ANNALS of Faculty Engineering Hunedoara – International Journal of Engineering
Tome XIII [2015] – Fascicule 2 [May]
ISSN: 1584-2665 [print]; ISSN: 1584-2673 [online]
a free-access multidisciplinary publication of the Faculty of Engineering Hunedoara

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ANALYSIS OF PERTURBATIONS IN HYDRAULIC TURBINES BY PROPER ORTHOGONAL DECOMPOSITION

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Abstract: The aim of this paper is to perform an analysis of perturbed swirling flow in a hydraulic turbine by the method of proper orthogonal decomposition (POD). We illustrate the kind of information which can be obtained from the computation of multivariable POD. The example is carried out on velocity field and pressure data from a turbulent test bench featuring a swirling flow inside the discharge cone of a Francis hydraulic turbine. The efficiency of the model is tested and a rigorous error analysis is performed.
Keywords: proper orthogonal decomposition, swirling flows, reduced order modeling

1. INTRODUCTION

The numerical simulation of swirling flows is a fundamental problem in hydrodynamics, with recent applications in physical sciences. In the field of modern hydraulic turbines, different numerical techniques of modeling the swirling flows dynamics have been proposed, each having its advantages and disadvantages. The methodology underlying the calculation of axisymmetrical turbulent flow in the diffuser cone of the hydraulic Francis turbine was presented by Resiga et al. [1] and a comparison with Laser Doppler Anemometry measurements have been performed in [2] to confirm the numerical results. Quite often, direct numerical simulation is not an appropriate solution for the simulation of hydraulic machines due to a tremendous computational effort, as demonstrated the work of Aeschlimann et al. [3], Guenette et al. [4], Nicolet et al. [5] for turbomachinery problems examples.

We propose in this paper a solution in form of a reduced order model (ROM) which accurately represents dominant features of the flow. In this paper we employ a data driven method, namely the Proper Orthogonal Decomposition (POD), that use sample stored calculated solutions or experimental data to generate the dominant modes of the perturbed velocity and pressure field in the hydraulic turbine. A benefit offered by the methodology further detailed in the paper is the computation of the fluctuating part of the swirling flow at theinlet of the discharge cone without knowing the detailed runner geometry and use it to assess the draft tube performances.

The remainder of this article is organized as follows. The description of the hydrodynamic model is presented in Section 2. The principles governing the Proper Orthogonal Decomposition are discussed in detail in Section 3. We devote the Section 4 to reveal the numerical results and to perform a rigorous error analysis for the ROM model. Summary and conclusions are drawn in the final section.

2. MATHEMATICAL FORMULATION OF THE HYDRODYNAMIC MODEL

We consider a bounded open domain $\Omega \subset \mathbb{R}^3$ and let $L^2(\Omega)$ be the Hilbert space of square integrable vector functions over Ω, associated with the energy norm $\|w\|_{L^2} = \langle w, w\rangle_{L^2}^{1/2}$ and the standard inner product $\langle v, w\rangle_{L^2} = \int_\Omega v\, w\, dz$.

Let H_∇ be the Hilbert space of divergence free functions given by $H_\nabla = \left\{ w \in L^2(\Omega) \,\middle|\, \nabla \cdot w = 0 \; in \; \Omega, \; w \cdot \vec{n} = 0 \; on \; \partial\Omega \right\}$, where $\vec{n}$ is the outward normal to the boundary. We define $H^d(\Omega) \subset L^2(\Omega)$ to be the Hilbert space of functions with d distributional derivatives that are all square integrable. Let V be the Hilbert space $V = \left\{ w \in H_\nabla \,\middle|\, w \in H^1(\Omega), \; w = 0, \; \frac{\partial w}{\partial \vec{n}} = 0, \; on \; \partial\Omega \right\}$, with norm $\|w\|_V = \langle w, w\rangle_V^{1/2}$ and the inner product $\langle v, w\rangle_V = \sum_{i=1}^{d} (\nabla v_i, \nabla w_i)$.

