ANNALS of Faculty Engineering Hunedoara – International Journal of Engineering
Tome XIV [2016] – Fascicule 2 [May]
ISSN: 1584-2665 [print; online]
ISSN: 1584-2673 [CD-Rom; online]
a free-access multidisciplinary publication of the Faculty of Engineering Hunedoara

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SOME NEW RESULTS ON FRACTIONAL ORDER CONTROL OF A ROBOTIC SYSTEM

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ABSTACT: This paper presents new optimal algorithms of fractional order PD/PID control based genetic algorithms (GA) in the control of a 3 DOF`s robotic system driven by DC motors. The optimal settings for a fractional order (FOPID) controller as well as integer order PID controller (IOPID) are done in two stage procedure, applying GA tuning approach and their extension for FOPID/IOPID controllers in a comparative manner. The effectiveness of the suggested optimal control is demonstrated with a given robotic system as an illustrative example.
Keywords: robotics; fractional order control; PID controller; genetic algorithms

1.INTRODUCTION

It is know that, due to its functional simplicity and performance robustness, PID controllers have been widely used in the process industries, [1]. However, in the recent years, the emergence of effective methods of solving differentiation and integration of non-integer order equations makes fractional-order systems more and more attractive for the control systems community. Fractional calculus (FC) has existed for over three centuries and the fractional integral-differential operators are a generalization of integration and derivation to non-integer order (fractional) operators [2,3]. It is remarkable the increasing number of studies related with the application of fractional controllers in many areas of science and engineering, where specially fractional-order systems are of interest for both modelling and controller design purposes.

Fractional order PID controller $PI^{\beta}D^{\alpha}$ (FOPID) is also the generalization of a standard (integer-order) PID (IOPID) controller, whereas its output is a linear combination of the input and the fractional integer/derivative of the input, where FOPID can improve both the stability and performance robustness of feedback control systems, [3] as well as are less sensitive to changes of parameters of a controlled system. In some of these works, it is verified that the fractional-order controllers can have better disturbance rejection ratios.

There are, today, many different forms of fractional integral operators, ranging from divided-difference types to infinite-sum types, Riemann-Liouville, Grunwald-Letnikov, Caputo's, Weyl's and Erdely-Kober, Jumarie's, etc, definitions of fractional derivative [2,3]. The left Riemann-Liouville (RL) definition of fractional derivative is given by

$$ {}_{a}^{RL}D_t^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dt^n} \int_a^t \frac{f(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau, \tag{1}$$

for $(n-1 \le \alpha < n)$ and for the case of $(\alpha > 0)$,the left fractional integral is defined as

$$ {}_{a}^{C}D_{t}^{\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_{a}^{t} \frac{f(\tau)}{(t-\tau)^{1-\alpha}} d\tau \tag{2} $$

where $\Gamma(.)$ is the well-known Euler's gamma function. Also, there is another definition of left fractional derivative introduced by Caputo as follows:

$$ {}_{a}^{C}D_{t}^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \int_{a}^{t} \frac{f^{(n)}(\tau)}{\left(t-\tau\right)^{\alpha-n+1}} d\tau, \quad n-1<\alpha<n, \tag{3} $$

Caputo and Riemann-Liouville formulation coincide when the initial conditions are zero.

Further research activities run towards defining new effective tuning techniques for non-integer order controllers. There is a need for an effective and efficient global approach to optimize these parameters automatically. An evolutionary computation technique has become gradually popular to obtain global optimal solution in many areas. Several evolutionary optimization algorithms such as the genetic algorithm (GA), differential evolution algorithm (DE), and particle swarm optimization (PSO) have been proposed to optimize the parameters of the several controllers, [4].

Here, we are interested in genetic algorithms (GA) based integer order as well as fractional (non-integer) order PID/PD control of a given RS with DC motors. Genetic algorithms have received much interest in recent years, [4] where the basic operating principles of GA are based on the principles of natural evolution. GA is a stochastic global adaptive search optimization technique based on the mechanisms of natural selection. GA can solve nonlinear multi-objective optimization problems and requires little knowledge of the problem itself and need not require that the search space is differentiable or continuous. GA don't suffer from the basic setback of traditional optimization methods such as getting stuck in local minima.

We propose time-domain criterion which involves integral absolute error (IAE), overshoot, as well as settling time. This will be done through a fitness function to achieve rise in the performance indices.

2. MATHEMATICAL MODEL OF A ROBOTIC SYSTEM WITH DC MOTORS

Robotic system (RS) is considered as an open linkage consisting of $n+1$ rigid bodies $[V_i]$ interconnected by n one-degree-of–freedom joints formed kinematical pairs of the fifth class, (Fig. 1a), where the robotic system possesses n degrees of freedom. Specially, the Rodriguez` method, [5] is proposed for modelling kinematics and dynamics of the robotic system.

