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SKIN FRICTION ANALYSIS ON THERMAL RADIATION AND CHEMICAL REACTION EFFECTS OF AN EXPONENTIALLY ACCELERATED VERTICAL PLATE WITH UNIFORM MASS DIFFUSION

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Abstract: Analysis of Skin friction on free convection flow field, of an incompressible viscous fluid past an exponentially accelerated vertical plate is considered under the following conditions (1) isothermal heat transfer at the plate (2) uniform mass transfer of the fluid. Effect of nature of flow in case of heating of the plate by analyzing skin friction is considered. Non-dimensional governing equations are converted to dimensional form and are solved by Laplace transform technique. Skin friction for various thermo physic parameters like Schmidt number, chemical reaction parameter, radiation parameter and accelerating parameter are discussed. It is observed that shear stress (skin friction) is increased as there is an increase in the values of Sc, K, a. There is retardation between fluid layers as there is a raise in the parameters, when the plate gets heated up due to conventional currents.
Keywords: exponential, radiation, isothermal, vertical plate, skin friction

1. INTRODUCTION

A chemical reaction effect depends on whether the reaction is homogeneous or heterogeneous. This depends on whether they occur at an interface or as a single phase volume reaction. In well-mixed systems, if it takes place at an interface, the reaction is heterogeneous; and homogeneous if it takes place in solution. Chambre and Young [1958] have analyzed a first order chemical reaction in the neighborhood of a horizontal plate. Das et al [1994] have studied the effect of homogeneous first order chemical reaction on the flow past an impulsively started infinite vertical plate with uniform heat flux and mass transfer. Again, mass transfer effects on moving isothermal vertical plate in the presence of chemical reaction studied by Das et al [1996].

Radiative heat and mass transfer play an important role in manufacturing industries for the design of fins, steel rolling, nuclear power plants, gas turbines and various propulsion devices for aircraft, missiles, satellites and space vehicles. England and Emery [1969] have studied the thermal radiation effects of a optically thin gray gas bounded by a stationary vertical plate. Radiation effect on mixed convection along a isothermal vertical plate was studied by Hossain and Takhar [1996]. Das et al [1999] have analyzed radiation effects on flow past an impulsively started infinite isothermal vertical plate.

Gupta [1979] studied free convection on flow past an linearly accelerated vertical plate in the presence of viscous dissipative heat using perturbation method. Kafousias and Raptis [1981] extended this problem to include mass transfer effects subjected to variable suction or injection. Free convection effects on flow past an exponentially accelerated vertical plate was studied by Singh and Naveen Kumar [1984]. The skin friction for accelerated vertical plate has been studied analytically by Hossain and Shayo [1986]. Basant Kumar Jha et al [1991] analyzed mass transfer effects on exponentially accelerated infinite vertical plate with constant heat flux and uniform mass diffusion. First order chemical reaction on exponentially accelerated vertical plate with mass diffusion was studied by R.Muthucumaraswamy, R.Rahul and P.Balachandran [2011].

However the study of skin friction on thermal radiation and chemical reaction effects of an unsteady flow past an exponentially accelerated vertical plate in the presence of chemical reaction of first order with uniform mass diffusion while the plate is cooled ($Gr > 0$, $Gc > 0$) has not been studied in the literature. It is proposed to study the friction of fluid layers for chemical reaction parameter, radiation parameter, accelerating parameter and Schmidt number. Such a study found useful in energy storage, food processing, freezing.

2. MATHEMATICAL ANALYSIS

The fluid is assumed to be in the direction of x′-axis which is taken along the vertical plate in the upward direction. The y′-axis is taken to be normal to the plate. Initially the temperature of the plate and the fluid is assumed to be same. Initially the temperature of the plate is T'_∞ and the concentration level in the fluid is assumed to be C'_∞. At time $t' > 0$, the plate temperature is raised to $T'w$ and the concentration level in the fluid is raised to $C'w$.

