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APPLICATION OF SIMILITUDE LAWS FOR EXPERIMENTAL INVESTIGATIONS OF DYNAMIC PROPERTIES OF TALL PROTOTYPE STEEL STRUCTURE

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ABSTACT: This paper elaborates the procedures for scaling prototype structure to scale model according to similitude laws keeping the dynamic properties of the original structure. The scaling procedures are very suitable for experiments where the purpose is not only to check the strength of the structure, but also to investigate the performance of other dynamic devices mounted on the primary structure such as dynamic absorbers. In such cases, the use of condensation methods comes to mind in order to reduce the original d.o.f. and make the model structure behave same in the first several modes as the original structure.
Keywords: structural dynamics, similitude laws, prototype, scale model, condensation methods, TMD, transducers, strain gauges

1. INTRODUCTION

Full scale testing of structures is accurate but it is time consuming and costly. Therefore, it is extremely useful if a full scale prototype can be replaced by scaled-down model which is much easier to work with.

One of the fundamental aspects of the vibration analysis is to obtain the natural frequencies of the studied system. This analysis becomes relevant not only for the prediction of displacements, and related deformations and tensions, but also for their control.

Table 1. Dynamic properties of prototype MRF

Slenderness	1 mode [Hz]	2 mode [Hz]	3 mode [Hz]
30500/80730= 1:2.6	0.26	0.75	1.30

The purpose of this paper is to present procedure for scaling real 80 m tall structure prototype, in order to perform scale model analysis of tuned mass damper –device used to suppress vibrations caused by wind load.

The prototype structure used for this study [1] is 20 story, 80 m high MRF. In order to make this structure prone to wind load, it should meet two conditions: first one according to Eurocode 1 [2] is the criteria for slenderness of the structure –width to high ratio at least 1:4, and second condition according to ASCE 7-05 standard [3] is the classification of structure as dynamically sensitive to wind, or “flexible” if $f_0 \leq 1$ Hz.

The MRF is analyzed in N-S direction which is susceptible to wind load. Table 1 gives the basic parameters of the unchanged structure. The structure is calculated with 3 degrees of freedom at each joint, horizontal, vertical and rotational, which summarize 418 degrees in total. This MRF is subject to further modifications and scaling procedures.

2. ADAPTATION OF THE ORIGINAL STRUCTURE

The first change in the original structure is reducing the degrees of freedom. Reduction of d.o.f. means increasing the stiffness of the MRF, and thus increasing the natural frequencies.

Table 2. Dynamic properties of prototype MRF-r1

Slenderness	1 mode [Hz]	2 mode [Hz]	3 mode [Hz]
30500/80730= 1:2.6	0.2879	0.8076	1.3414

First approximation is done by assigning to each joint two degrees of freedom, one horizontal (or shear stiffness), and other rotational [4]. All joints at same floor have same horizontal displacement. Therefore we get total of 140 degrees of freedom. Table 2 shows new values for natural frequencies.

Next step is simplifying the frame to less than 140 degrees by making it behave as shear type multi-story structure. This consideration is true if the structure is loaded in the elastic range and maintains maximal allowed deflection ≤H/200 at the top floor. Also, the mode shapes of the structure are almost identical.

Several expressions [5] have been proposed for calculation of the equivalent shear stiffness for a story in frame. The closest results in comparison to Table 1 are obtained from the method given by Muto (1974) shown in Table 3.

Table 3. Dynamic properties of prototype MRF-r2

Slenderness	1 mode [Hz]	2 mode [Hz]	3 mode [Hz]
30500/80730= 1:2.6	0.2860	0.7943	1.22066

As it can be seen from Table 3, the prototype structure is not completely susceptible to wind load according to codes. The missing criterion is lower or equal slenderness ratio to 1:4. Therefore, the N-S MRF is modified by shrinking the width from 30.5 m to 20.0 m in order to achieve the ratio. The floor mass is maintained, but the inter column beams are changed.

Table 4. Replacement beams in MRF-r3

Old beams	New beams
W21x50	W14x43
W24x62	W18x50
W27x84	W21x68
W30x99	W24x76
W30x108	W24x84

The change of beams is made according the principle: the columns stay same and carry same dead load, new beams are shorter but the stress from the load is same as the old ones. The replacement beams are shown in Table 4. The new beams affect the building by decreasing all natural frequencies, especially the first one down to 0.2265 Hz. The MRF is only considered as shear type. MRF-r3 is final version for further scaling and thus meets frequency criteria for wind susceptible structure shown on Figure 1.

