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CHEMICAL REACTION EFFECTS ON FLOW PAST A PARABOLIC STARTED VERTICAL PLATE WITH VARIABLE TEMPERATURE AND MASS DIFFUSION IN THE PRESENCE OF MAGNETIC FIELD

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Abstract: Closed form solution of unsteady hydromagnetic flow past a parabolic starting motion of the infinite vertical plate with variable temperature and mass diffusion, in the presence of homogeneous chemical reaction of first order has been studied. The plate temperature as well as concentration level near the plate are increased linearly with time. The dimensionless governing equations are solved using Laplace-transform technique. The effect of velocity profiles are studied for different physical parameters like chemical reaction parameter, thermal Grashof number, mass Grashof number, Schmidt number and time. It is observed that the velocity increases with increasing values the thermal Grashof number or mass Grashof number. The trend is just reversed with respect to the chemical reaction parameter as well as magnetic field parameter.
Keywords: parabolic, homogeneous, heat and mass transfer, chemical reaction, isothermal, first order, vertical plate, magnetic field

1. INTRODUCTION

Free convection flows are of great interest in a number of industrial applications such as fiber and granular insulation, geothermal systems etc. Buoyancy is also of importance in an environment where differences between land and air temperatures can give rise to complicated flow patterns. Magneto hydro-dynamic has attracted the attention of a large number of scholars due to its diverse applications. In astrophysics and geophysics, it is applied to study the stellar and solar structures, interstellar matter, radio propagation through the ionosphere etc. In engineering it finds its applications in MHD pumps, MHD bearings etc.

The phenomenon of mass transfer is also very common in theory of stellar structure and observable effects are detectable, at least on the solar surface. The study of effects of magnetic field on free convection flow is important in liquid metals, electrolytes and ionized gases. The thermal physics of hydro -magnetic problems with mass transfer is of interest in power engineering and metallurgy. A few representative fields of interest in which combined heat and mass transfer along with chemical reaction play an important role in chemical process industries such as food processing and polymer production.

Mass diffusion rates can be changed tremendously with chemical reactions. The chemical reaction effects depend whether the reaction is homogeneous or heterogeneous. This depends on whether they occur at an interface or as a single phase volume reaction. In well mixed systems, the reaction is heterogeneous, if it takes place at an interface and homogeneous, if it takes place in solution. In majority cases, a chemical reaction depends on the concentration of the species itself. A reaction is said to be of first order, if the rate of reaction is directly proportional to the concentration of only one reactant and is independent of others. Decomposition of nitrogen pentoxide in the gas phase as well in an organic solvent like CCl_4, conversion of N-chloroacetanilide into p- chloroacetanilide, hydrolysis of methyl acetate and inversion of cane sugar. The radioactive disintegration of unstable nuclei are the best examples of first order reactions.

Chambre and Young [3] have analyzed a first order chemical reaction in the neighborhood of a horizontal plate. Das et al [4] have studied the effect of homogeneous first order chemical reaction on the flow past an impulsively started infinite vertical plate with uniform heat flux and mass transfer. Again, mass transfer effects on moving isothermal vertical plate in the presence of chemical reaction studied by Das et al [5]. The dimensionless governing equations were solved by the usual Laplace-transform technique.

Pure heat transfer effects on impulsively started an infinite vertical plate in the presence of magnetic field was studied by Soundalgekar et al [12]. Again, mass transfer effects on MHD flow past an impulsively started an infinite isothermal vertical plate with uniform mass diffusion studied by Soundalgekar et al [11]. Rajesh Kumar et al [8] have studied exact solution of hydromagnetic flow on moving vertical surface with prescribed uniform heat flux. The effect of viscous dissipation on Darcy free convection flow over a vertical plate with an exponential temperature was analyzed by Magyari and Rees [6]. The combined effects of heat and mass transfer along a vertical plate in the presence of a transverse magnetic field were studied by Ramesh Babu, and Shankar [10]. Rajput & Kumar [9] studied the magnetic field effects on flow past an impulsively started vertical plate with variable temperature and mass diffusion. Recently, Muthucumarswamy et al [7] studied MHD effects on accelerated isothermal vertical plate with uniform mass diffusion using Laplace transform method.