Inertia terms are negligible as we are considering an infinite vertical plate. There is a first order chemical reaction between the diffusing species and the fluid. Under usual Boussinesq's approximation, the unsteady free convective flow of an incompressible fluid in dimensional form is given by

$$\frac{\partial u'}{\partial t'} = g\beta(T'-T'_\infty) + g\beta^*(C'-C'_\infty) + \nu\frac{\partial^2 u'}{\partial y'^2} \tag{1}$$

$$\rho\, C_p \frac{\partial T'}{\partial t'} = k\frac{\partial^2 T'}{\partial y'^2} - \frac{\partial q_r}{\partial y'} \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y'^2} - K_l(C'-C'_\infty) \tag{3}$$

with the following initial and boundary conditions:

$$t' \le 0: u'=0, T'=T'_\infty, C'=C'_\infty \quad for\ all \quad y'; t'>0: u'=u_0\exp(a't'), C'=C'_\infty \ at \ y'=0; u'=0, T'\to T'_\infty, C'\to C'_\infty,$$
$$as \quad y'\to\infty, where\ A=\frac{u_0^2}{\nu} \tag{4}$$

The local radiant for the case of an optically thin gray gas is expressed by

$$\frac{\partial q_r}{\partial y'} = -4a^*\,\sigma\left(T'^4_\infty - T'^4\right) \tag{5}$$

It is assumed that the temperature differences within the flow are sufficiently small such that T'^4 may be expressed as a linear function of the temperature. This is accomplished by expanding T'^4 in a Taylor series about T'_∞ and neglecting higher-order terms, thus

$$T'^4 \cong 4T'^3_\infty\, T' - 3T'^4_\infty \tag{6}$$

By using (5) and (6), (2) reduces to

$$\rho\, C_p \frac{\partial T'}{\partial t'} = k\frac{\partial^2 T'}{\partial y'^2} + 16a^*\sigma T'^3_\infty\,(T'_\infty - T') \tag{7}$$

On introducing the following non-dimensional quantities

$$U=\frac{u'}{u_0},\quad t=\frac{t'u_0^2}{\nu},\quad Y=\frac{y'u_0}{\nu},\quad \theta=\frac{T'-T'_\infty}{T'_w-T'_\infty},$$
$$Gr=\frac{g\beta\nu(T'_w-T'_\infty)}{u_0^3},\quad C=\frac{C'-C'_\infty}{C'_w-C'_\infty},\quad Gc=\frac{\nu g\beta^*(C'_w-C'_\infty)}{u_0^3}, \tag{8}$$
$$R=\frac{16a^*\nu^2\sigma T'^3_\infty}{ku_0^2},\quad Pr=\frac{\mu C_p}{k},\quad Sc=\frac{\nu}{D},\quad K=\frac{\nu K_l}{u_0^2},\quad a=\frac{a'\nu}{u_0^2}$$

in (1) to (4), leads to

$$\frac{\partial U}{\partial t} = Gr\,\theta + Gc\,C + \frac{\partial^2 U}{\partial Y^2} \tag{9}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr}\frac{\partial^2\theta}{\partial Y^2} - \frac{R}{Pr}\theta \tag{10}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2} - K\,C \tag{11}$$

The initial and boundary conditions in non-dimensional form are

$$\begin{array}{llllll} & U=0, & \theta=0, & C=0, & for\ all & Y,t\le 0 \\ t>0: & U=\exp(at), & \theta=1, & C=1, & at & Y=0 \\ & U=0, & \theta\to 0, & C\to 0 & as & Y\to\infty \end{array} \tag{12}$$

All the physical variables are defined in the nomenclature.

The solutions are obtained for flow field in the presence of first order chemical reaction and radiation. Equations (9) to (11), subject to the boundary conditions (12), are solved by the usual Laplace-transform technique and the expressions for temperature, velocity and concentration are as follows:

$$\theta = \frac{1}{2}\left[\exp(2\eta\sqrt{Rt})\, erfc(\eta\sqrt{Pr} + \sqrt{bt}) + \exp(-2\eta\sqrt{Rt})\, erfc(\eta\sqrt{Pr} - \sqrt{bt})\right] \tag{13}$$

$$C = \frac{1}{2}\left[\exp(2\eta\sqrt{KtSc})\, erfc(\eta\sqrt{Sc} + \sqrt{Kt}) + \exp(-2\eta\sqrt{KtSc})\, erfc(\eta\sqrt{Sc} - \sqrt{Kt})\right] \tag{14}$$

