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USING AUTOMATIC OPTIMAL SPLINE SMOOTHING METHOD TO OPTIMIZING EDGES OF MOROCCAN BOUGUER GRAVITY ANOMALY MAP

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Abstract: We present a singular method that is capable to filter out noise as well as suppress outliers of sampled real functions under fairly general conditions. From an a priori selection of the number of points that define the adjusting spline, but not their location in that curve, the automatic optimal spline smoothing method automatically determines the adjusting cubic spline in a least-squares optimal sense. The method is fast and easily allows for selection of various possible number of knots, adding a desirable flexibility to the procedure. As an illustration, we apply the AOSS method to Moroccan Bouguer gravity data map. The AOSS smoothing technique is an efficient tool in the interpretation of geophysical potential field data particularly suitable in denoising, filtering and analyzing gravity data singularities. The AOSS smoothing and filtering technique was found to be consistently useful for optimizing edges and contours of geophysical data maps as Moroccan Bouguer gravity anomaly data map.
Keywords: Smoothing, automatic, spline, gravity, Bouguer, Morocco

1. INTRODUCTION

In applied geophysics we are often required to represent a set of measured data in the form of a smooth curve from which desirable parameters or attributes are to be extracted. So sharpening and edge-enhancement filters are often applied to geophysical data that were collected with sample spacing. Consequently, the first step in any processing of such geophysical data is the "cleaning up" of the noise in a way that preserves the signal sharp variations. In the literature we find several methods that try to filter and/or smooth the data [1] [2]. The natural question is how we should choose the "interpolating points" from the data so as to construct the desired smoothing function. Normally, these points are extracted from the data, in a regular fashion or manually selected. An alternative methodology to the usual gridding/digital filtering processing pathway is the Automatic Optimal Spline Smoothing method (AOSS) [3][4][5][6] which recently developments have fascinated the scientific, engineering, and mathematics community with their versatile applicability. The AOSS method that optimally selects points to define a cubic spline that best represents the data in the least-squares sense is proposed and applied to Moroccan Bouguer gravity data map. The AOSS is constructed to optimize some property of the Moroccan Bouguer gravity anomaly data map such as smoothness. In this paper we propose to optimize the responses of the filtered Bouguer gravity anomaly data map of Morocco (figure 1 and figure 2). So the AOSS has become a powerful signal and image processing tool used to filtering geophysical data.

2. THE BOUGUER GRAVITY ANOMALY

A gravity anomaly is the difference between the measured gravity at a particular location and the theoretical gravity given by a reference Earth model (e.g. The International Gravity Formula/Geodetic Reference System 1980) for the same location. It is widely used in the study of density inhomogeneities inside the Earth. Measured gravity data contain the effects of latitude, Earth tides, instrumental drift, distance from the reference ellipsoid, and masses between the actual topography and the reference ellipsoid [7]. In order to obtain anomalies that are comparable over large areas, a number of corrections must be applied. These are commonly referred to as Earth tides, instrumental drift, latitude, free-air and topography corrections. When the first four corrections are applied to measured gravity data we obtain the free-air gravity anomaly Δg_{FA}, which at short wavelengths correlates strongly with topography. The end-product of all gravity correction is the Bouguer anomaly Δg_B, which should correlate mainly with lateral density variations within the crust and Moho topography. The Bouguer anomaly is readily obtained by applying the correction for the gravitational attraction of topography to the free-air anomaly. The main purpose of the complete Bouguer correction is to remove all non-geological components of the gravity anomalies enhancing subsurface mass variations [8][9].

Figure 1. Geographical map of Morocco **Figure 2**. Satellite map of Morocco

Source: http://visibleearth.nasa.gov/view_rec.php?id=673

Gravity measurements are used in studying the figure, composition, and structure of the Earth. Density variations of bedrock and soil in the immediate vicinity of measuring points influence the force of gravity in a discernible way. Quantities describing position, shape, and structure of geological formations can be interpreted from this local variation of gravity. Measuring gravity has thus become an important method in geological mapping and exploration for mineral resources. The area is limited between longitudes West $1°$ to $11°$, and latitudes North $28°$ to $38°$. The gravity data references used were obtained from the "Bureau Géodésique International"**.** The gravity data used for this study have been obtained from available worldwide databases and from local sources, and have been complemented with new measurements acquired in the High Atlas.

