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THE DYNAMIC MODELING OF A SUBSYSTEM FOR THE UPPER SUPPORTING STRUCTURE OF A BUCKET WHEEL EXCAVATOR

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ABSTACT: This paper presents the problem of forming a dynamic model for supporting the structure of a bucket wheel excavator. Application of the Finite Elements Method arrived at a local linearism of equations regarding a dynamic elastic line for supporting the structure of a bucket wheel excavator under real working conditions. During the formation of this dynamic model, rods of spatial truss as constructive elements were treated as support with continual masses. The case of forming a dynamic model for the subsystem is explained in detail, as well as the phase of choosing the generalized coordinates of the system. Then, the selected nodes of the reference system, the degrees of freedom and the dynamic parameters of a reduced model were determined. Verification of the reduced model was carried out by comparing the results of their frequencies with those obtained by Finite Element, then selecting the reference nodes and degrees of freedom, according to the requirements of the problem posed. This model was designed with twenty degrees of freedom which obtained results which were very similar to the reduced model with two degrees of freedom.
Keywords: Supporting structure, spatial truss, frequencies, and potential energy

1. INTRODUCTION

Over recent decades earthmovers, especially bucket wheel excavators as continuous excavation machines, being the first and main components in bucket wheel excavator-conveyors'-stacker systems influencing other components and the largest structures in earth-based technology, have become progressively larger and their mechanisms more efficient, Bošnjak et al [1].

Dynamic modelling of those bucket wheel excavators which are used during surface mining presents a very complex problem and a considerable number of scientific researchers have investigated this problem but very little research has been done until now on the supporting structure of the bucket wheel excavator. Volkov, Çerkasov [2], discussed a number of problems of the dynamics of bucket wheel excavators in system excavator-belt-folding. These authors also examined the problems of vibration regarding the supporting structures of excavator with unlimited numbers of degrees of freedom. Rubinstein [3], reviewed the form of acquiring dynamic elastic line. Vladimirov [3], based on the results of many experiments, gives analysis of bucked wheel excavator loads that caused during to the mine, for the same parcel of land.

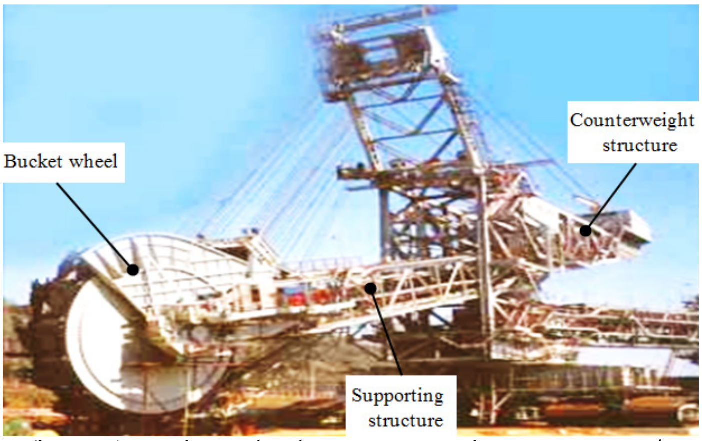

Figure 1. Bucket Wheel Excavator, SchRs 1760 x 32/5

The basic tasks of the upper supporting structure is to accept loads due to the resistance of mining and other weights of supporting structural parts, which are necessary for the work of the excavator and to transmit to the rotary platform, respectively to the driven equipment [4]. The upper supporting structure of the bucket wheel excavator can be realized constructively in different forms but in most cases can be in the form of a spatial truss. Through the spatial truss is a designed bucket wheel excavator from the Krupp manufacturer, type SchRs 1760 × 32/5, Figure 1. The upper supporting structure of the bucked wheel excavator is in the form of a spatial truss type G consisting of a vertical pillar with two spatial trusses related to each other with counter weight structures. The upper supporting structure consists of two upper vertical spatial trusses, upper horizontal spatial truss and lower spatial truss.

MODELING OF SUPPORT STRUCTURE IN SPATIAL TRUSS SHAPE

Modeling of the upper support structure allows the solving of concrete problems regarding vibration of the bucked wheel excavator.

Also, the developed model is of universal character and can be used even in cases of forming the dynamic models of spatial truss supports. Spatial truss Γ of the upper support structure is modeled with Finite Elements of rods type [5], Figure 2. The spatial model shown in Figure 2 consists of 84 nodes and 172 Finite Element type rods. In further consideration of this model, used for determining approximate values, respectively dynamic parameters (coefficients of inertia and rigidity coefficients) of a reduced dynamic model for the upper support structure.