$$\begin{aligned} U = {} & \frac{e^{at}}{2}\begin{bmatrix} e^{2\eta\sqrt{(M+a)t}}\, erfc\left(\eta + \sqrt{(M+a)t}\right) + \\ e^{-2\eta\sqrt{(M+a)t}}\, erfc\left(\eta - \sqrt{(M+a)t}\right) \end{bmatrix} + (e+f)\begin{bmatrix} e^{2\eta\sqrt{Mt}}\, erfc\left(\eta + \sqrt{Mt}\right) + \\ e^{-2\eta\sqrt{Mt}}\, erfc\left(\eta - \sqrt{Mt}\right) \end{bmatrix} - e\, e^{ct}\begin{bmatrix} e^{2\eta\sqrt{(M+c)t}}\, erfc\left(\eta + \sqrt{(M+c)t}\right) + \\ e^{-2\eta\sqrt{(M+c)t}}\, erfc\left(\eta - \sqrt{(M+c)t}\right) \end{bmatrix} \\ & - f\, e^{dt}\begin{bmatrix} e^{2\eta\sqrt{(M+d)t}}\, erfc\left(\eta + \sqrt{(M+d)t}\right) + \\ e^{-2\eta\sqrt{(M+d)t}}\, erfc\left(\eta - \sqrt{(M+d)t}\right) \end{bmatrix} - e\begin{bmatrix} e^{2\eta\sqrt{Rt}}\, erfc\left(\eta\sqrt{\Pr} + \sqrt{bt}\right) + \\ e^{-2\eta\sqrt{Rt}}\, erfc\left(\eta\sqrt{\Pr} - \sqrt{bt}\right) \end{bmatrix} + e\, e^{ct}\begin{bmatrix} e^{2\eta\sqrt{\Pr(b+c)t}}\, erfc\left(\eta\sqrt{\Pr} + \sqrt{(b+c)t}\right) + \\ e^{-2\eta\sqrt{\Pr(b+c)t}}\, erfc\left(\eta\sqrt{\Pr} - \sqrt{(b+c)t}\right) \end{bmatrix} \\ & - f\begin{bmatrix} e^{2\eta\sqrt{KtSc}}\, erfc\left(\eta\sqrt{Sc} + \sqrt{Kt}\right) + \\ e^{-2\eta\sqrt{KtSc}}\, erfc\left(\eta\sqrt{Sc} - \sqrt{Kt}\right) \end{bmatrix} + f\, e^{dt}\begin{bmatrix} e^{2\eta\sqrt{Sc(K+d)t}}\, erfc\left(\eta\sqrt{Sc} + \sqrt{(K+d)t}\right) + \\ e^{-2\eta\sqrt{Sc(K+d)t}}\, erfc\left(\eta\sqrt{Sc} - \sqrt{(K+d)t}\right) \end{bmatrix} \end{aligned} \tag{15}$$

where, $b = \frac{R}{Pr}, c = \frac{R}{1-Pr}, d = \frac{K\,Sc}{1-Sc}, e = \frac{Gr}{2c(1-Pr)}, f = \frac{Gc}{2d(1-Sc)}$ and $\eta = Y/2\sqrt{t}$ where erfc is called complementary error function.

3. SKIN FRICTION AND SKIN FRICTION PROFILES

We now study skin-friction from velocity. It is given in non-dimensional form as

$$\tau = -\left.\frac{dU}{d\eta}\right|_{\eta=0} \tag{16}$$

From equations (15) and (16), we have

$$\begin{aligned} \tau = {} & \sqrt{a}\, erf\left(\sqrt{at}\right)\exp(at) + \frac{1}{\sqrt{t\pi}} - \sqrt{c}\, erf\left(\sqrt{ct}\right)\exp(ct) - \sqrt{d}\, erf\left(\sqrt{dt}\right)\exp(dt) - 2e\sqrt{R}\, erf\left(\sqrt{bt}\right) + 2e\sqrt{\Pr(b+c)}\, erf\left(\sqrt{\Pr(b+c)t}\right)\exp(ct) \\ & -2f\sqrt{KSc}\, erf\left(\sqrt{Kt}\right) + 2e\sqrt{Sc(K+d)}\, erf\left(\sqrt{(K+d)t}\right)\exp(dt) \end{aligned} \tag{17}$$

where $b = \frac{R}{Pr}, c = \frac{R}{1-Pr}, d = \frac{K\,Sc}{1-Sc}, A = \frac{Gr}{c(1-Pr)}, B = \frac{Gr}{c^2(1-Pr)}, C = \frac{Gc}{d(1-Sc)}, D = \frac{Gc}{d^2(1-Sc)}$

