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1. Emina PETROVIĆ, 2. Vlastimir NIKOLIĆ, 3. Ivan ĆIRIĆ, 4. Miloš SIMONOVIĆ,  
5. Saša PAVLOVIĆ, 6. Marko MANČIĆ, 7. Boban RAJKOVIĆ

## KINEMATIC MODEL AND CONTROL OF MOBILE ROBOT FOR TRAJECTORY TRACKING

<sup>1-4</sup>University of Niš, Faculty of Mechanical Engineering, Department for Mechatronic and Control Systems, Aleksandra Medvedeva 14 Street, 18000 Niš, SERBIA

<sup>5-6</sup>University of Niš, Faculty of Mechanical Engineering, Department for Energetics and Process Technique, Aleksandra Medvedeva 14 Street, 18000 Niš, SERBIA

<sup>7</sup>Železara Smederevod.o.o., Radinac bb, 11300 Smederevo, SERBIA

**ABSTRACT:** Trajectory tracking has been an extremely active research area in robotics in the past decade. In this paper, a kinematic model of two wheel mobile robot for reference trajectory tracking is analyzed and simulated. For controlling the wheeled mobile robot PID controllers are used. For finding the optimal parameters of PID controllers, in this work particle swarm optimization (PSO) is used. The proposed methodology is shown to be a successful solution for solving the problem.

**Keywords:** trajectory tracking, Particle swarm optimization, kinematic model, mobile robot

### 1. INTRODUCTION

Trajectory tracking is an essential part for modern robots. The primary task for every mobile robot is to move along a defined trajectory. In recent years a large number of researchers are dealing with researches on tracking control of wheeled mobile robot which is a typical nonholonomic system. Kanayama and Yuta proposed a method using straight line reference for the robot's locomotion instead of a sequence of points [1]. The control of a mobile robot with nonholonomic constraints on a reference path is done in [2]. Also many control algorithms were proposed in the path-tracking framework, such as adaptive controllers [3], fuzzy controllers [4], fuzzy neural networks [5], sliding mode control [6], etc. Among all these control methods, PID control is the most used widely, as well as it's in the robot field. In addition, there are control methods combining PID control and some other control methods. A PID controller where the velocity is the control objective for controlling the guidance of mobile robot is designed by Wu at all [7]. Xu et al designed a fuzzy PID controller for trajectory tracking of mobile robot [8].

In this paper, PID controllers are used for controlling the wheeled mobile robot. The control of the robot is solved by considering its first order kinematics model. The consideration of only first order kinematics is very common in theory as well as in practice. This is mainly because, at lower speeds, the system dynamics can be neglected [9].

The Particle Swarm optimization method was used for numerical calculation of optimal PID controller gains which is used to adjust the speed of the wheels of mobile robot.

### 2. KINEMATIC MODEL OF MOBILE ROBOT

The goal of the robot kinematic modelling is to find the speed of the robot in the inertial frame as a function of the wheels speeds and the geometric parameters of the robot (configuration coordinates) [10].

Suppose the posture vector of mobile robot is presented as  $p = [x \ y \ \theta]^T$ , where  $x$  and  $y$  present the position of mobile robot and  $\theta$  is defined as the angle between the X-coordinate and the heading direction(Figure 1). Mobile robot is controlled by the angular velocities of the wheels  $\omega_L, \omega_R$ . Between the angular velocities  $\omega_L, \omega_R$  and circumferential speeds  $V_R, V_L$  there are the relations [11]:

$$V_L = R\omega_L, V_R = R\omega_R \tag{1}$$

where  $R$  is radius of the wheels.

A kinematic model of mobile robot can be based on the following equations:

$$\begin{aligned} \dot{x} &= V\cos(\theta) \\ \dot{y} &= V\sin(\theta) \rightarrow \\ \dot{\theta} &= \omega \end{aligned} \quad V = \frac{V_L + V_R}{2} \tag{2}$$

$$\omega = \frac{V_L - V_R}{D}$$

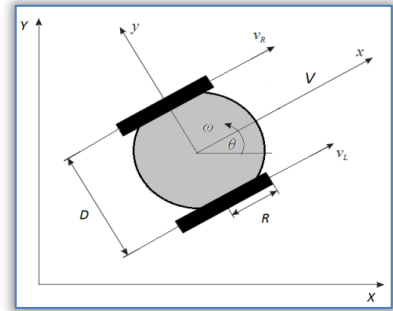


Figure 1. Simulation scheme of mobile robot – kinematics

According to the kinematics, the relationship between the posture vector  $p$  expressed in the X-Y coordinate and the velocities vector has derived as:

$$\dot{p} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \frac{\cos(\theta)}{2} & \frac{\cos(\theta)}{2} \\ \frac{\sin(\theta)}{2} & \frac{\sin(\theta)}{2} \\ \frac{1}{D} & -\frac{1}{D} \end{bmatrix} \begin{bmatrix} V_L \\ V_R \end{bmatrix} \tag{3}$$

In this paper we wanted to determinate the wheel speeds ( $V_L, V_R$ ) for a given desired position of the mobile robot  $p = [x \ y \ \theta]$ .