Moroccan Bouguer gravity data come from the Moroccan gravity gridded data sets [10] (figure 3).

Additional gravity measurements have been acquired during 2002 and 2003 using a Scintrex CG 3 micro-gravimeter. All measurements were brought back to the level of reference of the international network of gravimetric standardization of 1971. The theoretical values of gravity were calculated using the gravimetric formula of the geodetic System of reference (I.A.G, 1971). The Moroccan Bouguer anomaly was calculated using a vertical gradient of the gravity of $0.3086\ \mathrm{mgal.m^{-1}}$ and a density of $2.67\ \mathrm{g.cm^{-3}}$ for crustal lithologies. If θ represents the geographical latitude of the station in degrees of a point given to the surface of the Earth, the theoretical value of gravity g_T in this point is provided by the following international gravimetric formula:

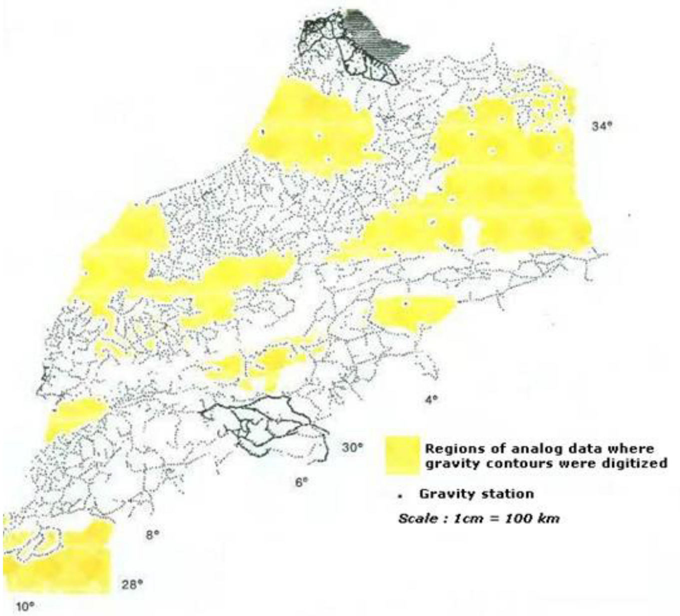

Figure 3. Moroccan gravity gridded data

$$g_T = 978031.85\left(1 + 0.005278895\ \sin^2\theta + 0.000023462\ \sin^4 2\theta\right)$$

Bouguer anomaly (Δg_B) for each station is calculated using the following expression:

$$\Delta g_B = g_{obs} + (0.3086 - 2\pi G\rho)H - g_T$$

where g_{obs} is the observed gravity, H is the orthometric altitude in meters, ρ is the average density of the crust ($2.67\ \mathrm{g.cm^{-3}}$) and G the universal gravitational constant which value is $6.673 \times 10^{-11}\ \mathrm{N.m^2.kg^{-2}}$ [11][12]. We applied this method to the gravimetric map of the area of Morocco. Topography and free air gravity data are taken from the TOPEX gravity data set [13]. A map of the Bouguer anomaly gives a good impression of subsurface density. Low (negative) values of Bouguer anomaly indicate lower density beneath the measurement point. High (positive) values of Bouguer anomaly indicate higher density beneath the measurement point. The Moroccan Bouguer gravity anomaly map obtained after all these corrections (figure 4) was build starting from 5075 data station references and 4661 data measurements which made it possible to calculate a regular square grid of step 450m with about 1 mgal precision [14]. This process gives gravity information better than the pre-existing data sets, where the spacing between samples was around 5 km in the plains and much larger in mountain areas. The Bouguer anomaly reflects the lateral variations of the density of the rocks. Bouguer anomaly maps, which are the most common ones in geological applications, display best the sub-surface density variations of soil and bedrock.