Agrawal et al [1] studied free convection due to thermal and mass diffusion in laminar flow of an accelerated infinite vertical plate in the presence of magnetic filed. Agrawal et al [2] further extended the problem of unsteady free convective flow and mass diffusion of an electrically conducting elasto-viscous fluid past a parabolic starting motion of the infinite vertical plate with transverse magnetic plate. The governing equations are tackled using Laplace transform technique.

In this paper, an investigation is carried out to study the effects of on flow past an infinite vertical plate subjected to parabolic motion with variable temperature and variable mass diffusion, in the presence of applied transverse magnetic field and chemical reaction of first order. The dimensionless governing equations are solved using the Laplace-transform technique. The solutions are derived in terms of exponential and complementary error functions.

2. MATHEMATICAL ANALYSIS

In this problem, we consider the unsteady hydromagnetic flow of a viscous incompressible fluid past an infinite vertical plate with variable temperature and mass diffusion, in the presence of chemical reaction of first order and applied transverse magnetic field. The x'-axis is taken along the plate in the vertically upward direction and the y -axis is taken normal to the plate. At time $t' \le 0$, the plate and fluid are at the same temperature T_∞ and concentration C'_∞. At time t'>0 , the plate is started with a velocity $u = u_0 t'^2$ in its own plane against gravitational field. The plate temperature as well as concentartion near the plate are raised linearly with time. A chemically reactive species which transforms according to a simple reaction involving the conecntration is emitted from the plate and diffuses into the fluid. The plate is also subjected to a uniform magnetic field of strength B_0 is assumed to be applied normal to the plate. The reaction is assumed to take place entirely in the stream. Then under usual Boussinesq's approximation for unsteady parabolic starting motion is governed by the following equations:

$$\frac{\partial u}{\partial t'} = g\beta(T - T_\infty) + g\beta^*(C' - C'_\infty) + \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho}u \tag{1}$$

$$\rho C_p \frac{\partial T}{\partial t'} = k\frac{\partial^2 T}{\partial y^2} \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y^2} - k_1(C' - C'_\infty) \tag{3}$$

With the following initial and boundary conditions:

$$\begin{gathered} u = 0, \quad T = T_\infty, \quad C' = C'_\infty \quad \text{for all } y, t' \le 0 \\ t' > 0: u = u_0 . t'^2, \; T = T_\infty + (T_W - T_\infty)At', \; C' = C'_\infty + (C'_W - C'_\infty)At' \text{ at } y = 0 \\ u \to 0 \quad T \to T_\infty, \quad C' \to C'_\infty \quad \text{as } y \to \infty \end{gathered} \tag{4}$$

where $A = \left(\frac{u_0^2}{\nu}\right)^{1/3}$.

On introducing the following non-dimensional quantities:

$$\begin{gathered} U = u\left(\frac{u_0}{\nu^2}\right)^{1/3}, \quad t = \left(\frac{u_0^2}{\nu}\right)^{1/3} t', \quad Y = y\left(\frac{u_0}{\nu^2}\right)^{1/3}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \; C = \frac{C' - C'_\infty}{C'_w - C'_\infty} \\ Gr = \frac{g\beta(T_w - T_\infty)}{(\nu u_0)^{1/3}}, \; Gc = \frac{g\beta(C'_w - C'_\infty)}{(\nu u_0)^{1/3}}, \; K = K_1\left(\frac{\nu}{u_0^2}\right)^{\frac{1}{3}}, \; Pr = \frac{\mu C_p}{k}, \; Sc = \frac{\nu}{D} \\ M = \frac{\sigma B_0^2}{\rho}\left(\frac{\nu}{u_0^2}\right)^{1/3} \end{gathered} \tag{5}$$