Simulation scheme of kinematics of mobile robot has shown in Figure2.

### 3. TRACKING TARGET TRAJECTORY BY THE MOBILE ROBOT USING PID CONTROLLER

In this paper, we proposed a control structure to ensure that the mobile robot can track target trajectory. Two PID controllers are used for motion control of mobile robot. The first one of PID controller is used to control the velocity and another for controlling azimuth of the mobile robot.

The problem of controlling, with given a desired reference position, is reduced to get the distance and deviation angle equal to zero, to achieve the objective of position control.

We get error distance from following formula:

$$e = \sqrt{(x_{target} - x_{robot})^2 + (y_{target} - y_{robot})^2} - distance \tag{4}$$

Deviation angle we get from:

$$\varphi = \tan^{-1} \frac{y_{target} - y_{robot}}{x_{target} - x_{robot}} \tag{5}$$

Parameters of PID controllers has optimized by using the Particle swarm optimization algorithm.

### 4. PARTICLE SWARM OPTIMIZATION

Particle swarm optimization is a population based stochastic search algorithm that is the most recent development in the category of combinatorial meta-heuristic optimization. In the basic particle swarm optimization,

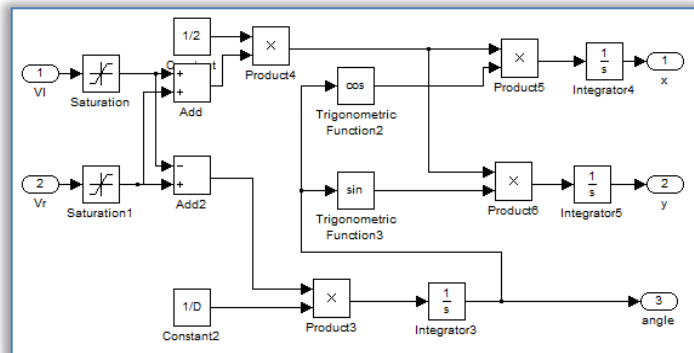


Figure 2. Simulation model of kinematics of the mobile robot

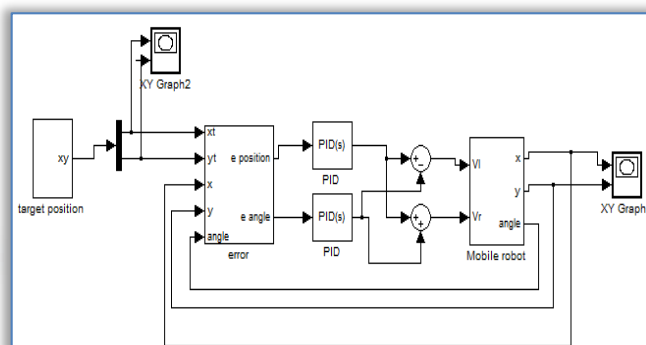


Figure3. Simulation model for PID based mobile robot target tracking control

particle swarm consists of  $n$  particles, and the coordinates of each particle represent a possible solution called particles associated with position and velocity vector in  $D$ -dimensional space. At each iteration particle moves towards an optimum solution, through its current velocity, personal best solution obtained by themselves so far and global best solution obtained by all particles.

The position of the particle of the swarm we represent by a  $D$  - dimensional vector  $\mathbf{x}_i = (x_1, x_2, \dots, x_D)$ . The velocity (position change per generation) of the particle  $\mathbf{x}_i$  can be represented by another  $D$  -dimensional vector  $\mathbf{v}_i = (v_1, v_2, \dots, v_D)$ . The best position previously visited by the the particle is denoted as  $\mathbf{b}_i = (b_1, b_2, \dots, b_D)$ . If the topology is defined in the way that all particles are assumed neighbours and  $g$  as the index of the particle visited the best position in the swarm, then  $\mathbf{p}_g$  becomes the best solution found so far, and the velocity of the particle and its new position will be determined according to the following two equations:

$$\mathbf{v}_i^{(m+1)} = w\mathbf{v}_i^{(m)} + c_1r_1(\mathbf{b}_i^{(m)} - \mathbf{x}_i^{(m)}) + c_2r_2(\mathbf{p}_g^{(m)} - \mathbf{x}_i^{(m)}) \quad (6)$$

$$\mathbf{x}_i^{(m+1)} = \mathbf{x}_i^{(m)} + \mathbf{v}_i^{(m+1)} \quad (7)$$

$r_1$  and  $r_2$  are random variables in the range  $[0,1]$ ;  $c_1$  and  $c_2$  are acceleration coefficients for regulating the relative velocity towards global and local best.