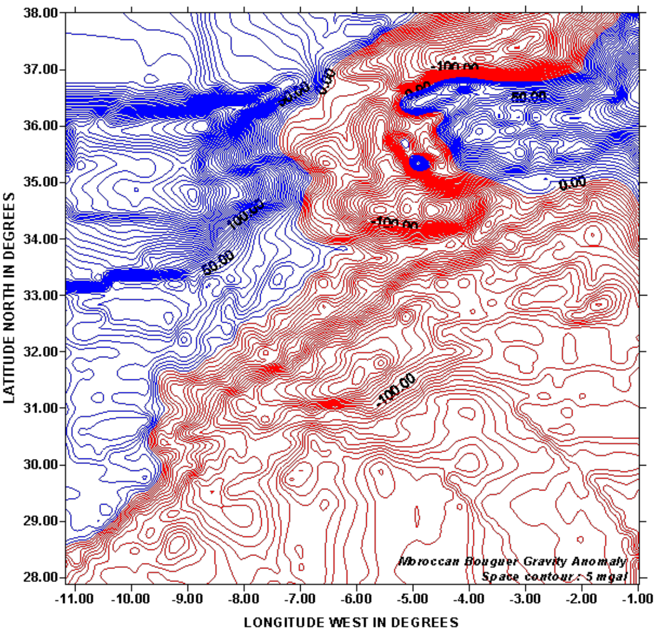

Figure 4. Map of the Moroccan Bouguer Gravity Anomaly Data

Gravity data collected are often contaminated with noise and artifacts coming from various sources. The presence of noise distorts the characteristics of the geophysical signal resulting in poor quality of any subsequent processing. Consequently the first step in any processing of such geophysical data is the "cleaning up" of the noise in a way that preserves the signal sharp variations.

The present paper deals with filtering and smoothing Moroccan Bouguer gravity anomaly map using the AOSS method which is a widely used technique that is applicable to smoothing geophysical data map. All that is needed is a criterion for separating the noise component from the total signal, leaving us with the 'true' or smoothed data. The results show a significant suppression of the noise and a very good recovery of the Bouguer anomaly signal map.

3. THE AUTOMATIC OPTIMAL SPLINE SMOOTHING (AOSS) METHOD FORMULATION

Consider a noisy data $\Omega=\left\{(x_i,y_i)\in R^2 \middle| j=1,...,M\right\}$ be the number of interpolating points and $\Gamma=\left\{(X_j,Y_j)\in R^2 \middle| X_{j-1}\le X_j, j=1,...,N\right\}$ be the set of these points that the sought of cubic spline. To obtain the best set Γ in the least-sqares sense, we must solve the $2N$ -variable problem.

$$\min_{\Gamma}\sum_{i=1}^{M}\left|y_i-s(x_i)\right|^2$$

where s is the cubic spline defined by Γ, $X_L\ge \min_i x_i$ and $X_N\le \max_i x_i$ .To solve this problem, the Gencan optimization method was applied [15] which is an active-set method for smooth box-constrained minimization. The method combines an unconstrained method, including a line search which aims to add many constraints to the working set at a single iteration, with a recently introduced technique (spectral projected gradient) for dropping constraints from the working set. As usual, the optimization process needs an initial approximation. For this purpose, we chose the initial set as composed by N regularly sampled pairs on the originally given set Ω, that is $(X_j,Y_j)=(x_i,y_i)$, with $i=\lfloor 1+(j-1).(M-1)/(N-1)\rfloor$ for $j=1,...,N$ where $\lfloor p\rfloor$ denotes the greater integer less than or equal to p. We have not made any consideration on how to choose the number N of interpolating points. The method is designed to automatically find, in the least-squares sense, the best cubic spline for the specified number N [16]. For a small number of points N, the obtained spline will not be able to represent more than the general trend of the curve [17]. On the other extreme, for large values of N, the spline will tend to fit even the outliers. Since the method is fast, it is reasonable to estimate the cubic spline for several choices of N [18][19]. This flexibility can be very useful to the user or interpreter, in the sense that a number of inexpensive trials can be implemented before a final decision on which level of smoothness is the best choice for the problem. The main advantage of this approach is that any related statistical information that may exist, either a priori from the experimental data recording conditions, or a posteriori from the noise content of the data. is not utilised in the smoothing procedure. It is thus possible to 'oversmooth' or 'undersmooth'. Oversmoothing is almost certain to take place when the signal has a rather high frequency content while undersmoothing occurs in the presence of 'outliers', i.e. points that are far out from the general trend of neighbouring data points [20].