Starting from the incompressible Navier-Stokes equations in cylindrical coordinates formulation, we assume the swirling flow downstream the Francis runner as a steady columnar vortex $V=(V_r,V_\theta,V_z)$, depending on the radial coordinate r in form of a helical rope of circular cross-section. The circumferentially averaged components of the velocity profiles V_r , V_θ , V_z and the pressure state p are exact solutions of the Navier-Stokes equations.Due to a significant variation in the axial direction of the flow under consideration, we employ a normal mode form in time and in the tangential coordinate θ. In the axialcoordinate, we express the fluctuation wave as a rapidlyvarying wave-like part scale by a relatively slowly varyingfunction

$$\vec{u}=\vec{\Phi}(r,z)\exp\left(\int_0^z\left(k_r(s)+ik_i(s)\right)ds+i\left(m\theta-\omega t\right)\right), \tag{1}$$

where the function of the fluctuating waves $\vec{\Phi}(r,z)=\left(\hat{u}_r(r,z),\hat{u}_\theta(r,z),\hat{u}_z(r,z)\right)$ is complex and must be computed considering the radial variation and also a weak dependence on the axial coordinate.

Here the complex axial wavenumber $k(z)=k_r(z)+ik_i(z)$ is suitably subdivided into real and imaginary part, where $k_r(z)$ represents the fluctuation growth rate and $k_i(z)$ is the axial wavenumber, respectively, m is the azimuthal wavenumber (or the mode number) and ω represents the frequency. Injecting the modes decomposition (1) in the velocity field and also in the pressure field yields the parabolized stability model of perturbed flow, in the inviscid case

$$i\omega\hat{u}_r+\left(\frac{2V_\theta}{r}+\frac{imV_\theta}{r}\right)\hat{u}_\theta-\frac{\partial\hat{p}}{\partial r}-kV_z\hat{u}_r=0,\quad \left(\frac{V_\theta}{r}+\frac{\partial V_\theta}{\partial r}\right)\hat{u}_r+\left(i\omega-im\frac{V_\theta}{r}\right)\hat{u}_\theta-\frac{im}{r}\hat{p}-kV_z\hat{u}_\theta=0, \tag{2a-b}$$

$$\left(\frac{\partial}{\partial r}+\frac{1}{r}\right)\hat{u}_r+\frac{im}{r}\hat{u}_\theta+k\hat{u}_z=0,\quad \left(\frac{\partial V_z}{\partial r}+\frac{imV_z}{r}\right)\hat{u}_r+i\omega\hat{u}_z-kV_z\hat{u}_z-k\hat{p}=0. \tag{2c-d}$$

We consider that the reference computational configurationis the longitudinal section of the cylindrical surface $\left\{(r,\theta,z)\middle|r\in[0,R_{\max}],z\in[0,4R_{\max}],\theta=const\in[0,2\pi)\right\}$ with radius $R_{\max}=1.063$. The boundary value problem (2a)-(2d) is associated with the mixed boundary conditions

$$\hat{u}_r(0,z)=\hat{u}_\theta(0,z)=\partial_r\hat{u}_z(0,z)=\partial_r\hat{p}(0,z)=0, \tag{3a}$$

$$\hat{u}_r(R_{\max},z)=\partial_r\hat{u}_\theta(R_{\max},z)=\partial_r\hat{u}_z(R_{\max},z)=\partial_r\hat{p}(R_{\max},z)=0. \tag{3b}$$

The variation of the perturbed velocity field under the mentioned boundary conditions is computed using a two-directional differential quadrature technique presented in detail in [6] and a grid resolution $N\times M=6\times 6$, in form

$$\vec{\Phi}(r,z,t)=real\left\{\exp\left(\int_0^z k(s)ds+i(m\theta-\omega t)\right)\right\}\sum_{k=1}^{N}\sum_{s=1}^{M}a_{ks}\varphi_k(r)e^{-imz}, \tag{4}$$

where $\varphi(r)$ represents the set of orthogonal polynomial base. For this example, a number of $N=360$ equally distributed snapshots of the velocity field have been captured on the time domain $t\in[0,2520]$, Δt is set to 7 following the Nyquist criterion [7] and therefore captures all frequencies in the range $[0,0.44]$ correctly.