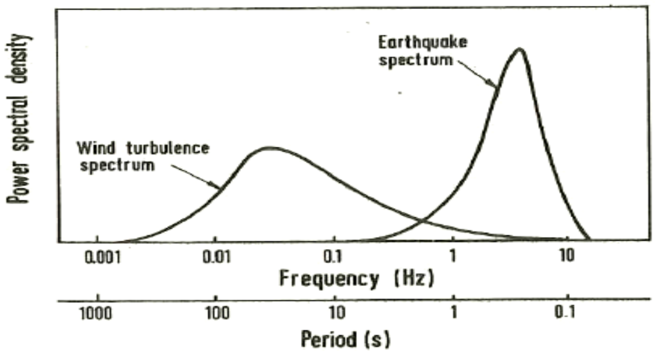

Figure 1. Spectral densities of winds and earthquakes

3. CONDENSATION OF 20 DEGREES TO LOWER 4 EQUIVALENT DEGRESS

Condensation methods [6] are usually applied to d.o.f. where mass or inertia can be neglected i.e. rotational joints. The focus of investigation is not the structure itself from strength point of view, but dynamic behavior and the purpose to serve as base to install and test the capacity of tuned mass damper.

Next simplification is reduction of 20 story masses to 4 reduced equivalent floors and columns at each level. All elements together represent structure with equivalent dynamic behavior from frequency point of view, displacement point of view and mode shape point of view. The model is condensed in a way that every fifth floor is kept and the height is the same as the 20 story frame. The comparative values of the 20 story and 4 story model are given in Table 4.

Table 4. Comparative values of 20 to 4 story building

20 story MRF natural frequencies		
1 mode [Hz]	2 mode [Hz]	3 mode [Hz]
0.2265	0.6340	1.0270
4 story condensed MRF		
1 mode [Hz]	2 mode [Hz]	3 mode [Hz]
0.2444	0.6461	0.8937
% difference in frequencies		
1 mode [Hz]	2 mode [Hz]	3 mode [Hz]
7.9%	1.1%	13%

Another comparison is given with normalized mode shape vectors (Table 5 and Figure 2).

Table 5. Normalized mode vectors with √Mn

20 story MRF			4 story MRF			Error	
mode			mode			[%]	
1st	2nd	3th	1st	2nd	3th		
0	0	0	0	0	0		
0.2117	-0.4777	-0.5338	0.2343	-0.4897	-0.4607	modes	
0.3934	-0.4794	0.1287	0.4326	-0.4692	-0.0051	1	7-10.7
0.5395	0.0721	0.5044	0.5879	0.1217	0.5353	2	2-18
0.6269	0.7476	-0.8269	0.6707	0.8835	-0.8240	3	0.3-13

Parameters of the equivalent mass of floors and stiffness of the columns are given in Table 6.

Based on the formula (1) for lateral stiffness of shear type story:

$$K = N_c \frac{12\,E\,I}{h^3} \tag{1}$$

where N_c is number of columns per story, can be calculated equivalent moment of inertia I_{EK}. Calculated values are also shown in Table 7.

The condensed matrix is not diagonal, but has offset members. This is result of condensation done over members which have significant inertia, which in real case can't be neglected. Consequently the sum of masses in Table 6 is lower than the one of structure with 20 stories. In order to keep the sum consistent, it will be assumed that the difference in mass is equally distributed to the floors. Therefore, we get final structure with natural frequencies lower than one in Table 4, given in Table 7. The error introduced in the modal shape vectors with the last assumption is ≤20%.