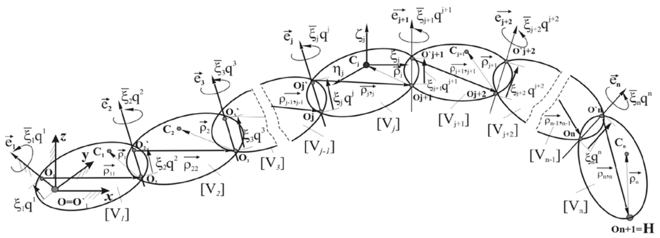

Fig.1a. Open-chain structure of the robotic multi-body system

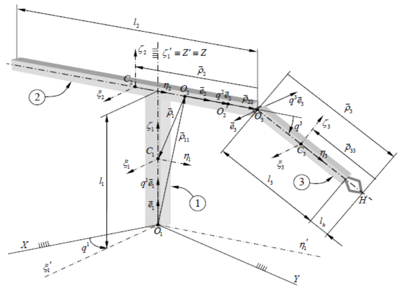

Fig. 1b. Robotic system with 3 DOF's

The configuration of the RS can be defined by the vector of joint generalized coordinates (q) = $(q^1,q^2,\ldots,q^n)^T$. The geometry of the system has been defined by unit vectors $\vec{e}_i$, $i=1,2,\ldots,j,\ldots,n$ where unit vectors $\vec{e}_i$ are describing the axis of rotation (translation) of the i-th segment with respect to the previous segment and as well as vectors $\vec{\rho}_i$ and $\vec{\rho}_{ii}$, usually expressed in local coordinate systems connected with bodies, $\left(\vec{\rho}_i^{(i)}\right),\left(\vec{\rho}_{ii}^{(i)}\right)$. The parameters $\xi_i, \bar{\xi}_i = 1-\xi_i$ denote parameters for recognizing joints $\xi_i, \bar{\xi}_i = 1-\xi_i$, $\xi_i = 1 - prismatic$, $0 - revolute$. For the entire determination of this RS, it is necessary to specify masses m_i and tensors of inertia J_{Ci} expressed in local coordinate systems. In order that the kinematics of the robotic system may be described, points O_i, O_i' are noticed somewhere at the axis of the corresponding joint (i) such that they coincide in the reference configuration. Equations of motion of the RS can be expressed in the identical covariant form as follows

$$ \sum_{\alpha=1}^{n} a_{\alpha i}(q)\ddot{q}^{\alpha} + \sum_{\alpha=1}^{n}\sum_{\beta=1}^{n} \Gamma_{\alpha\beta,i}(q)\dot{q}^{\alpha}\dot{q}^{\beta} = Q_i \qquad i=1,2,\ldots,n. \tag{4} $$

where coefficients $a_{\alpha\beta}$ are covariant coordinates of basic metric tensor $\left[a_{\alpha\beta}\right] \in R^{n\times n}$ and $\Gamma_{\alpha\beta,\gamma}$ $\alpha,\beta,\gamma = 1,2,\ldots,n$ presents Christoffel symbols of first kind and Q_i generalized forces.

Here, it is used RS with 3 DOF's, (Fig. 1b), driven by three DC motors, and taking into account dynamics of DC motors, where inductivity is $L \approx 0$, one may obtain dynamical model of RS in extended state space, [6].

3. FURTHER RESULTS ON FRACTIONAL ORDER CONTROL OF A ROBOTIC SYSTEM

3.1 Optimal conventional and fractional order PID control algorithm based on GA

Unlike conventional PID controller, there is no systematic and rigor design or tuning method existing for $PI^{\beta}D^{\alpha}$ controller and it affords more flexibility in PID controller design due to the selection of five controller parameters, ($K_p, K_d, K_i, \alpha, \beta$), and FOPID is given as follows:

$$u(t) = K_p e(t) + K_d\, {}_0D_t^{\alpha} e(t) + K_i\, {}_0D_t^{-\beta} e(t) \tag{5}$$

There are many different methodologies have been introduced in the literature for the design of FOPID controllers where some of these are based on an extension of the IOPID and other are based on treated FOPID as a multi-objective optimization problem. Here, we propose using genetic algorithms (GA), [4] for determine the optimal parameters FOPID/IOPID controllers. In this study, it is introduced next optimality criterion which involves besides steady state error e, i.e IAE, overshoot P_o, as well as settling time T_s.

$$J = |P_0| + T_s + \int |e|dt \to min \tag{6}$$

All the GA parameters are arranged as follows: population size: N=100; crossover probability: pc=0.75; mutation probability: p_m=pm_0 min (1,l/g), pm_0=0.1 - initial mutation probability, l=25 - generation threshold, g - current number of generation, generation gap gr=0.35. Here, remainder stochastic sampling with replacement as selection method is used. For calculation of fractional order derivative and integral the Crone approximation of second order was used. The idea was to determine the optimal parameters for the conventional PID control algorithm (its gains) first and then to use these optimal parameters (gains) as known parameters for the fractional PID control algorithm in order to determine the optimal exponents of differentiation and integration of fractional order. In our case, each individual vector has the FOPID parameters (five parameters) where for reducing the time of optimization, the ranges of FOPID parameters are selected as

$$K_p \in [10,200],\ K_i \in [0,100],\ K_d \in [10,200],\ \alpha \in (0.2,1],\ \beta \in [0,1].$$