3. POD METHODOLOGY FOR REDUCED ORDER MODELING

The model order reduction using the method of POD has been illustrated on a variety of examples ranging from fluid mechanics [8], turbulent flows [9]and oceanography [10], with potential applications in electrical engineering [11] and computational physics [12]. We concentrate in this section to approximate the velocity components of the swirling flow fluctuation $\vec{\Phi}(r,z,t)$ as a finite sum of form

$$\vec{\Phi}(r,z,t)\approx\sum_{j=1}^{p}a_j(t)\phi_j(r,z), \tag{5}$$

expecting that this approximation becomes exact as $p\to+\infty$. A solution to solve the approximation problem (5) is given by the spectral methods detailed in [13] when orthogonal polynomials are used for the basisfunctions $\phi_j(r,z)$. In Proper Orthogonal Decomposition approach, we seek for an orthonormal basis functions, i.e. $\langle\phi_i(r),\phi_j(r)\rangle_{L^2}=\delta_{ij}$, where δ_{ij} is the Kronecker delta symbol

$$\delta_{ij}=\begin{cases}1 & if\quad i=j\\ 0 & if\quad i\neq j\end{cases} \tag{6}$$

and the coefficients a_j are computed by projecting the velocity field onto the POD modes

$$a_j(t) = \left\langle \vec{\Phi}(r,z,t), \phi_j(r,z) \right\rangle_{L^2}. \tag{7}$$

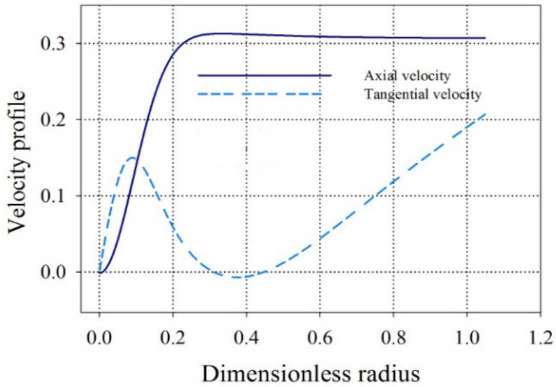

Figure 1. Velocity profiles at inlet of the computational domain

The POD problem reduces to find the subspace $X = span\{\phi_1, \phi_2, ..., \phi_p\}$ spanned by the sequence of orthonormal functions $\phi_j(r,z)$ such that the p -approximation of $\vec{\Phi}(r,z,t)$ is as good as possible in the least square sense. The approximation problem (5) is then equivalent to the following constrained minimization problem

$$\min_{\phi_1,\phi_2,...,\phi_p} \int_\Omega \left\| \vec{\Phi}(r,z,t) - \sum_{j=1}^{p} \left\langle \vec{\Phi}(r,z,t), \phi_j(r,z) \right\rangle_{L^2} \phi_j(r,z) \right\|_{L^2}^2 dr$$
$$s.t. \quad \left\langle \phi_i, \phi_j \right\rangle_{L^2} = \delta_{ij}, 1 \le i \le j \le p. \tag{8}$$

4. NUMERICAL RESULTS

The flow field is characterized in terms of nondimensional variables, in cylindrical coordinates, by the inlet velocity field depicted in Figure 1. Typical distribution of velocity field in the axial-radial domain is displayed in Figure 2.

