK_c}.erf\left(\sqrt{K_ct}\right)+\frac{\sqrt{S_ct}}{\sqrt{\pi}}e^{-(K_ct)} \tag{18}$$

Numerical method (Crank-Nicolson scheme)

The momentum equation (9) is coupled with temperature and concentration equations (10) and (11) respectively. Therefore, we solved the coupled equation numerically by finite difference method. In the present problem, we solve the equation using Crank-Nicolson scheme. Moreover, the first and second partial derivatives terms with respect to 'y' are discretized using central difference scheme. We consider a finite fluid domain [0, 2] and discretized using uniform grid. Let the grid points y_i, i = 1, 2, 3 …… N with ($y_{i-1} < y_i$ and $y_i = 0$ and $y_N = 2$) be equispaced with cell width Δy at the time level t^n, n = 0, 1, 2 , …… with uniform time step Δt.

Let u_i^n, θ_i^n and C_i^n be the discrete values at time t^n and at position y_i.

For the given solution u_i^n, θ_i^n and C_i^n at the time level t^n, the solutions at the next time level t^{n+1} are obtained from the following discrete equations.

$$\frac{u_i^{n+1}-u_i^n}{\Delta t}+\frac{1}{2}\left(M+\frac{1}{kp}\right)\left(u_i^{n+1}+u_i^n\right)$$

$$=\frac{1}{2(\Delta y)^2}\left[\left(u_{i+1}^{n+1}-2u_i^{n+1}+u_{i-1}^{n+1}\right)+\left(u_{i+1}^n-2u_i^n+u_{i-1}^n\right)\right]+\frac{1}{2}G_r\left(\theta_i^{n+1}+\theta_i^n\right)+\frac{1}{2}G_m\left(C_i^{n+1}+C_i^n\right) \tag{19}$$

$$\frac{\theta_i^{n+1}-\theta_i^n}{\Delta t}=\frac{1}{2P_r(\Delta y)^2}\left[\left(\theta_{i+1}^{n+1}-2\theta_i^{n+1}+\theta_{i-1}^{n+1}\right)+\left(\theta_{i+1}^n-2\theta_i^n+\theta_{i-1}^{n+1}\right)\right]-\frac{(R+H)}{2P_r}\left(\theta_i^{n+1}+\theta_i^n\right) \tag{20}$$

$$\frac{C_i^{n+1}-C_i^n}{\Delta t}=\frac{1}{2S_C(\Delta y)^2}\left[\left(C_{i+1}^{n+1}-2C_i^{n+1}+C_{i-1}^{n+1}\right)+\left(C_{i+1}^n-2C_i^n+C_{i-1}^n\right)\right]-\frac{K_c}{2}\left(C_i^{n+1}+C_i^n\right) \tag{21}$$

The discretized equations (19) – (21) are solved by using the scientific computing program MATLAB *fsolve* routine. This routine uses a non-linear least-squares algorithm to solve a system of non-linear equations. The simulation was performed using N = 401 grid points and time step of $\Delta t = 0.1$.

4. RESULT DISCUSSION

During the discussion, the effects of the permeability of the medium, heat source and chemical reaction are to be focused as these factors contribute to the novelty of the present study. The Darcy model accounts for the permeability of the medium and two linear models having constant coefficients are incorporated in energy and mass transfer equations to account for the heat source and chemical reaction.

It is evident from the boundary conditions that the impulsive oscillatory motion of the plate, with constant amplitude u_0 and frequency ω having linear distribution of temperature and concentration, engender the motion of the fluid resulting velocity, temperature and concentration distribution in the flow domain. The common characteristic of the velocity distribution is oscillatory and the oscillation is confined to a few layers near the plate and then the velocity attains free stream state asymptotically.