4. APPLICATION

The method is designed to handle datasets composed by samples of rather complicated real functions. It is to be stressed that, by construction, the obtained function is naturally smooth up to second-order derivative. The proposed method is applied to smooth Moroccan Bouguer gravity data anomaly. Well data as gravity data are, in general, very noisy, so it may be desirable to consider the parameters as smooth functions. The AOSS method efficiently handles and removes white noise. The optimization AOSS method represents the gravity data very well. The method is robust enough to provide good results.

5. METHODOLOGY AND PROCEDURE

The AOSS problem is reduced to finding the spline curve to smooth and denoise the Moroccan Bouguer gravity anomaly data map. We calculated the output AOSS filtered signal using an automatic routine [21] for each gravity profile (figure 5 and figure 6). Then we deferred all the results to built a regular map which represent in fact the AOSS model filtering and denoising Moroccan Bouguer gravity anomaly data map (figure 3). The advantage of the AOSS model filter is the ability to preserve higher moments in the gravity data and thus reduce smoothing on peak heights. It is a powerful tool particularly suitable in denoising, filtering and analyzing problems and potential singularities. Moreover this property is crucial for performing the reconstruction of the filtered geophysical signal corresponding to gravity data anomaly map of Moroccan. So the present paper deals with analyzing gravity data map using the Automatic Optimal Spline Smoothing to denoise anomalous zones map of Moroccan Bouguer gravity data anomaly.