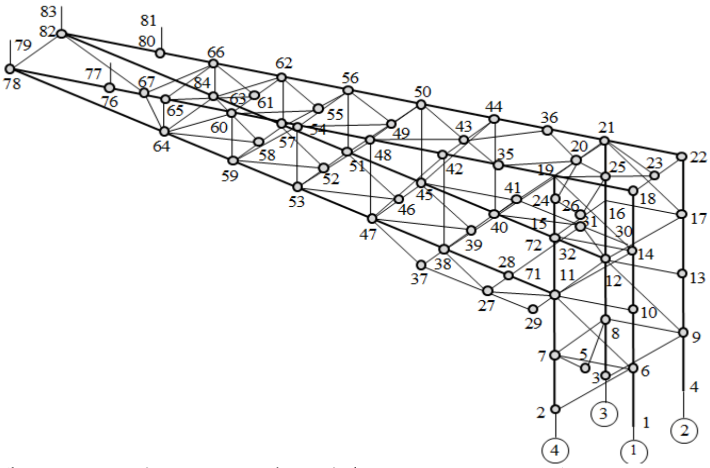

Figure 2. Reference nodes of the upper supporting structure in spatial form

3. MATHEMATIC MODELING OF THE UPPER SUPPORTING STRUCTURE

When choosing generalized coordinates of the upper supporting structure which describe the vibration motions they are conditioned by two requests. The first request consists in its description of the motion, while the second consists of describing the motions of the excavator parts which holds. Generalized coordinates of the upper supporting structure and degrees of freedom are given in Figure 3.

Generalized coordinates $q_1, q_2, \dots, q_{20}$, allow the necessary description and precise engineering calculation, as the motion of the upper supporting structure of the excavator, as well as the motions of all parts of the excavator which holds.

Generalized coordinates $q_{13}, q_{14}, \dots, q_{20}$ are not only determined for describing the vibration of the constructive parts of the upper supporting structure but also for creating concrete pairs of the substructure of bucked wheel arm.

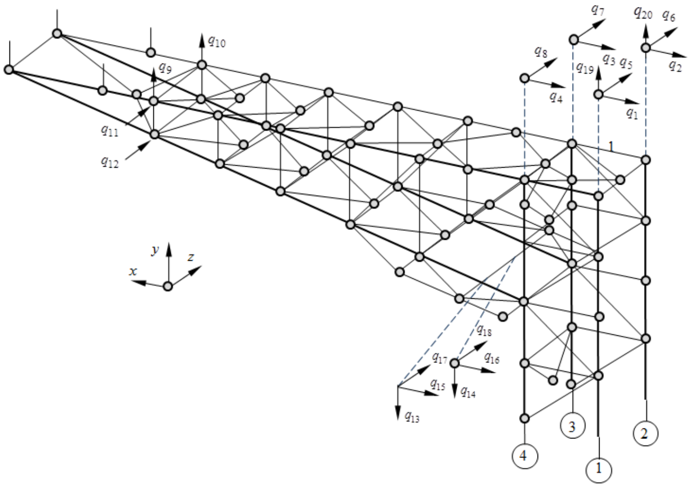

Figure 3. Generalized coordinates of the upper supporting structure in spatial form

The pillar rods of the upper supporting structure are treated as rods with loads distributed in continual form. The nodes of the pillar are loaded with concentrated loads, which were obtained by reducing masses throughout the element.

Dynamic lines of flexibility of the pillar rods are approximated by applying the linear method. However, in the roads segment examined, the elastic lines are approximated in a straight line, which corresponds to the elastic line. The equation of the straight line is determined by the application of the Finite Elements Method. A part of the upper supporting structure which belongs to the pillar is shown in Figure 4.