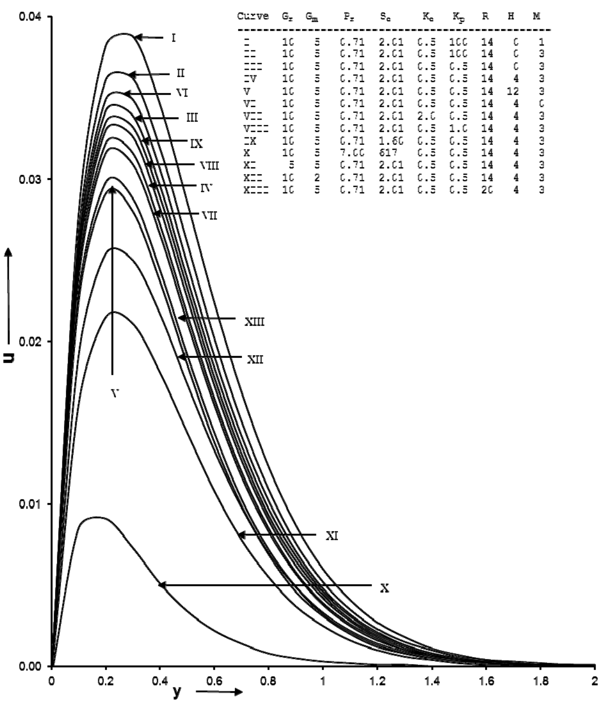

Fig. 2. Velocity distribution (cooling of the plate) when *t = 0.2 and ωt = π/2*

The figs. 2(a), 3(a), 5(a) and 6 (a) depict the numerical solutions by finite different method with their analytical counter parts given by figs.2, 3, 5 and 6 respectively. It is observed that there is an excellent agreement between two results establishing the validity of the numerical scheme and conformity of the results detailed below.

Figs.2 and 3 exhibit the cases of cooling and heating of the plate respectively. Fig.2 exhibits the velocity distribution under the influence of the cooling of the plate. It is interesting to remark that the permeability parameter (k_p), thermal Grashof number (G_r) and modified Grashof number (G_m) accelerate the velocity whereas heat source parameter (H), magnetic parameter (M), radiation parameter (R), Chemical reaction parameter (K_c), Schmidt number (S_c) and Prandtle number (P_r) decelerate at all points of the flow domain.

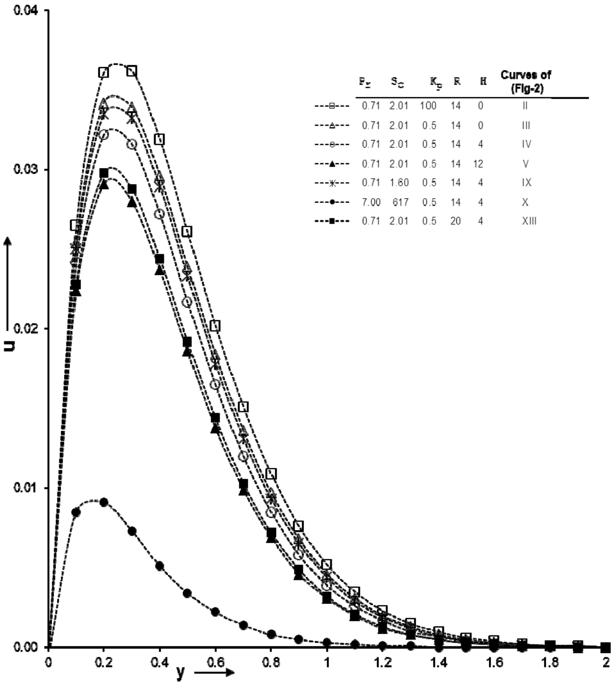

Fig- 2(a). Velocity distribution (cooling of the plate) for G_r = 10, G_m = 5, K_c = 0.5, M = 3,t = 0.2 and ωt = π/2 (Crank Nicolson method)