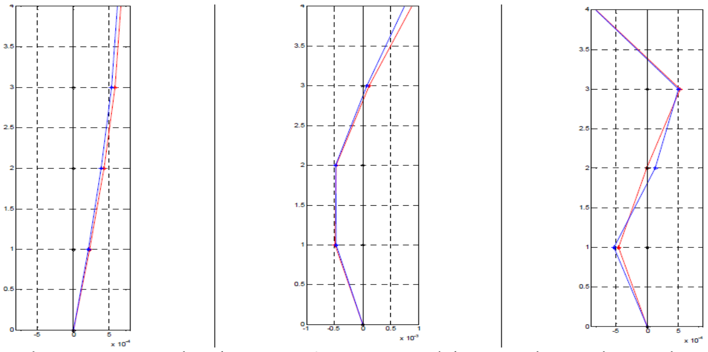

Figure 2. Mode shapes of 20 story-blue and condensed 4 story-red structure

Table 6. Condensed model of 4 story MRF-r4

Story	Mass[kg]*10⁶	Stiffness[N/m]*10⁷	Iek [m⁴]
1	1.2566	1.9871	0.080349
2	1.2621	1.9974	0.064603
3	1.3418	1.7214	0.055676
4	0.5137	0.9818	0.03175

Table 7. Four story condensed MRF-r5

1 mode [Hz]	2 mode [Hz]	3 mode [Hz]
0.2007	0.5026	0.7975

4. SCALING THE CONDENSED MODEL TO RATIO OF 1:80

In structural problems, two types of similitude are most often considered [7]. One of them is Cauchy similitude, based on Cauchy number and expressed as $Cn = \rho v^2/E$, which should be the same on the prototype and the model [8]. This similitude is adequate for cases in which restitution forces are essentially elastic.

The other type of similitude is the Froude similitude, adequate for situations in which gravity action plays a primary role. Similitude relationships are given in Table 8.

Table 8. Similitude relationships

Quantity	Symbol	Cauchy similitude	Froude similitude
Length	L	$L_P = \lambda L_M$	$L_P = \lambda L_M$
Modulus of elasticity	e	$E_P = e E_M$	$E_P = e E_M$
Specific mass	ρ	$\rho_P = \rho \rho_M$	$\rho_P = \rho \rho_M$
Area	A	$A_P = \lambda^2 A_M$	$A_P = \lambda^2 A_M$
Volume	V	$V_P = \lambda^3 V_M$	$V_P = \lambda^3 V_M$
Mass	m	$m_P = \rho \lambda^3 m_M$	$m_P = \rho \lambda^3 m_M$
Velocity	v	$v_P = e^{1/2} \rho^{-1/2} v_M$	$v_P = \lambda^{1/2} v_M$
Acceleration	a	$a_P = e \rho^{-1} \lambda^{-1} a_M$	$a_P = a_M$
Force	F	$F_P = e \lambda^2 F_M$	$F_P = \rho \lambda^3 F_M$
Moment	M	$M_P = e \lambda^3 M_M$	$M_P = \rho \lambda^4 M_M$
Stress	σ	$\sigma_P = e \sigma_M$	$\sigma_P = \lambda \rho \sigma_M$
Strain	ε	$\varepsilon_P = \varepsilon_M$	$\varepsilon_P = \lambda e^{-1} \rho \varepsilon_M$
Time	t	$t_P = \lambda e^{-1/2} \rho^{1/2} t_M$	$t_P = \lambda^{1/2} t_M$
Frequency	f	$f_P = \lambda^{-1} e^{1/2} \rho^{-1/2} f_M$	$f_P = \lambda^{-1/2} f_M$

For scale modeling, there are two categories of modeling: with (a) artificial mass simulation or (b) gravity loads are neglected. The general rule for modeling is given with expression (2):

$$\frac{S_g S_L S_\rho}{S_E} = 1 \quad (2)$$

where g:gravity; L:length; ρ: density; E: stiffness. If we accept that gravity scaling factor is 1, than the expression (2) takes shape:

$$S_\rho = \frac{S_E}{S_L} \quad (3)$$

which means that we cannot scale density and stiffness in the same time. This means we need to add additional artificial masses.

Table 9. Scaling factors

S_L	S_ρ	S_E
80:1	0.75:1	61:1

Therefore, we select first two important features of our scale model: S_L and S_ρ. Then we select the third scale factor – S_E which determines the material for the columns. The scaling factors are given in the Table 9.

5. SELECTION OF MATERIALS AND DEFINING DIMENSIONS

Since the scale factors are defined, according to Froude similitude, next step is to select materials and geometrical properties of the model structure.

Because S_ρ is different from 1, it means that the model will be built from different material of the prototype and additional masses will be added.

The prototype structure is analyzed in wind susceptible direction which is N-S frame only. It means that the model should behave same as the prototype and the weak axis of columns should be normal to the oscillating plane Figure 3. Also, to make columns more susceptible to oscillate in same direction for higher modes, we select high ratio of b:d ≥ 3.