The first rods of the pillar of the upper supporting structure [6] are defined by nodes 1, 6, 10, 14 and 18, Figure 5. The descriptions of the above nodes are derived at from the model of the upper supporting structure, Figure 3. Node 18 expresses the unit displacement in the direction of the generalized coordinate q_1, respectively $q_1 = 1.0$ [cm], which creates the limiter element which corresponds to the model of the upper supporting structure. Displacement of the reference node 18 causes displacements of other nodes 14, 10 and 6 of the pillar, Figure 5. These displacements are determined by applying the Finite Elements Method. In order to clarify the methods of local linearism, the function of the pillar rod acquired a segment, located between nodes 10 and 14. With displacement of node 18, nodes 10 and 14 go to the positions 10' and 14'. The equation of the straight line which replaced the dynamic lines of flexibility, is determined by the following expression:

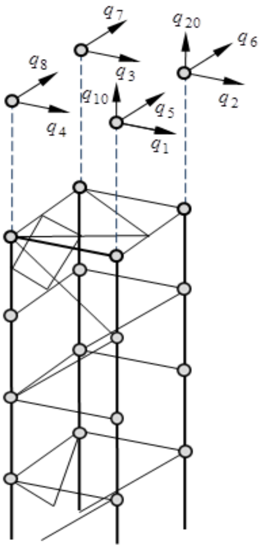

Figure 4. Pillar of the upper supporting structure

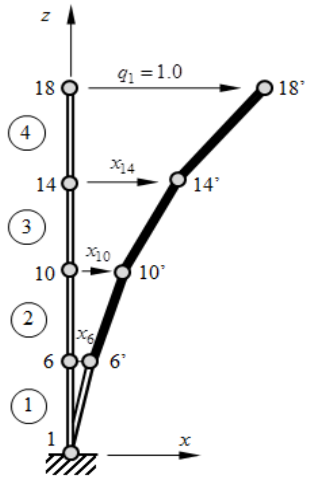

Figure 5. Generalized coordinates q_1

$$\frac{x - x_{10}}{x_{14} - x_{10}} = \frac{z - z_{10}}{z_{14} - z_{10}}. \tag{1}$$

Direction coefficients are finally derived at from the equation of the straight line:

$$x = k_{3x} z + b_{3x}. \tag{2}$$

By applying supporting structure of the excavator, the numerical values of the displacements in the nodes of the first rod were obtained, which are shown in Table 1.

Table 1. Nodes displacement of first rod

Nodes	Transitional, x [cm]	Transitional, y [cm]	Transitional, z [cm]
18	1.00	1.00	1686
14	0.65067	0.68919	1200
10	0.43443	0.32022	712
6	0.19327	0.09441	356
1	0.00	0.00	0.00

Numerical values of the direction coefficient and cross-section in the respective axes, as well as constants of the equation of straight line to all segments of the first rod, being given in Table 2.

Table 2. Numerical values of the direction coefficient and constants of the equation of the straight line

Segments	Direction coefficient		Constants of equation of straight line	
	k_x	k_y	b_x	b_y
1 (1 - 6)	$5.43 \cdot 10^{-4}$	$2.65 \cdot 10^{-4}$	0	0
2 (6 - 10)	$6.74 \cdot 10^{-4}$	$6.34 \cdot 10^{-4}$	-0.048	-0.131
3 (10-14)	$4.43 \cdot 10^{-4}$	$7.56 \cdot 10^{-4}$	0.119	-0.218
4 (14-18)	$7.19 \cdot 10^{-4}$	$6.40 \cdot 10^{-4}$	-0.212	-0.078

Given the numerical values of the directions' coefficients and cross-sections in the respective axes the constants of the straight line equation (Table 2), are calculated for generalized coordinates: $q_1 = 1.0$ [cm], $q_5 = 1.0$ [cm] and $q_{19} = 0$, the final equation of the straight line within the segment of i^{th} of the rod, has the form:

$$x_i = (k_{ix} z + b_{ix})\, q_1, \tag{3}$$

$$y_i = (k_{iy} z + b_{iy})\, q_5, \tag{4}$$

$$z_i = (k_{iz} z + b_{iz})\, q_{19}. \tag{5}$$

where: k_{ix} – direction coefficient, and b_{ix} – constants of equation of straight line (cross section in the respective axes).

By applying the approximate function described previously, the kinetic energy of the rod pillar can be determined. However, reducing the masses of the upper supporting structure that are associated with the rod should be done first. The final dynamic model regarding the rod pillar is shown in Figure 6.

Concentrated masses m_6, m_{10}, m_{14} and m_{18} which are located in the nodes 6, 10, 14 and 18 were obtained by the reductions of the construction elements which are connected with the first rod. Continual masses m_1', m_2', m_3' and m_4', represent the masses on the unit of the pillar's length, within the segment being considered.