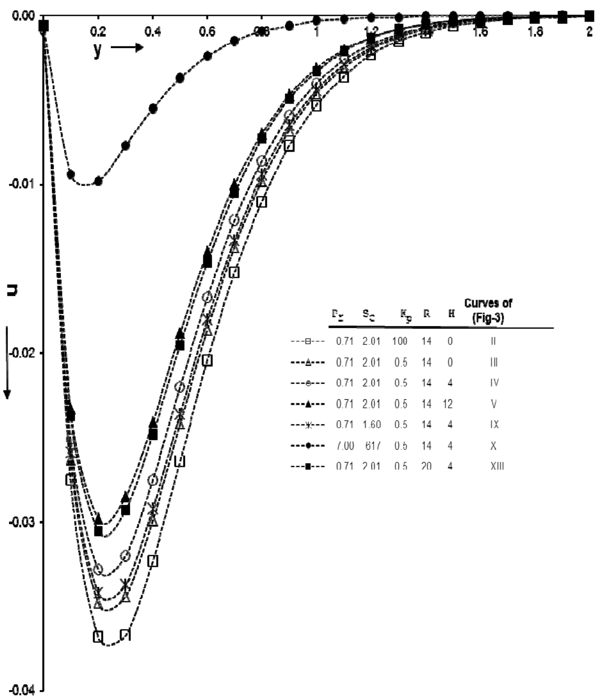

Fig. 3(a) Velocity distribution (heating of the plate) for G_r = -10, G_m = -5, K_c = 0.5, M = 3,t = 0.2 and ωt = π/2 (Crank Nicolson method)

On careful observation it is revealed that the rise in velocity is higher in the presence of lighter gas (curve IV & IX). The value of S_c in water medium is always very high for all types of concentration and the presence of Cl_2 in water medium reduces the velocity significantly than in air medium curve IV (S_c = 2.01, P_r = 0.71) and curve X (S_c = 617, P_r = 7.0, Chlorine through water medium). This shows that the presence of porous medium, with heavier species exert retarding effect on the velocity and the retarding influence is increased significantly in case of water medium and cooling of the plate. This observation agrees with Mahato [7].

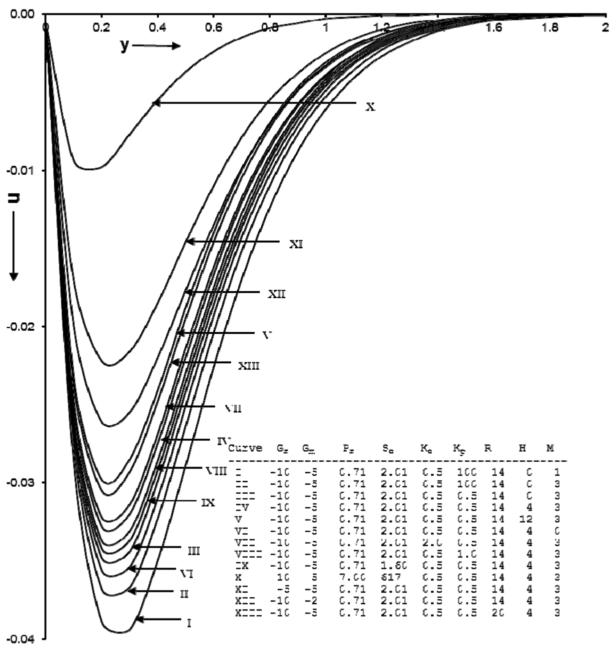

Fig. 3. Velocity distribution (heating of the plate) when t = 0.2 and ωt = π/2

The reverse effect is observed in case of heating of the plate (Fig.3). Further it is observed from Fig.3 that the velocity profiles appear to be the mirror image of the cooling of the plate (Fig.2), i.e. symmetrical about the y-axis. The influence of free convective currents due to Grashoff numbers G_r and G_m remain same under various flow conditions. An increase in $|G_r|$ and $|G_m|$ leads to increase the absolute velocity, $|u|$ in both the cases. Moreover, the effect of radiation is to decrease the velocity (Curves IV and XIII).

Fig.4. presents the velocity profiles for various values of ωt (angle of oscillation), permeability parameter (k_p) and heat source parameter (H). Curve I, represents the flow due to impulsive motion of the plate without oscillation, heat source and porosity of the medium. It is evident that

the velocity increases and attains high value in this case. The same state prevails even in the presence of heat source (Curve II). This shows that heat source is not that effective without oscillation and without porous medium to modify the velocity field but the curve III (presence of porous medium) shows the reduction of velocity due to permeability parameter (k_p= 0.5). Thus, it is concluded that heat source has no effect on velocity in case of impulsive motion without oscillation but permeability has a retarding effect. It is interesting to examine the curve III $(\omega t = 0)$ and curve VII ($\omega t = \pi$) resulting from impulsive motion of the plate in opposite direction (u_0 and $-u_0$). The flow is almost symmetrical about y-axis showing a back flow when $\omega t = \pi$; The radiative oscillatory motion in the presence of porous medium for $\omega t = \pi/3$ (curve V) and $\omega t = 2\pi/3$ (curve VI) are also symmetrical about the main flow direction. Thus it is remarked that the direction of impulsive motion has a significant effect on reversing the flow irrespective of oscillation or without oscillation. The curve VIII is a special case representing without the motion of the plate but a radiative flow with permeability. This represents a linearly varying slow flow with almost vanishing boundary layer.