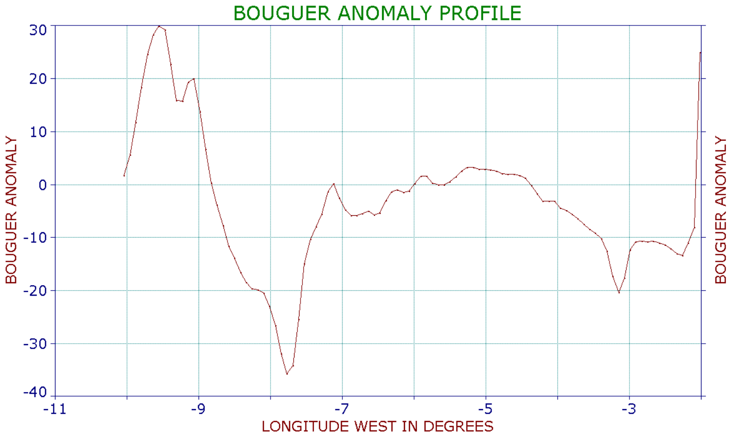

Figure 5. Example of real Bouguer gravity anomaly profile

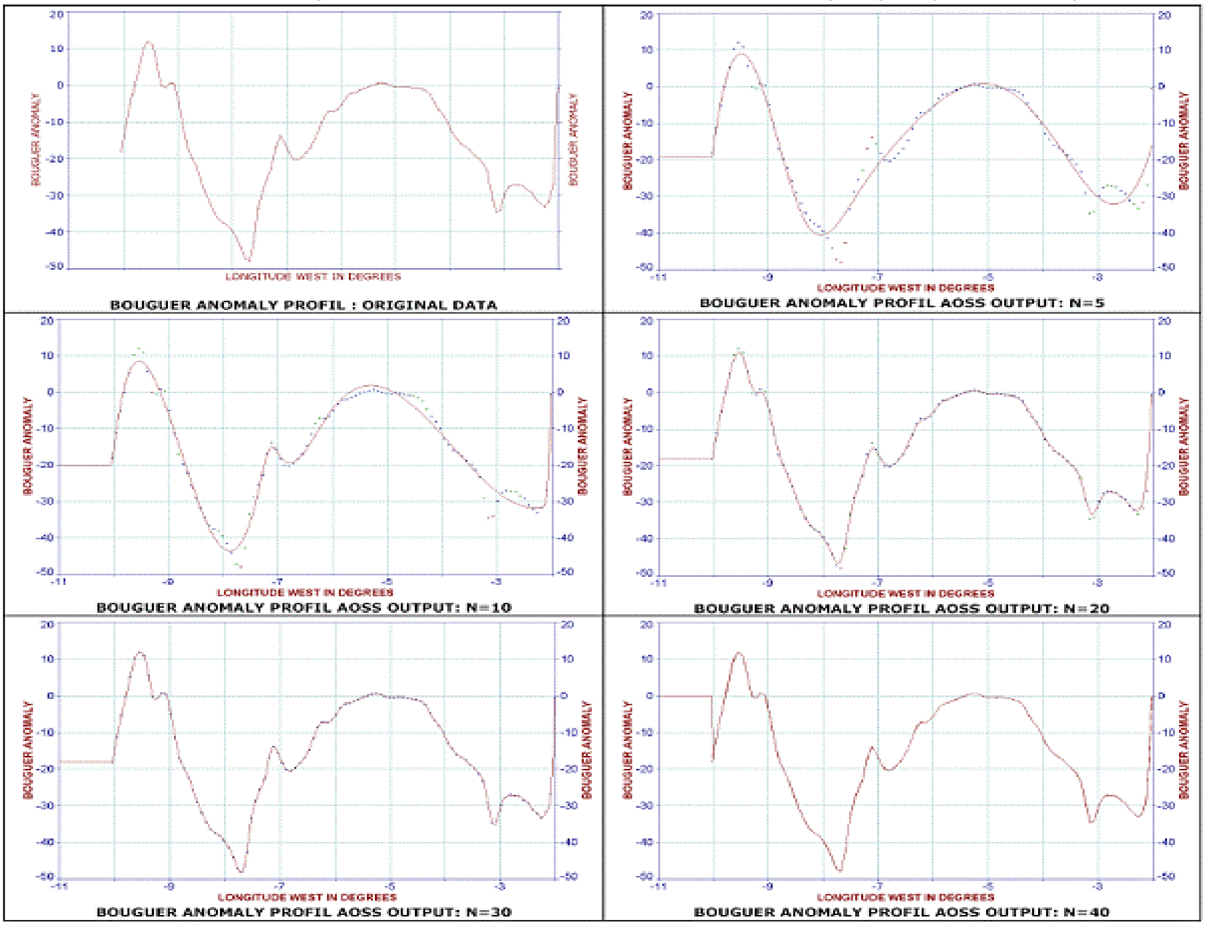

Figure 6. Examples of Bouguer gravity anomaly profile AOSS output

6. RESULTS & CONCLUSIONS

Figure 7 represents an indicator of the "smoothed" level of variation of the contrast of density between the disturbances and the normal phosphate-bearing rock. The AOSS output maps corresponding to Moroccan Bouguer gravity anomaly data were obtained by AutoSignal routine. This singular method enables us to define more efficiently the anomalous and disturbed zones.