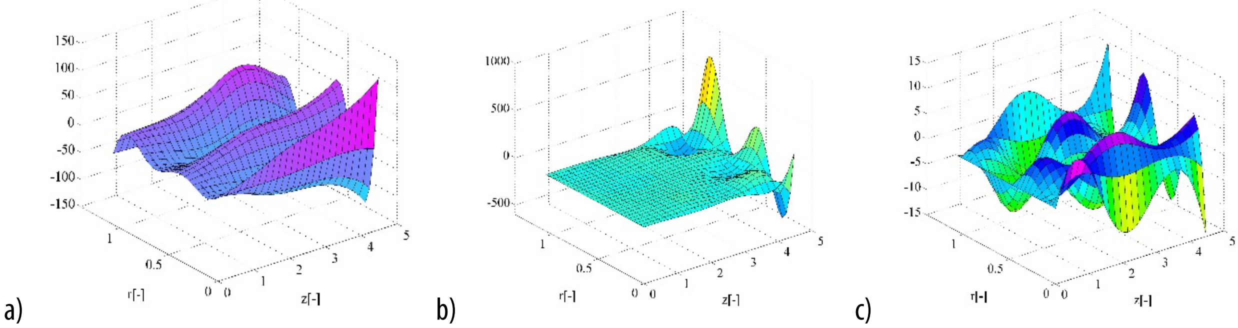

Figure 2. a) Axial velocity perturbation; b) Radial velocity perturbation; c) Tangential velocity perturbation

A comparison of the full solution of the velocity field and reduced order models solutions is provided in Figure 3. The swirling flow field computed at time level $t = 350$ by the reduced order model exhibits an overall good agreement with that from the full model.

In the POD-ROM framework, the velocity field was constructed by using $p = 10$ POD modes which recover a percent of 99.93% of the flow energy. The local error between the full solution and POD-ROM solutions, at time $t = 350$ is presented in Figure 4.

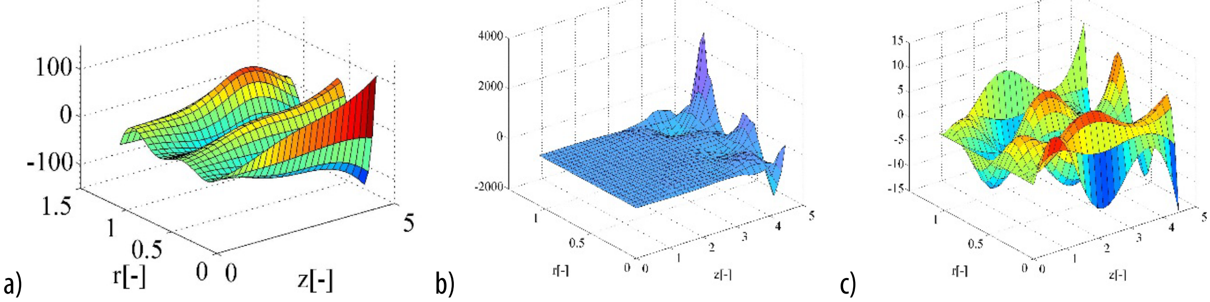

Figure 3. a) POD computed axial velocity perturbation; b) POD computed radial velocity perturbation;
c) POD reconstruction of tangential perturbation field.

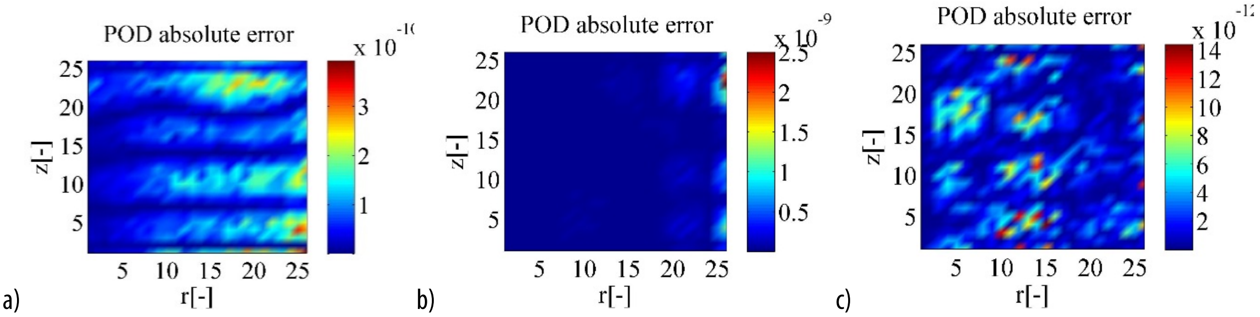

Figure 4. POD error at reconstruction at time $t = 350$ of: a) axial velocity field;
b) radial velocity field; c) tangential velocity field

Figure 5 presents the first three dominant POD modes for the swirling flow decomposition. These are associated to the most energetic coherent structures in the flow perturbations.