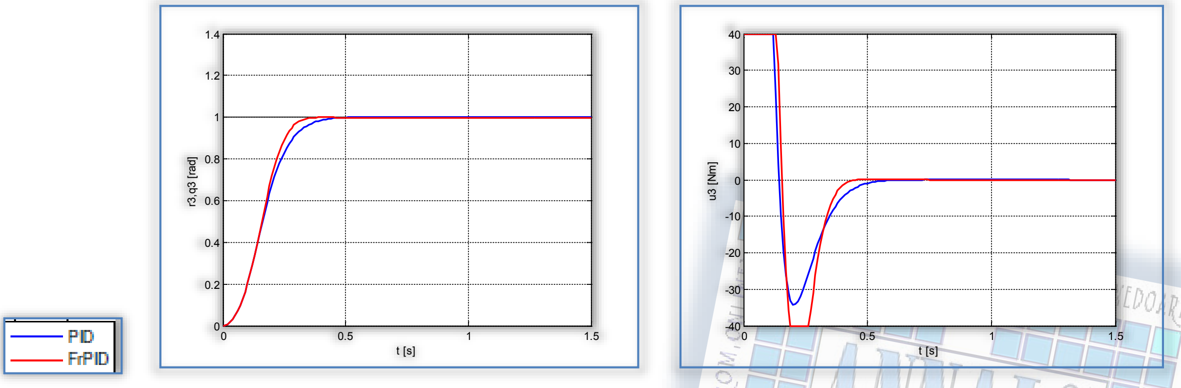

Figure 2a. The step responses of the $q_3(t)\ [rad]$, **Figure 2b**. The control action $u_3(t)\ [Nm]$

In Table 1 they are presented the optimal parameters of the FOPID as well as IOPID controller using GA.

Table 1. The optimal parameters of the FOPID controller and the IOPID controller based on GA

controller		K_p	K_i	K_d	β	α	J_{opt}
PID	1.	199	2	24	-	-	0.98651
	2.	212	2	26	-	-	0.84875
	3.	246	1	28	-	-	0.68718
FOPID	1.	169	0	57	0.074	0.621	0.69887
	2.	254	80	95	0.142	0.656	0.72954
	3.	197	55	78	0.161	0.605	0.56187

Table 2. The values of steady state error e, i.e IAE, overshoot P_o, as well as T_s.

controller		$T_s[s]$	$\Pi[\%]$	$\int\lvert e\rvert dt$	J
PID	1.	0.37	0.3347	0.2818	0.98651
	2.	0.39	0.1770	0.2818	0.84875
	3.	0.38	0.078	0.2292	0.68718
FrPID	1.	0.35	0.0018	0.1753	0.52316
	2.	0.33	0.0030	0.1704	0.50342
	3.	0.31	0.0012	0.1658	0.47705

In simulations they are compared step response,(here only for $q_3(t)[rad]$)of these two optimal FOPID/IOPID controllers, presented in Fig. 2a as well as control action $u_3(t)$, Fig. 2b.

As can be seen from the Fig. 2a and Table 1, 2, better performance for robot control can be achieved using FOPID. From this comparison we conclude that the optimal FOPID controller gives better performance for robot control as compared to optimal IOPID controller method.

ACKNOWLEDGEMENT
This work is partially supported by the Ministry of Science and Environmental Protection of Republic of Serbia as Grant No-35006.

Note: This paper is based on the paper presented at The 12th International Conference on Accomplishments in Electrical and Mechanical Engineering and Information Technology – DEMI 2015, organized by the University of Banja Luka, Faculty of Mechanical Engineering and Faculty of Electrical Engineering, in Banja Luka, BOSNIA & HERZEGOVINA (29th – 30th of May, 2015), referred here as[7].

REFERENCES

[1] K. J. Astrom, T. Hagglund,(1995). PID controller: Theory, design and tuning, Instrument Society of America.
[2] Podlubny, I. (1999). Fractional Differential Equations. Academic Press, San Diego.
[3] Sabatier, J., O. P. Agrawal and J. A. Tenreiro Machado, (2007). Advances in Fractional Calculus, Springer, Netherlands.
[4] Haupt, R. and S. E. Haupt, (2004). Practical Genetic Algorithms, Wiley-IEEE, New York.
[5] Čović, V. Lazarević, M. (2009). Mechanics of Robots, Book. Fac. of Mech. Eng., Univ. of Belgrade, Serbia, (in Serbian).
[6] Batalov, S. (2011). Nonlinear robust control of a multivariable robotic system with the application of Fractional Calculus and GA,MSc work, Fac. of Mech. Eng. Belgrade, (in Serbian).
[7] Mihailo LAZAREVIĆ, Srecko BATALOV, Nikola ĐUROVIĆ, Some new results on fractional order control of a robotic system, Electrical and Mechanical Engineering and Information Technology – DEMI 2015, BOSNIA & HERZEGOVINA

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– International Journal of Engineering

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