The criteria for final determination of dimensions of columns should be bucking analysis with eccentric load. Final dimensions of columns are given in Table 10. The 4th–level column size is only different from the condition b:d ≥ 3 because of buckling.

Table 10. Column dimensions

Story	1	2	3	4
[mm]	6x25	6x23	6x18	6x12

Material selection for columns is determined by the stiffness scaling factor S_E. In this case, the model should have columns made out of material with approx. 61 time's smaller modulus of elasticity than the steel. One options for large size scaling in experimental techniques is PMMA. The commercial name of this product is Plexiglas.

For this experiment two types of products were tested: extruded and cast PMMA Figure 4. The cast PMMA came to be more appropriate, and less stiff than the extruded one. The columns were made from cast PMMA and cutted with laser.

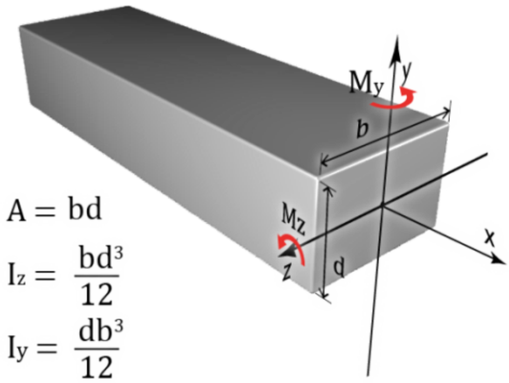

Figure 3. Weak axis of the column Iz

Figure 4. Tensile test and bending test

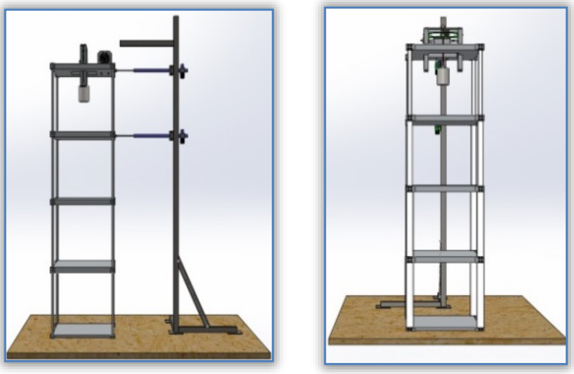

Figure 5. CAD model

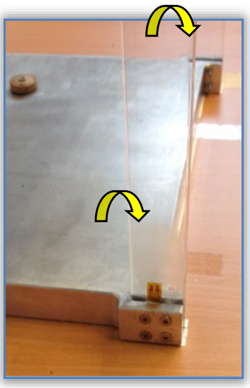

Figure 6. Restrains

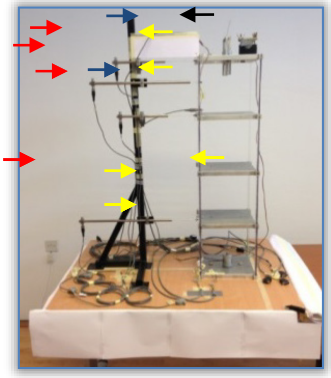

Figure 7. Experimental layout

Specimens were were prepared according to ISO 3167 – multipurpose specimens from polymers, and the modulus of elasticity was confirmed by tensile and bending test [9]. It came to be min. 3000, and average 3250 MPa. Total mass of the model according to scaling density scaling factor S_ρ is calculated to be total of 14 kg. Average floor mass without columns included is calculated to be equal to 3.36 kg. The sum of mass of all plexiglas columns is considered to be equally distributed to each floor of the structure. Review is given in Table 11.

Table 11. Review of masses

Model mass [kg]	Floor mass [kg]	Column mass [kg]
14	3.36	4x0.14

Aluminum alloy plates thick 20 mm are being used as floor weight. The shape of the plates is made with CNC machine according to CAD drawings of the model (Figure 5). The external dimensions from centerline to centerline of columns are maintained in accordance with the slenderness ratio.

The connection of column to floor plate is done in order to achieve shear type of building by restraining the joint from bending. This assumption should work enough accurate for small inter-story drifts (Figure 6).

6. EXPERIMENTAL LAYOUT

The prototype structure is being prepared in order to investigate the capacity of the of vibration absorption device.