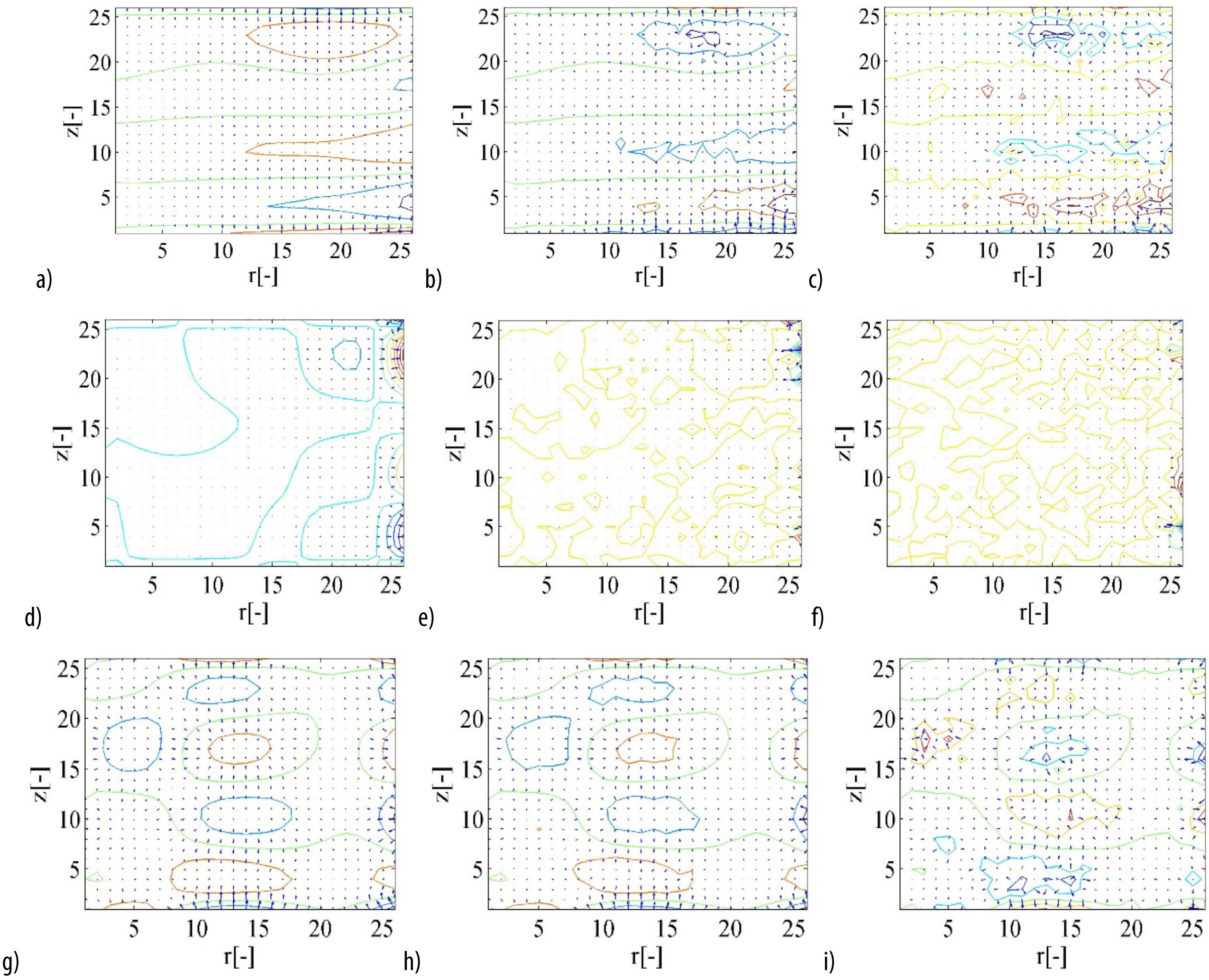

Figure 5. First three POD modes for swirling flow field: a)-c) axial velocity field decomposition;
d)-f) radial velocity decomposition; g)-i) tangential velocity field decomposition.