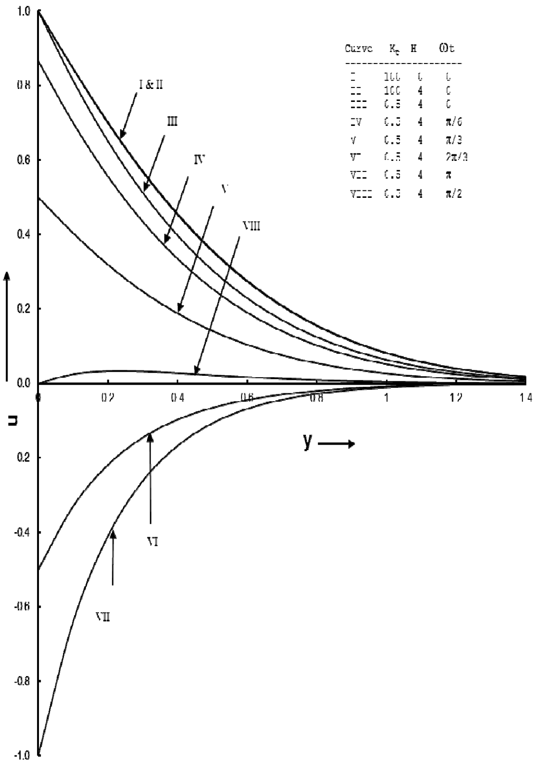

Fig.4. Velocity distribution for different phase angle when $G_r = 10$, $G_m = 5$, $P_r = 0.71$, $S_c = 2.01$, $K_c = 0.5$, $R = 14$, $M = 3$, $t = 0.2$

Fig.5. exhibits the temperature distribution for different values of P_r, R and H. The presence of heat source causes a fall in temperature uniformly throughout the flow domain and further increase in heat source decreases the temperature. This indicates that fall of temperature commensurate with the increase in strength of heat source parameter.

It is also remarked that an increase in radiation (R) in the absence of heat source (curves I and II) reduces the temperature the same observation as Vijay Kumar and Varma [18]. The same effect is observed in the presence of heat source also. Thus, the radiation parameter decreases the temperature irrespective of presence/absence of heat source. Prandtle number (P_r) has the same effect as that of radiation parameter (R) but t, time parameter has an opposite effect. This remark agrees well with Vijaya Kumar & Varma [18]. On careful observation it is further observed that higher plate temperature causes a sharp fall in temperature within a few layers of the fluid flow close to the plate.