Since the generalized velocity is derived at by the time of the generalized displacement, then from expressions (3), (4) and (5), the projections of the velocity of the basic masses of the first segment on the axis *x*, *y*, and *z*, are determined by the following expression:

$$v_{1x} = (k_{1x}z + b_{1x}) \cdot \dot{q}_1\,, \qquad (6)$$

$$v_{1y} = (k_{1y}z + b_{1y}) \cdot \dot{q}_5\,, \qquad (7)$$

$$v_{1z} = (k_{1z}z + b_{1z}) \cdot \dot{q}_{19}\,. \qquad (8)$$

The square of the absolute velocities of the basic masses will be:

$$v_1^2 = v_{1x}^2 + v_{1y}^2 + v_{1z}^2\,. \qquad (9)$$

The kinetic energy of the masses of the first rod of the pillar is now is written as follows:

$$E_k = \frac{1}{2} m \cdot v_1^2 = \frac{1}{2} \begin{bmatrix} (k_{1x}z + b_{1x})^2 \cdot \dot{q}_1^2 \ldots \\ + (k_{1y}z + b_{1y})^2 \cdot \dot{q}_5^2 \ldots \\ + (k_{1z}z + b_{1z})^2 \cdot \dot{q}_{19}^2 \end{bmatrix}. \qquad (10)$$

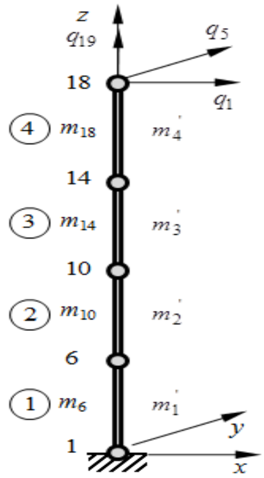

Figure 6. Concentrated and continual masses of the first rod of pillar

The kinetic energy of the upper supporting structure with twenty degrees of freedom is written as follows:

$$E_k = \frac{1}{2}\left(a_{1,1} + a_{2,1} + a_{3,1} + \ldots + a_{20,1}\right) \cdot \dot{q}_1^2 \ldots + \cdots + \frac{1}{2}\left(a_{1,20} + a_{2,20} + a_{3,20} + \ldots + a_{20,20}\right) \cdot \dot{q}_{20}^2. \qquad (11)$$

The potential energy of the upper supporting structure with twenty degrees of freedom is written as follows:

$$E_p = \frac{1}{2}\left(c_{1,1} + c_{2,1} + c_{3,1} + \ldots + c_{20,1}\right) \cdot q_1^2 + \ldots \cdots + \frac{1}{2}\left(c_{1,20} + c_{2,20} + c_{3,20} + \ldots + c_{20,20}\right) \cdot q_{20}^2. \qquad (12)$$

where: a_{ij} – coefficients of inertia (i = 1, 2, ... , 20, j = 1, 2, ..., 20), and c_{ij} – stiffness coefficients (i = 1, 2, ... , 20, *j* = 1, 2, ..., 20).

4. DYNAMIC MODELING OF THE UPPER SUPPORTING STRUCTURE WITH THE FINITE ELEMENTS METHOD

From Lagrange's Differential Equation of the second order:

$$\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{q}_r}\right) + \frac{\partial E_p}{\partial q_r} = 0, \qquad (r = 1, 2, 3, \ldots, 20)\,. \qquad (13)$$

The determinant of the stiffness matrix will be obtained for the system with twenty degrees of freedom of the upper supporting structure using the following expression:

$$\Delta(\omega^2) = \begin{vmatrix} (c_{1,1} - a_{1,1}\omega^2) & (c_{1,2} - a_{1,2}\omega^2) & (c_{1,3} - a_{1,3}\omega^2) & \ldots & (c_{1,20} - a_{1,20}\omega^2) \\ (c_{2,1} - a_{2,1}\omega^2) & (c_{2,2} - a_{2,2}\omega^2) & (c_{2,3} - a_{2,3}\omega^2) & \ldots & (c_{2,20} - a_{2,20}\omega^2) \\ (c_{3,1} - a_{3,1}\omega^2) & (c_{3,2} - a_{3,2}\omega^2) & (c_{3,3} - a_{3,3}\omega^2) & \ldots & (c_{3,20} - a_{3,20}\omega^2) \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ (c_{20,1} - a_{20,1}\omega^2) & (c_{20,2} - a_{20,2}\omega^2) & (c_{20,3} - a_{20,3}\omega^2) & \ldots & (c_{20,20} - a_{20,20}\omega^2) \end{vmatrix} = 0. \qquad (14)$$