The correlation coefficient defined below is used as an additional metric to validate the quality of the reduced order model

$$C_i(\Phi) = \frac{\left(\left\|\Phi_i(r,z)\cdot\Phi_i^{POD}(r,z)\right\|_F\right)^2}{\left\|\left(\Phi_i(r,z)\right)^H\cdot\Phi_i(r,z)\right\|_F \left\|\left(\Phi_i^{POD}(r,z)\right)^H\cdot\Phi_i^{POD}(r,z)\right\|_F}, \mathrm{i}=0,\ldots,\mathrm{N}, \tag{9}$$

where $\Phi_i(r,z)$ means the solution of the full model at time i , $\Phi_i^{POD}(r,z)$ represents the computed solution at time i by means of the reduced order model, $(\cdot)$ represents the Hermitian inner product and $"H"$ stands for the conjugate transpose. We denote by $\|\cdot\|_F$ the Frobenius matrix norm in the sense that for any matrix $A \in \mathbb{C}_{m\times n}$ having singular values $\sigma_1,\ldots,\sigma_n$ and SVD of the form $A = U\Sigma V^H$, then

$$\|A\|_F = \|U^H AV\|_F = \|\Sigma\|_F = \sqrt{\sigma_1^2 + ... + \sigma_n^2}. \tag{10}$$

A comparison of the correlation coefficient between the full and reduced order model is provided in Figure 6.

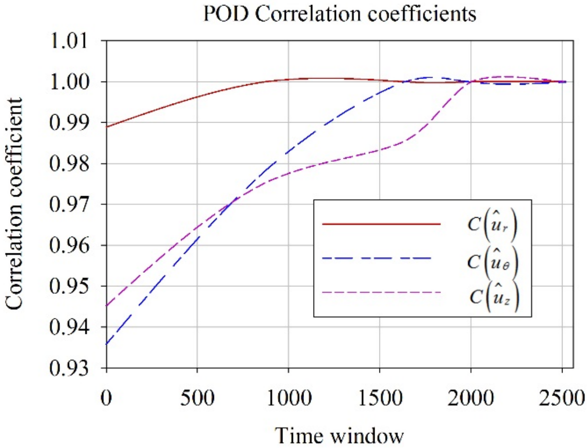

Figure 6. Correlation coefficients for the velocity field variables.

The values of the correlation coefficients are greater than 93.5% and confirm the efficiency of the POD reduced order model.

5. CONCLUSIONS

We have proposed a framework for proper orthogonal decomposition of 2D flows, when numerical data snapshots are captured at the inlet of the draft tube of a hydraulic turbine. We presented a rigorous error analysis for the ROM model obtained by POD. We found a very close agreement between the swirling flow reconstruction computed with the ROM model and the solution provided by the high fidelity model. The benefit of employing the POD method prevails when the main interest is to find coherent structures in the POD modes which are energetically ranked.We have identified the most energetic coherent structures in the flow perturbations.

The modal decomposition method presented in this paper leads to reduced order models with dramatically reduced numbers of equations and unknowns. A future extension of this research will address an efficient numerical approach for modal decomposition of swirling flows, where the full mathematical model implies more sophisticated relations at domain boundaries that must be satisfied by the reduced order model also.

ACKNOWLEDGEMENT

The author acknowledges the partially support of strategic grant POSDRU/159/1.5/S/137070 (2014) of the Ministry of National Education, Romania, co-financed by the European Social Fund–Investing in People, within the Sectorial Operational Program Human Resources Development 2007-2013.

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