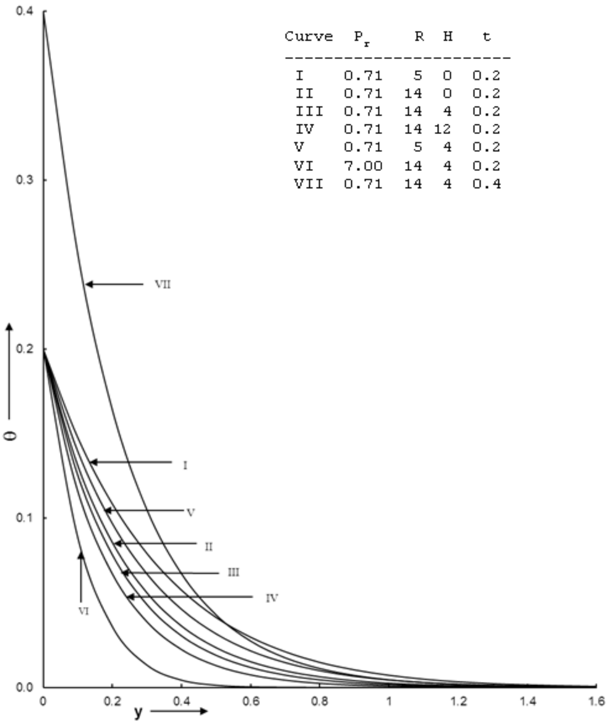

Fig. 5 Temperature distribution

In Fig.6, concentration profiles for K_c and S_c show the same characteristic as that of H and P_r in temperature distribution respectively. Profiles exhibit mass transfer of Cl_2, H_2O, CO_2, NH_3 in air and Cl_2 in water medium. Thus it is concluded that higher Prandtl number fluid (low thermal diffusivity) and heavier species (low mass diffusivity) causes a fall in temperature and role of heat source and chemical reaction remains same as that of temperature distribution. Most interesting aspect of concentration distribution is displayed by the curves IX and X. The concentration falls rapidly in case of water medium (Cl_2 through water) than the air medium setting aside the influence of chemical reaction and time.

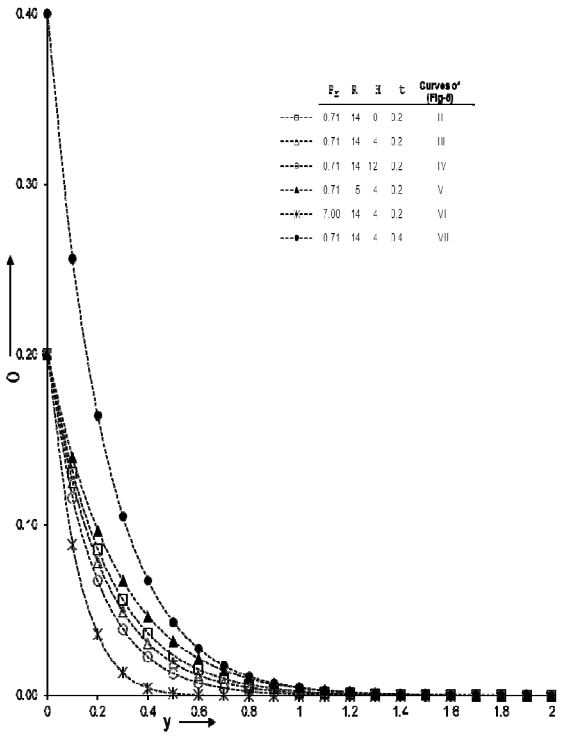

Fig- 5(a). Temperature distribution (Crank Nicolson method)