Expression (14) represents the frequency equations. When expression (14) takes place the polynomial equation of the twentieth order is determined by the following expression:

$$A_{20}(\omega^2)^{20} + A_{19}(\omega^2)^{19} + A_{18}(\omega^2)^{18} \ldots + \ldots + A_1\omega^2 + A_o = 0. \qquad (15)$$

The polynomial presented by equation (15) has twenty roots of frequency (ω^2) which are given in Table 3. The potential energy in the form of a matrix, is given by the expression, [7]:

$$E_p = \frac{1}{2}\{q_s\}^T \cdot [c] \cdot \{q_s\}; \quad s = 1, 2, \ldots, 20. \qquad (16)$$

The elements of the stiffness matrix in reality are the influence coefficients of the structure displacement. These coefficients are in this case determined by applying the Finite Element Method regarding the previous model of the upper supporting structure to the respective nodes, loaded with unit forces. The differential equations of vibration in matrix form are given in the following form:

$$\begin{bmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,20} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,20} \\ \cdots & \cdots & \cdots & \cdots \\ a_{20,1} & a_{20,2} & \cdots & a_{20,20} \end{bmatrix} \cdot \begin{Bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \cdots \\ \ddot{q}_{20} \end{Bmatrix} \ldots + \begin{bmatrix} c_{1,1} & c_{1,2} & \cdots & c_{1,20} \\ c_{2,1} & c_{2,2} & \cdots & c_{2,20} \\ \cdots & \cdots & \cdots & \cdots \\ c_{20,1} & c_{20,2} & \cdots & c_{20,20} \end{bmatrix} \cdot \begin{Bmatrix} q_1 \\ q_2 \\ \cdots \\ q_{20} \end{Bmatrix} = \{0\}. \tag{17}$$

The particular solution of the system with homogeneous linear equations is presented in the following form:

$$\{q\} = \{A\} \cdot \cos(\omega t - \alpha) \tag{18}$$

From the Lagrange's Equations of the second order, the resulting system of homogeneous equations in the algebraic form is:

$$([c] - \omega^2 [a]) \cdot \{A\} = \{0\}. \tag{19}$$

Equation (19) represents the system of equations regarding the frequencies by differential equations. With its solutions, with concrete values of coefficients of inertia and stiffness of the system, the software package Structural Analysis Program [8], by using input values which are given in Table 1, will be obtained as circular frequencies of system ($\omega_1, \omega_2, \ldots, \omega_{20}$) and vibrational frequencies ($f_1, f_2, \ldots, f_{20}$), as shown in Table 3.

The vibrational frequencies (f_i) *are* determined by the following expression:

$$f_i = \frac{\omega_i}{2\pi} \quad [\mathrm{Hz}]. \tag{20}$$

Table 3. Values of the circular and vibrational frequencies

Degree of freedom	Circular frequencies ω_i [s^{-1}]	Vibration frequency f_i [Hz]	Degree of freedom	Circular frequencies ω_i [s^{-1}]	Vibration frequency f_i [Hz]
1	2.64	0.42	11	194.62	30.98
2	5.84	0.93	12	254.11	40.44
3	11.29	1.80	13	273.38	43.51
4	24.70	3.93	14	319.98	50.93
5	28.07	4.47	15	466.46	74.24
6	63.09	10.04	16	474.06	75.45
7	75.92	12.08	17	474.15	75.46
8	80.76	12.85	18	2022.70	321.92
9	104.35	16.61	19	2894.37	460.65
10	159.29	25.35	20	2906.06	462.51

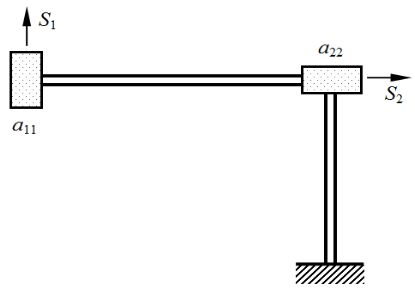

Figure 7. Reduced dynamic model of the upper supporting structure in the vertical plane

5. REDUCED MODEL

Analysis of the model of the upper supporting structure with 20 degrees of freedom, notes that the first frequency of the system in the vertical plane respond to the generalized coordinates q_9 and q_{10}, while the second frequencies correspond to the generalized coordinates q_1 and q_2. Given the fact that the upper supporting structure has a vertical plane of symmetry, it then adopts a reduction of the two generalized coordinates S_1 and S_2 Figure 7.