The equations (1) to (3) reduces to the following dimensionless form:

$$\frac{\partial U}{\partial t} = Gr\theta + GcC + \frac{\partial^2 U}{\partial Y^2} - MU \tag{6}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial Y^2} \tag{7}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2} - KC \tag{8}$$

The corresponding initial and boundary conditions in non-dimensionless form are as follows:

$$\begin{gathered} U=0,\quad \theta=0,\quad C=0 \quad \text{for all } Y,t\le 0 \\ t>0:\ U=t^2,\quad \theta=t,\quad C=t \quad \text{at} \quad Y=0 \\ U\to 0,\quad \theta\to 0,\quad C\to 0 \quad \text{as} \quad Y\to\infty \end{gathered} \tag{9}$$

The dimensionless governing equations (6) to (8) and the corresponding initial and boundary conditions (9) are solved by the usual Laplace-transform technique and the solutions are derived as follows:

$$\theta = t\left[\left(1+2\eta^2 Pr\right)\mathrm{erfc}(\eta\sqrt{Pr}) - \frac{2\eta\sqrt{Pr}}{\sqrt{\pi}}\exp(-\eta^2 Pr)\right] \tag{10}$$

$$\begin{aligned} C = {} & \frac{t}{2}\left[\exp\left(2\eta\sqrt{ScKt}\right)\mathrm{erfc}\left(\eta\sqrt{Sc}+\sqrt{Kt}\right) + \exp\left(-2\eta\sqrt{ScKt}\right)\mathrm{erfc}\left(\eta\sqrt{Sc}-\sqrt{Kt}\right)\right] \\ & - \frac{\eta\sqrt{Sc}\sqrt{t}}{2\sqrt{K}}\left[\exp\left(-2\eta\sqrt{ScKt}\right)\mathrm{erfc}\left(\eta\sqrt{Sc}-\sqrt{Kt}\right) - \exp\left(-2\eta\sqrt{ScKt}\right)\mathrm{erfc}\left(\eta\sqrt{Sc}+\sqrt{Kt}\right)\right] \end{aligned} \tag{11}$$

$$\begin{aligned} U = {} & \left[\frac{\left(\eta^2 + Mt(1+2ac+2bd)\right) + 2M(c+d)}{2M}\right]\left[\exp\left(2\eta\sqrt{Mt}\right)\mathrm{erfc}\left(\eta+\sqrt{Mt}\right) + \exp\left(-2\eta\sqrt{Mt}\right)\mathrm{erfc}\left(\eta-\sqrt{Mt}\right)\right] \\ & + \left[\frac{\eta\sqrt{t}\left(1-4M(t+ac+bd)\right)}{4M^{3/2}}\right]\left[\exp\left(-2\eta\sqrt{Mt}\right)\mathrm{erfc}\left(\eta-\sqrt{Mt}\right) - \exp\left(2\eta\sqrt{Mt}\right)\mathrm{erfc}\left(\eta+\sqrt{Mt}\right)\right] \\ & - \frac{\eta t}{M\sqrt{\pi}}\exp(-(\eta^2+Mt)) - 2c\,\mathrm{erfc}(\eta\sqrt{Pr}) - 2act\left[\left(1+2\eta^2 Pr\right)\mathrm{erfc}(\eta\sqrt{Pr}) - \frac{2\eta\sqrt{Pr}}{\sqrt{\pi}}\exp(-\eta^2 Pr)\right] \\ & + c\exp(at)\left[\exp\left(2\eta\sqrt{(M+a)t}\right)\mathrm{erfc}\left(\eta+\sqrt{(M+a)t}\right) + \exp\left(-2\eta\sqrt{(M+a)t}\right)\mathrm{erfc}\left(\eta-\sqrt{(M+a)t}\right)\right] \\ & - d\exp(bt)\left[\exp\left(2\eta\sqrt{(M+b)t}\right)\mathrm{erfc}\left(\eta+\sqrt{(M+b)t}\right) + \exp\left(-2\eta\sqrt{(M+b)t}\right)\mathrm{erfc}\left(\eta-\sqrt{(M+b)t}\right)\right] \\ & + c\exp(at)\left[\exp\left(2\eta\sqrt{Prat}\right)\mathrm{erfc}\left(\eta\sqrt{Pr}+\sqrt{at}\right) + \exp\left(-2\eta\sqrt{Prat}\right)\mathrm{erfc}\left(\eta\sqrt{Pr}-\sqrt{at}\right)\right] \\ & + d\exp(bt)\left[\exp\left(2\eta\sqrt{Sc(K+b)t}\right)\mathrm{erfc}\left(\eta\sqrt{Sc}+\sqrt{(K+b)t}\right) + \exp\left(-2\eta\sqrt{Sc(K+b)t}\right)\mathrm{erfc}\left(\eta\sqrt{Sc}-\sqrt{(K+b)t}\right)\right] \\ & - (1+bt)d\left[\exp\left(2\eta\sqrt{ScKt}\right)\mathrm{erfc}\left(\eta\sqrt{Sc}+\sqrt{Kt}\right) + \exp\left(-2\eta\sqrt{ScKt}\right)\mathrm{erfc}\left(\eta\sqrt{Sc}-\sqrt{Kt}\right)\right] \\ & + \frac{bd\eta\sqrt{Sc}\sqrt{t}}{\sqrt{K}}\left[\exp\left(-2\eta\sqrt{ScKt}\right)\mathrm{erfc}\left(\eta\sqrt{Sc}-\sqrt{Kt}\right) - \exp\left(2\eta\sqrt{ScKt}\right)\mathrm{erfc}\left(\eta\sqrt{Sc}+\sqrt{Kt}\right)\right] \end{aligned} \tag{12}$$