In this paper, for the optimization of parameters of PID controllers, position of the particle is represented by six-dimensional vector  $\mathbf{x}_i = (P_1, I_1, D_1, P_2, I_2, D_2)$ .

The PSO flowchart can be described as following:

- » Generate the initial particles by randomly generating the position and velocity for each particle.
- » Evaluate each particle's fitness.
- » For each particle, if its fitness is smaller than its previous is the best ( $\mathbf{b}_i$ ), update  $\mathbf{b}_i$ .
- » For each particle, if its fitness is smaller than the best one ( $\mathbf{p}_g$ ) of all particles, then update  $\mathbf{p}_g$ .
- » For each particle generate a new particle  $t$  according to the formula (6) and (7).
- » If the stop criterion is satisfied, then stop, else go to Step 3.

## 5. RESULTS AND CONCLUSION

Optimized parameters of PID controller are applied for tracking the two types of target trajectories – trajectory in shape of a circle and a straight line. The resulting tracking of the defined trajectory has shown in Figure 4 and Figure 5.

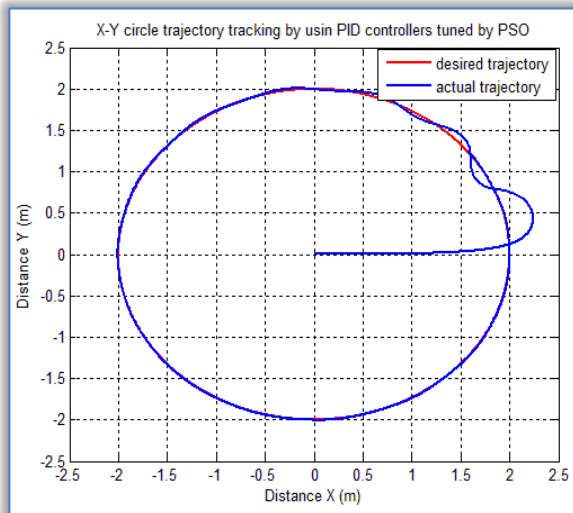


Figure 4. Circle trajectory tracking using PID controllers

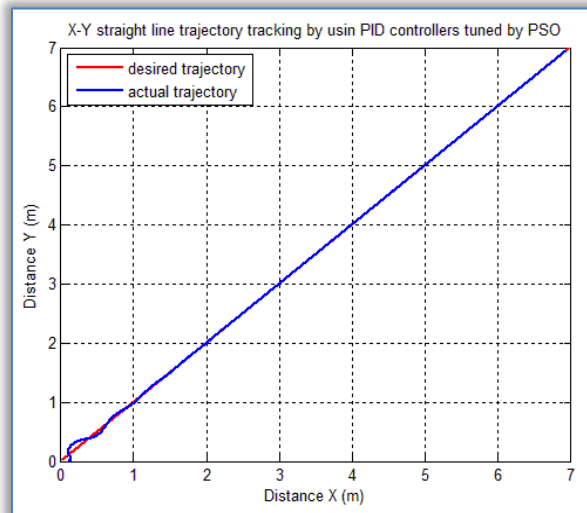


Figure 5. Straight line trajectory tracking using PID controllers

In this paper we have discussed the problem of target trajectory tracking. This problem has been solved by using the well known method of classical theory of control and stochastic search algorithm. The proposed Particle swarm algorithm optimizes parameters of PID controller. The obtained optimized parameters then have been applied on simulation scheme in purpose to track target trajectory. The results of our simulations present that proposed method is suitable to solve problem of target tracking.

**Note:** This paper is based on the paper presented at The 12th International Conference on Accomplishments in Electrical and Mechanical Engineering and Information Technology – DEMI 2015,

organized by the University of Banja Luka, Faculty of Mechanical Engineering and Faculty of Electrical Engineering, in Banja Luka, BOSNIA & HERZEGOVINA (29th – 30th of May, 2015), referred here as [12].

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