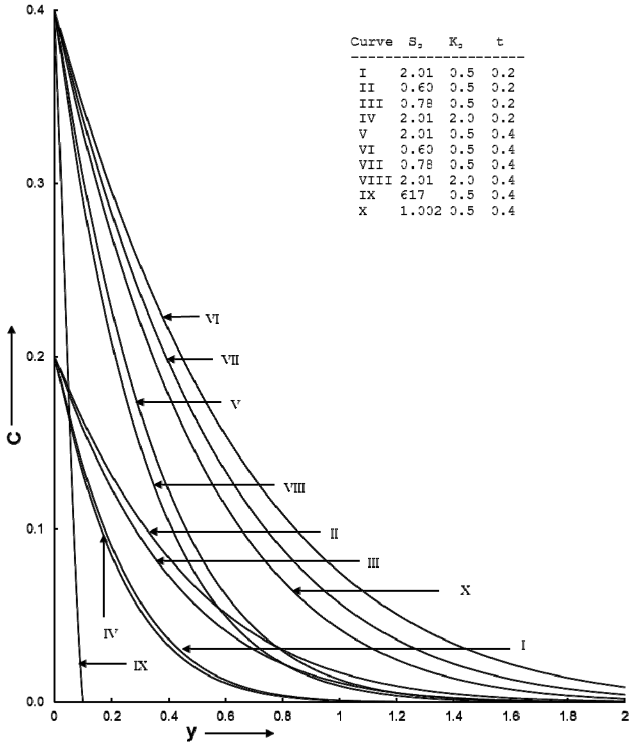

Fig. 6. Concentration distribution

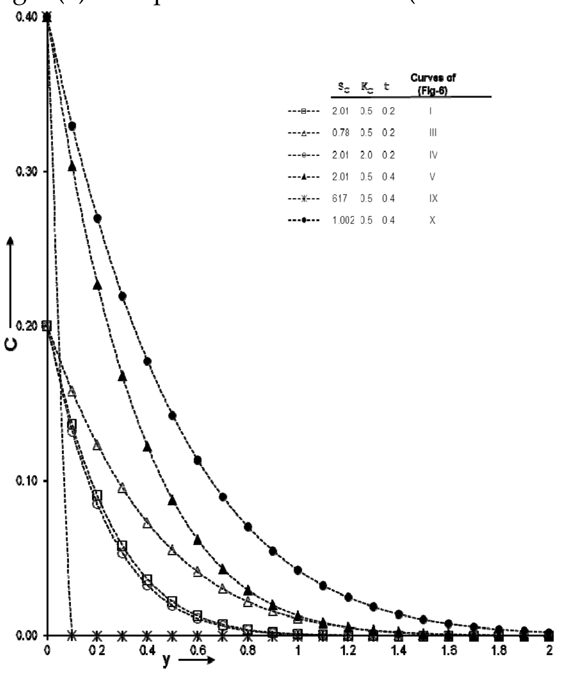

Fig. 6(a). Concentration distribution (Crank Nicolson method)

Thus, it is remarked that the medium of diffusion plays a vital role in concentration distribution.Table 1 and 2 shows that the rates of heat transfer and mass transfer, increase with an increase in respective characterizing parameters H, P_r and K_c, S_c. The elapse of time also increases the temperature.

Table 3(a) and 3(b) present the skin friction for cooling and heating of the plate respectively. It is seen that cooling of the plate experiences increase in skin friction with an increase in H, K_c, k_p, S_c, P_r and R but the reverse effect is observed in case of M, G_r, G_m, ωt and t. In case of heating of the plate the skin friction increases with an increase in k_p and t, where as reverse effect is observed in case of all other parameters. Thus it is concluded that in case of cooling of the plate, the presence of stronger free-convection current due to heat and mass transfer and oscillation of the plate reduce the skin friction on the other hand in case of heating of the plate except permeability parameter (k_p) and time t, all other parameters reduce the skin friction. Thus, in case of heating of the plate presence of porous matrix is not desirable for the reduction of the skin friction whereas presence of chemical reaction and heat source do not favour the reduction in case of cooling of the plate.

It is further remarked that in aqueous medium skin friction decreases in case heating of the plate whereas reverse effect is observed in case of cooling of the plate.

Thus, the liquid medium is more suitable for the reduction of the skin friction than the gaseous medium.

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TABLE - 1 Nusselt number (Nu)

P_r	R	H	t	N_u
0.71	5	0	0.2	0.6014
0.71	14	0	0.2	0.8431
0.71	14	4	0.2	0.9322
0.71	14	12	0.2	1.0894
0.71	10	4	0.2	0.8431
7	14	4	0.2	1.5529
0.71	14	4	0.4	1.7807

TABLE - 2 Sherwood number (S_h)

S_c	K_c	t	S_h
2.010	0.5	0.2	0.7390
0.600	0.5	0.2	0.4037
0.780	0.5	0.2	0.4603
1.600	0.5	0.2	0.6593
1.002	0.5	0.2	0.5218
2.010	2.0	0.2	0.8072
2.010	0.5	0.4	1.0778

TABLE - 3(a) Skin friction (τ_w) (Cooling of the plate)

G_r	G_m	P_r	S_c	K_c	K_p	R	H	M	t	ωt	τ_w
10	5	0.71	2.01	0.5	100.0	14	0	1	0.2	π/2	0.5958
10	5	0.71	2.01	0.5	100.0	14	0	3	0.2	π/2	0.2930
10	5	0.71	2.01	0.5	0.5	14	0	3	0.2	π/2	0.0781
10	5	0.71	2.01	0.5	0.5	14	4	3	0.2	π/2	0.0909
10	5	0.71	2.01	0.5	0.5	14	12	3	0.2	π/2	0.1110
10	5	0.71	2.01	0.5	0.5	14	4	0	0.2	π/2	0.4549
10	5	0.71	2.01	2.0	0.5	14	4	3	0.2	π/2	0.0949
10	5	0.71	2.01	0.5	1.0	14	4	3	0.2	π/2	0.1881
10	5	0.71	1.60	0.5	0.5	14	4	3	0.2	π/2	0.0829
10	5	7.00	617	0.5	0.5	14	4	3	0.2	π/2	0.2859
5	5	0.71	2.01	0.5	0.5	14	4	3	0.2	π/2	0.2128
10	2	0.71	2.01	0.5	0.5	14	4	3	0.2	π/2	0.1675
10	5	0.71	2.01	0.5	0.5	20	4	3	0.2	π/2	0.1065
10	5	0.71	2.01	0.5	0.5	14	4	3	0.4	π/2	-0.7615
10	5	0.71	2.01	0.5	0.5	14	4	3	0.2	π/3	1.3179
10	5	0.71	2.01	0.5	0.5	14	4	3	0.2	π	-3.9266
10	5	0.71	2.01	0.5	100.0	8	0	3	0.2	π/2	0.2671
10	5	0.71	2.01	0.5	0.5	14	4	3	0.2	2π/3	-1.5601
10	5	0.71	2.01	0.5	0.5	14	4	3	0.2	5π/6	-3.0936