where, $a=\dfrac{M}{Pr-1}$, $b=\dfrac{M-KSc}{Sc-1}$, $c=\dfrac{Gr}{2a^2(1-Pr)}$, $d=\dfrac{Gc}{2b^2(1-Sc)}$ and $\eta=\dfrac{Y}{2\sqrt{t}}$.

3. RESULTS AND DISCUSSION

In order to get physical understanding of the problem numerical computations are carried out for different physical parameters K(chemical reaction parameter), M (Magnetic field parameter), Pr (Prandtl number), Gr (Thermal Grashof parameter), Gc (Mass Grashof parameter), Sc (Schmidt number) and t(time) upon the nature of the flow and transport. The value of the Schmidt number Sc is taken to be 0.6 which corresponds to water-vapor. Also, the values of Prandtl number Pr are chosen such that they represent water (Pr = 7.0). The numerical values of the velocity are computed for different physical parameters like chemical reaction parameter, magnetic field parameter, Prandtl number, thermal Grashof number, mass Grashof number, Schmidt number and time.

Figure 1 represents the effect of the concentration profiles for different values of the chemical reaction parameter (K=0.2, 2, 5, 10) at t=0.4. The effect of chemical reaction parameter is important in concentration field. The profiles have the common feature that the concentration decreases in a monotone fashion from the surface to a zero value far away in the free stream. It is observed that the concentration increases with decreasing chemical reaction parameter.

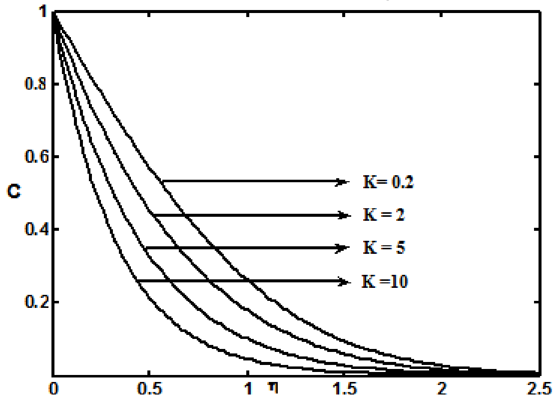

Figure 1. Concentration profiles for different values of chemical reaction parameter (K)