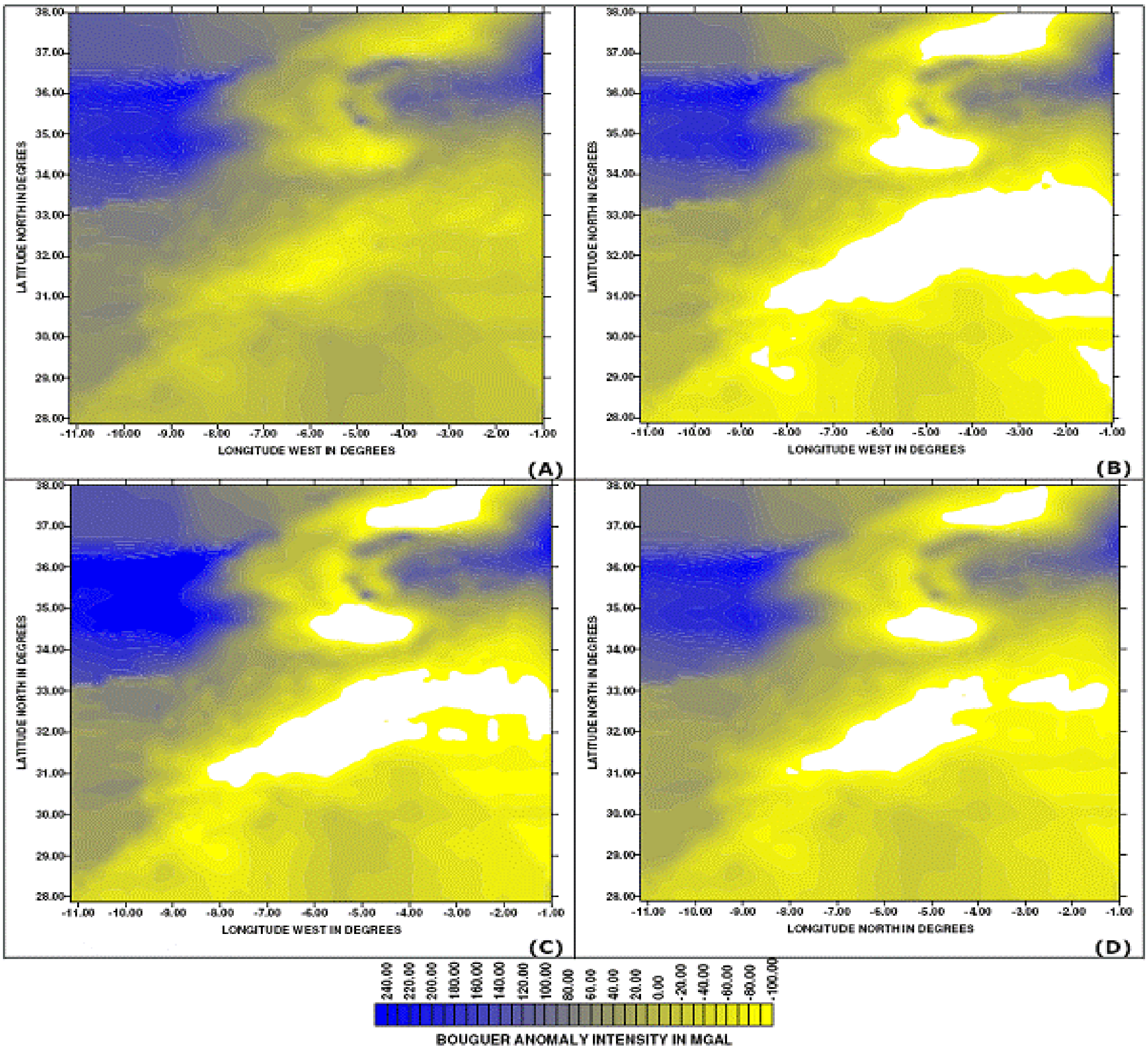

Figure 7. The AOSS output results of the Moroccan Bouguer Gravity Anomaly data map. [A: Original data, B: N=20, C: N=30, D: N=40].