The kinetic energy of the reduced system of the dynamic model in the matrix form is expressed as follows:

$$E_k = \frac{1}{2} [\dot{S}_1 \ \ \dot{S}_2] \cdot \begin{bmatrix} a_{11} & 0 \\ 0 & a_{22} \end{bmatrix} \cdot \begin{Bmatrix} \dot{S}_1 \\ \dot{S}_2 \end{Bmatrix}. \tag{21}$$

The potential energy from the system shown in Figure 7 is written as follows:

$$E_p = [S_1 \ \ S_2] \cdot \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} \cdot \begin{Bmatrix} S_1 \\ S_2 \end{Bmatrix} = \; = \frac{1}{2} [S_1 \ \ S_2] \cdot \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix}^{-1} \cdot \begin{Bmatrix} S_1 \\ S_2 \end{Bmatrix}. \tag{22}$$

By applying the Lagrange's Differential Equation of the second order, the following expression is obtained:

$$\frac{d}{dt} \frac{\partial E_k}{\partial \dot{S}_i} + \frac{\partial E_p}{\partial S_i} = 0 \; ; \; i = 1, 2. \tag{23}$$

By solving equation (22), in this case the following results are obtained for circular frequencies:

$$\omega_1 = 5.872 \left[s^{-1}\right]; \quad \omega_2 = 28.086 \left[s^{-1}\right] \tag{24}$$

Therefore, given the relationship between the circular and vibrational frequencies, the values of the vibrational frequency of the system with two degrees of freedom in the vertical plane are:

$$f_1 = \frac{\omega_1}{2\pi} = 0.94 \left[Hz\right]; \; f_2 = \frac{\omega_2}{2\pi} = 4.472 \left[Hz\right]. \tag{25}$$

6. DISCUSSION OF RESULTS

In Table 4 are presented comparison between obtained results with twenty and two degrees of freedom as well as difference. Table 4. Comparison between the results of the system with twenty and two degrees of freedom.

Table 4. Comparison between the results of the system with twenty and two degrees of freedom

System with twenty degrees of freedom		System with two degrees of freedom		Difference [%]
Circular Frequ. ω_i [s^{-1}]	Vibration Frequ. f_i [Hz]	Circular Frequ. ω_i [s^{-1}]	Vibration Frequ. f_i [Hz]	
5.84	0.93	5.872	0.94	1.064
28.07	4.47	28.07	4.472	0.045

These results are consistent with the values of the first and second frequencies of the system with vertical plane, which are obtained in the model with 20 degrees of freedom [6].

By comparing the obtained results with the case of twenty degrees of freedom and the case of the reduction model with two degrees of freedom, the results are within an accuracy of 0,045 %, respectively 1,064 %. This high accuracy for the reduced dynamic model enables the calculating of very fast characteristic parameters of the upper supporting structure of the bucket wheel excavator. So, in the reduced dynamic model of the upper supporting structure with two degrees of freedom in the vertical plane, the coefficients of inertia and stiffness which are set above reflect the frequency spectrum of the system in the vertical plane.

7. CONCLUSIONS

The upper supporting structure is an important subsystem of the bucked wheel excavator and as such is treated in this paper. Developing of a dynamic model through the Finite Element Method of the upper supporting structure, by entirely considering the spatial truss, enables analyses of all nodes, elements, rods, and pillars of the structure. Based on the upper supporting structure of the treated model, adequate displacement for loads are gained by those acting on the nodes of the supporting structure.

The displacement values obtained through the Structural Analysis Program, were taken as the basis value for determining of potential and kinetic energy. The equation of motion of the upper supporting structure is determined by Lagrange's Equations of the second order.

The upper supporting structure is treated as a spatial truss with twenty degrees of freedom, with finite elements of rod type, by analysing their frequencies it can be converted into a mathematical model with two degrees of freedom. Based on small errors between two elaborated methods, it can be drawn the conclusion that the developed method with reduction model with two degree of freedom as shown in this paper is useful, simple and there are sufficient precision. This approach enables the calculating of very fast characteristic of supporting structure of the bucket wheel excavator, tower cranes as well as spatial trusses.

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