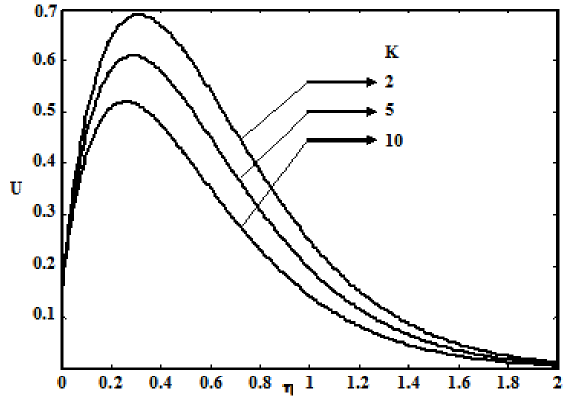

Figure 2. Velocity profiles for different values of chemical reaction parameter (K)

The velocity profiles for different values of the chemical reaction parameter (K=2, 5, 10), Gr=5, Gc=40, Pr=7, M=2 and t=0.4 are shown in Figure 2. It is observed that the velocity increases with decreasing chemical reaction parameter.

Figure 3 demonstrates the effect of velocity for different values of the magnetic field parameter (M=1.2, 1.8, 2), Gr=5, Gc=40, Pr=7, K=4 and t=0.4 . It was observed that the velocity increases with decreasing values of the magnetic field parameter. This agrees with the expectations, since the magnetic field exerts a retarding force on the free convective flow.

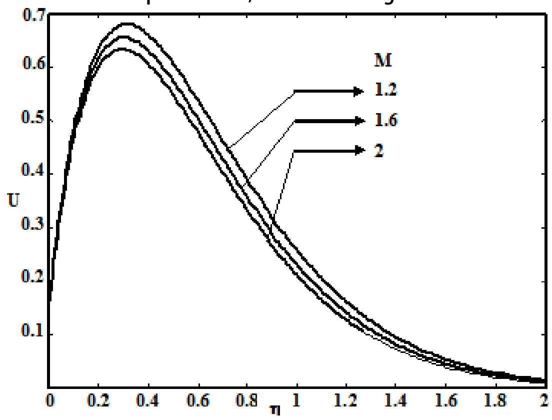

Figure 3. Velocity profiles for different magnetic field parameter (M)

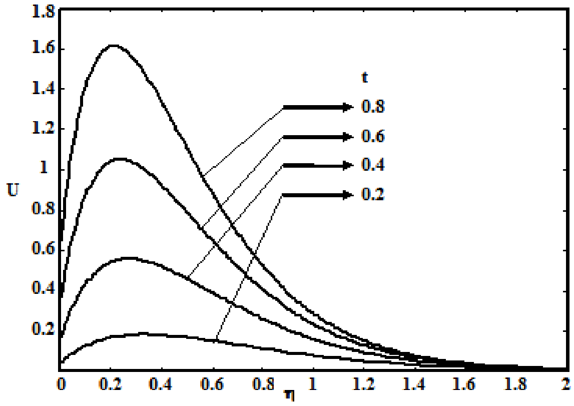

Figure 4. Velocity profiles for different time (t)

The velocity profiles for different values of the time (t=0.2, 0.4, 0.6, 0.8), Gr=5, Gc=40, K=4 and M=4 are presented in figure 4. The trend shows that the velocity increases with incrasing values of the time t. The effect of velocity profiles for different values of the Schmidt number (Sc=0.16, 0.3, 0.6), Gr=5, Gc=40, Pr=7, M=4 and t=0.4 are shown in figure 5. It is observed that the velocity increases with decrasing values of the Schmidt number.