For researchers, it's much easier to work around with a small scale model and control the parameters of the experiment than with large size structure.

The main parameters that needs to be measured during the experiment are: Symmetry of the model; Displacements of stories and vibration device; Accelerations of stories; Natural frequencies of the physical model; Phase difference between structure and TMD. In order to achieve the above stated targets, sensors are being used shown on Figure 7.

Inductive transducers W50 HBM are used to quantify the motion of model structure. Acceleration transducers B12 HBM are mounted on top 2 floors because at higher floors acceleration affects the comfort of living.

Strain gauges from Micro-Measurement 350.0±0.2Ω are connected in half bridge to sense only bending of columns at bottom and top connection where strain would be maximal. Also, they serve to give conclusion for the symmetry of the model, and also the range in which the structure behaves as shear type frame.

With different color of arrows, on Figure 7 is shown: red arrows-displacement transducers; blue arrows-acceleration transducers; yellow arrows-bending strain gauges; black arrow-excitation with eccentric mass.

A simple yet effective method of simulating wind induced vibrations in one direction was developed to satisfy the requirements for excitation (Figure 8).

An eccentric rotating mass, provided by a small DC motor with adequate range of rpm's, was mounted rigidly on the top of the 4-th floor plate. By slowly increasing the voltage input, the rotational frequency varied from 0 to 4 Hz.

By observing the displacement of the structure over the duration of frequency sweep, it was possible to identify the first mode of natural frequency.

Kit of 3 eccentric masses was built in order to achieve 3 different displacement values in the elastic range of the structure with amplitude value of 3÷7 mm.

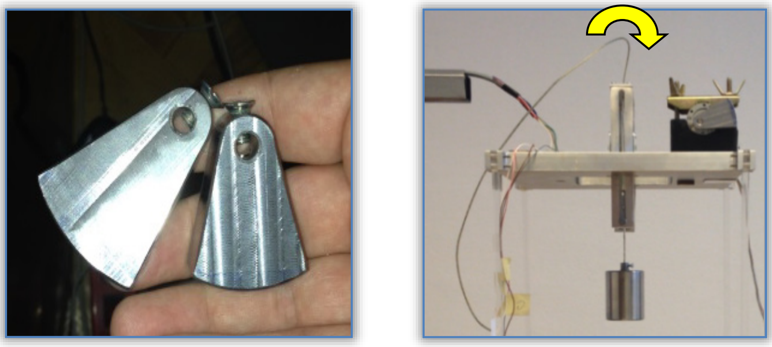

Figure 8. Excitation eccentric mass

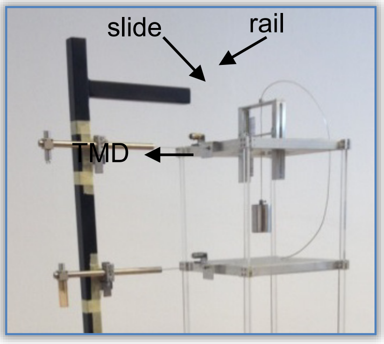

Figure 9. Location of TMD

The device for vibration suppression shown on Figure 9 (whose performance is not presented in this paper) was calculated earlier according to known procedures [10].

7. CONCLUSION

The main objective of conducting experiments on structures at reduced scale is to reduce the cost of experimentation. Cost is reduced due to the reduction in the loading equipment and reduction in the cost of test structure fabrication and testing.

Similitude law is generally applied to define a specimen for scale model tests. A proper similitude law should be selected for satisfying a specific test objective or method. Overall, the method of scaling is valid and justifies further research.

The modeling accuracy depends from the model material properties, fabrication accuracy, loading techniques, measurement methods and interpretation of results. Elastic models can be easily built to give high correlation with the prototype, if the model is fabricated and loaded perfectly.

This paper gives example of enough accurate, relatively easy and low cost approach for creating scaled structure that will be used as base for experimental testing of devices for vibration absorption. It also presents the experimental layout of measuring points, measuring equipment and simple system for external harmonic excitation.

Note:

This paper is based on the paper presented at The 12th International Conference on Accomplishments in Electrical and Mechanical Engineering and Information Technology – DEMI 2015, organized by the

University of Banja Luka, Faculty of Mechanical Engineering and Faculty of Electrical Engineering, in Banja Luka, BOSNIA & HERZEGOVINA (29th – 30th of May, 2015), referred here as[11].

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