The use of AOSS filtering method represent an effective filtering method which makes it possible to attenuate considerably the noise represented by the minor dispersed and random Bouguer anomaly signal. Comparatively to classical approaches used in filtering and smoothing geophysical data maps, the advantage of the AOSS filtering method result in the fact that it doesn't introduce significant bias to the shape of the original gravity signal. The proposed AOSS method tends to give a real configuration of the distribution of the densities on the surface with a significant suppression of the noise. So, we presented an automatic method for smoothing and outlier suppression of data sets that consist of sampled real function points. After an a priori selection of the number of knots, the procedure automatically finds the location of the interpolating points, in such a way that the resulting smoothing function is optimal in the least-squares sense. As the method is fast, it allows the user to apply the procedure to different numbers of knots, so as to choose the degree of smoothness that best fits the data. A particular feature of the method is its ability to adjust to abrupt discontinuities on the data. The results show a significant suppression of the noise and a very good recovery of the Bouguer gravity anomaly signal. So, the AOSS filtering method is thought to be a good approach to geophysical anomaly filtering. This technique of filtering and denoising gravity anomaly data map was found to be consistently useful and the corresponding maps may be used as auxiliary tools for decision making under field conditions.

References

[1.] Dierckx, P. Curve and Surface Fitting with Splines, New York, NY: Oxford Science Publications, 1993, 285 p.
[2.] Berghaus, D. Numerical methods for experimental mechanics, Ed Lavoisier, 1996, 312 p.
[3.] Koo, J.-Y. Spline estimation of discontinuous regression functions, J. Comp. Graph. Statist. 6, 1996, 266-284.
[4.] Lee, T. C. M. Robust fitting of discontinuous regression functions, Proceedings of the Interface, 31, 1999, 476-481.

[5.] Lee, T. C. M. Regression spline smoothing using the minimum description length principle. Statist. Probab. Letters, 48, 2000, 71-82.

[6.] Lee, T. C. M, Automatic smoothing for discontinuous regression functions, Statistica Sinica, 12, 2002, 823-842.

[7.] Telford, W. M., Geldart L.P. and Sheriff, R. E. Applied geophysics, Cambridge University Press, Cambridge, 1991, 770 p.

[8.] Blakely, R. J. Potential theory in gravity and magnetic applications, Cambridge University Press, 1995, 441 p.

[9.] Griffin, W.P. Residual gravity in theory and practice, Geophysics, 14, 1989, 39-56.

[10.] Bakkali, S., Amrani, M. and L.Bahi. About spatial filtering responses of the Bouguer gravity anomalies map of the North of Morocco, Revista Ingeniería y Universidad, Vol. 11(1), 2007, 7-15.

[11.] LaFehr, T.R. Standardization in gravity reduction: Geophysics, 57, 1991, 1170-1178.

[12.] Nettleton, L.L. Determination of density for reduction of gravimeter observations: Geophysics, 4, 1942, 176-183.

[13.] Smith, W. H. F., Sandwell, D.T. Global seafloor topography from satellite altimetry and ship depth soundings. *Science*, 277, 1997, 195-196. (http://topex.ucsd.edu/marine_topo/mar_topo.html).

[14.] Bakkali, S., Amrani, M. Analysing the enhancement edges of the Bouguer gravity anomaly map using sunshading method (area of the Tangier-Tetuan, Morocco), Revista Facultad de Ingeniería, 39, 2007, 69-78.

[15.] Birgin, E.G., Martinez, J.M. Large scale active set box constrained optimization method with spectral projected gradients, Computational Optimization and Applications, 23, 2002, 101-125.

[16.] Ruppert, D., Carroll, R. Spatially-adaptive penalties for spline fitting, Australian and New Zealand Journal of Statistics, 42, 2000, 205-224.

[17.] Rupert, D. Selecting the Number of Knots for Penalized Splines, Journal of Computational and Graphical Statistics, 11, 2002, 735–757.

[18.] Biloti, R., Santos, L.T., Martin, T. Automatic smoothing by optimal splines, Rev. Bras. Geof, Vol. 21 (2), 2003, 173-177.

[19.] Wand, M. Smoothing and mixed models," Computational Statistics, 18, 2003, 223-250.

[20.] Ngo, L., Wand, M. Smoothing with Mixed Model Software, Journal of Statistical Software, Volume 9, 2004, 1-54.

[21.] Systat , About AutoSignal V1.6 software, Copyrignt 2002 AISN Software Inc, 2002.

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