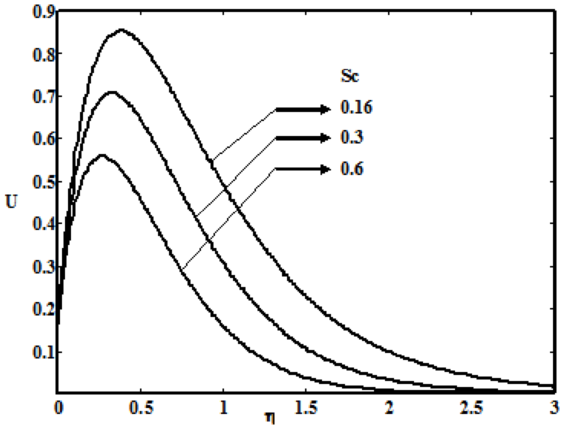

Figure 5. Velocity profiles for different Schmidt number (Sc)

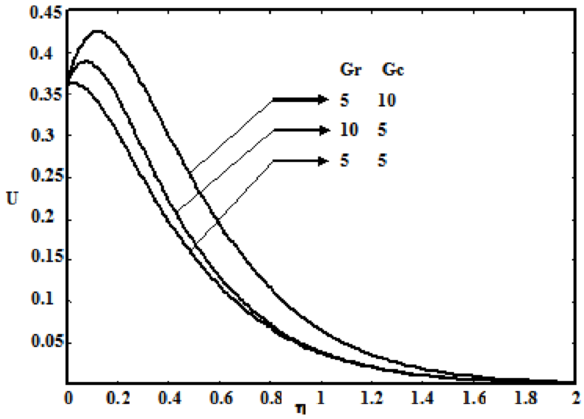

Figure 6. Velocity profiles for different thermal Grashof number (Gr) and mass Grashof number (Gc)

Figure 6 demonstrates the effects of different thermal Grashof number (Gr = 5, 10), mass Grashof number (Gc = 5, 10), K = 4, M = 4 and Pr = 7 on the velocity at t = 0.6. It is observed that the velocity increases with increasing values of the thermal Grashof number or mass Grashof number.

4. CONCLUSIONS

An exact solution of the MHD flow past a parabolic started an infinite vertical plate with variable temperature and mass diffusion, in the presence of chemical reaction of first order has been studied. The dimensionless governing equations are solved by the usual Laplace transform technique. The effect of the temperature, the concentration and the velocity fields for different physical parameters like chemical reaction parameter, magnetic field parameter, thermal Grashof number, mass Grashof number and t are studied graphically. It is observed that the velocity increases with increasing thermal Grashof number or mass Grashof number and time t in the presence of magnetic field parameter. But the trend is just reversed with respect to the chemical reaction parameter or magnetic field parameter. The plate concentration increases with decreasing values of the chemical reaction parameter.

NOMENCLATURE

A - Constants
B_0 - uniform magnetic field normal to the plate
C' - species concentration in the fluid, kg m^{-3}
C - dimensionless concentration
Cp - specific heat at constant pressure, J kg^{-1}k
D - mass diffusion coefficient, m^2s^{-1}
G_C - mass Grashof number
G_r - thermal Grashof number
G - acceleration due to gravity, m s^{-2}
k - thermal conductivity, W $m^{-1}k^{-1}$
M - dimensionless magnetic field parameter
P_r - Prandtl number
S_C - Schmidt number
T - temperature of the fluid near the plate K
t' – time, s
u - velocity of the fluid in the x' -direction, m s^{-1}
u_o - velocity of the plate, m s^{-1}
u - dimensionless velocity
y - coordinate axis normal to the plate m
Y - dimensionless coordinate axis normal to the plate

Greek symbols

β - volumetric coefficient of thermal expansion, K^{-1}
β^* - volumetric coefficient of expansion with concentration, K^{-1}
μ - coefficient of viscosity, Ra.s
ν - kinematic viscosity, kg m^{-3}
ρ - density of the fluid, kg m^{-3}
σ - Stefan-Boltzmann constant
τ - dimensionless skin-friction, kg m^{-1} s^2
θ - dimensionless temperature
η - similarity parameter
erfc - Complementary error function

Subscripts

ω - conditions at the wall
∞ - free stream conditions

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