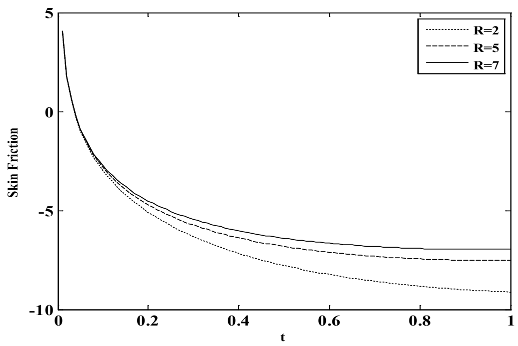

Figure 1. Skin friction for different values of R

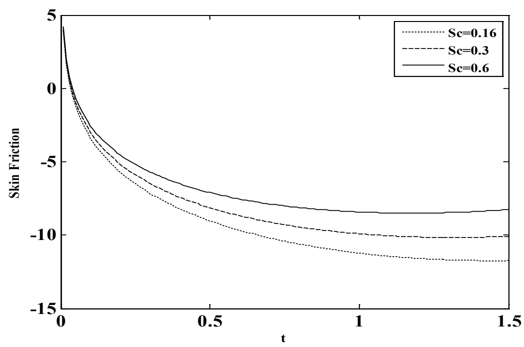

Figure 2. Skin friction for different values of Sc

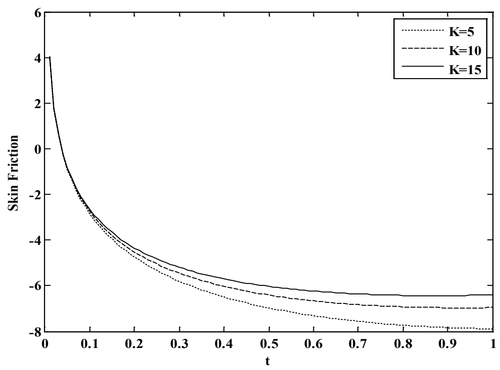

Figure 3. Skin friction for different values of K

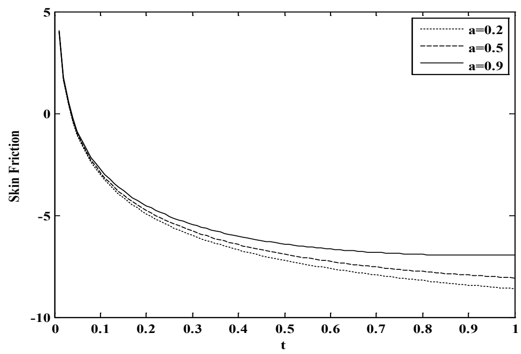

Figure 4. Skin friction for different values of a

4. DISCUSSION AND RESULTS

(1) From Figure 1 (where a=0.9, Sc=0.16, Gr= 15, Gc=10, Pr=0.71, K=10), it is observed that whenever there is an increase in the intensity of radiation (R =2, 5, 7), the fluid flow is slowed as there is an increase in the skin friction, whenever the plate is being cooled due to convective currents.

(2) Schmidt number (Sc) relates mass transfer boundary layer and hydrodynamic boundary layer. Figure 2 signifies that application of various species like hydrogen (0.16), Helium (0.3) and Water vapor (0.6) , the fluid flow is retarded slowly. Here pr=0.71, R=5, K=2, a=0.9, Gr=10, Gr=15.

(3) Contact between fluid layers is more as there is an increase in the chemical reaction parameter (K = 5, 10, 15) and it affects the convectional flow as it is seen from Figure 3 where Sc=0.16, Gr=15, Gc=10, Pr=0.71, a=0.9, R=7

(4) Figure 4 shows that by increasing the acceleration (a = 0.2, 0.5, 0.9) of the plate, it is observed that the skin friction increases, when Gr and Gc are positive. Sc=0.16, Gr=15, Gc=10, Pr=0.71, R=7, K=10.

NOMENCLATURE

C' species concentration in the fluid
C dimensionless concentration
C_p specific heat at constant pressure
D mass diffusion coefficient
Gc mass Grashof number
Gr thermal Grashof number
g acceleration due to gravity
K thermal conductivity
Pr Prandtl number
Sc Schmidt number
T temperature of the fluid near the plate
t' time
u velocity of the fluid in the x-direction
u_0 velocity of the plate
u dimensionless velocity
y coordinate axis normal to the plate
Y dimensionless coordinate axis normal to the plate

Greek symbols

β volumetric coefficient of thermal expansion
β^* volumetric coefficient of expansion with concentration
μ coefficient of viscosity
ν kinematic viscosity
ρ density of the fluid
τ dimensionless skin-friction
θ dimensionless temperature
η similarity parameter
$erfc$ complementary error function

Subscripts

w conditions at the wall
∞ free stream conditions

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