TABLE - 3(b) Skin friction (τ_w) (Heating of the plate)

G_r	G_m	P_r	S_c	K_c	K_p	R	H	M	t	ωt	τ_w
−10	−5	0.71	2.01	0.5	100.0	14	0	1	0.2	π/2	1.4621
−10	−5	0.71	2.01	0.5	100.0	14	0	3	0.2	π/2	1.0853
−10	−5	0.71	2.01	0.5	0.5	14	0	3	0.2	π/2	0.8462
−10	−5	0.71	2.01	0.5	0.5	14	4	3	0.2	π/2	0.8335
−10	−5	0.71	2.01	0.5	0.5	14	12	3	0.2	π/2	0.8133
−10	−5	0.71	2.01	0.5	0.5	14	4	0	0.2	π/2	1.2328
−10	−5	0.71	2.01	2.0	0.5	14	4	3	0.2	π/2	0.8295
−10	−5	0.71	2.01	0.5	1.0	14	4	3	0.2	π/2	0.9418
−10	−5	0.71	1.60	0.5	0.5	14	4	3	0.2	π/2	0.8414
−10	−5	7.00	617	0.5	0.5	14	4	3	0.2	π/2	0.7185
−5	−5	0.71	2.01	0.5	0.5	14	4	3	0.2	π/2	0.7116
−10	−2	0.71	2.01	0.5	0.5	14	4	3	0.2	π/2	0.7569
−10	−5	0.71	2.01	0.5	0.5	20	4	3	0.2	π/2	0.8178
−10	−5	0.71	2.01	0.5	0.5	14	4	3	0.4	π/2	0.9998
−10	−5	0.71	2.01	0.5	0.5	14	4	3	0.2	π/3	2.0604
−10	−5	0.71	2.01	0.5	0.5	14	4	3	0.2	π	-3.1841
−10	−5	0.71	2.01	0.5	100.0	8	0	3	0.2	π/2	1.1111
−10	−5	0.71	2.01	0.5	0.5	14	4	3	0.2	2π/3	-0.8176
−10	−5	0.71	2.01	0.5	0.5	14	4	3	0.2	5π/6	-2.3510

To sum up, in case of a heated plate, reduction of skin friction is favoured by almost all the parameters except permeability and time. It is due to transfer of heat energy to fluid layers near the plate which accelerates the fluid particle and reduces the skin frictions. The appearance of negative sign can be attributed to the presence of larger phase angles $\omega t = 2\pi/3$ and $5\pi/6$ which set the plate in opposite direction but when $\omega t = \pi/2$ oscillation of the plate stops.

5. CONCLUSION

- ✓ There is an excellent agreement between the solution obtained by Laplace Transform and finite difference method (Crank-Nicolson scheme).
- ✓ Porous medium and heavier species exert retarding effect on the velocity field. The retarding effect is accelerated further in aqueous medium and cooling of the plate.
- ✓ Heat source has no noticeable effect in case of impulsive motion without oscillation but permeability of the medium has a retarding effect.
- ✓ The radiation has a retarding effect on the velocity field as well as temperature field.
- ✓ The fall of temperature commensurate with the increase in heat source parameter.
- ✓ Concentration decreases with an increase in chemical reaction parameter.
- ✓ Rates of heat and mass transfer at the plate increase with rise in heat source and chemical reaction parameter respectively.
- ✓ Aqueous medium in the presence of heated plate is suitable for reducing the skin friction.

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