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INSTABILITY OF RAYLEIGH-BERNARD CONVECTION AFFECTED BY INCLINED WALL TEMPERATURE VARIATION

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ABSTACT: In this paper the direct numerical simulation of the full Navier-Stokes equation in vorticity-stream function form for the case of an inclined fluid layer with temperature modulation on the upper plate is presented. The fluid flows between parallel plates which are inclined with some angle α in respect to the horizontal plane. The lower side of the viscous fluid layer is constant temperature surface. Since the temperature of the upper plate is higher than the lower plate, Rayleigh-Benard convection appears mostly in the upper zone. We investigate the stability of this convection for water as a working fluid. The results of direct numerical simulation are presented as fields of temperature, vorticty, stream function and velocity. This paper is concerned by the National Program of Energy Efficiency, project number: III42008, funded by the Government of Republic of Serbia.
Keywords: Direct numerical simulation; Navier-Stokes equations; Rayleigh-Bénard convection; Non-linear flow instability; Viscous fluid flow

1. INTRODUCTION

Rayleigh-Bernard (R-B) convection is a classical problem of fluid mechanics, where the viscous fluid is flowing in between two parallel walls, while upper wall is usually cooled and lower is heated. The reason for flow appearance is temperature gradient in vertical direction which causes instability of density distribution in layers of the fluid, and thus movement. Solution to this problem has been described by Rayleigh. It is related for case where fluid is in gravitational field limited at the top and bottom sides, by horizontal walls with constant but respectively different temperatures. As a result, he got critical value of dimensionless parameter at which flow of the fluid starts. This parameter is called Rayleigh number and it is determined as:

$$R_a = \frac{g\beta(T_1 - T_2)H^3}{a\nu}$$

where g is gravitational acceleration, β Thermal expansion coefficient, T_1 upper plate temperature, T_2 lower plate temperature, H distance between two plates, ν kinematic viscosity and a thermal diffusion coefficient. In the above equation, fluidproperties are applicablefor average fluid temperature $T_m=(T_1+T_2)/2$, because this is the best reference temperature. In this case T_1 is the variable in time and in x-direction, and so is the Ra number. Ra is dimensionless parameter and it represents relation between thrust and diffusion forces.

Critical value of Rayleigh number is, according to linear stability theory, $R_{ac}=1708$, for critical wave number $q_c=3.117$. Below this number fluid starts to flow and forms two-dimensional vorticity cells with approximately squared cross section. As Ra rises, cell flow becomes significantly complex. Two-dimensional cells break apart into tree-dimensional cells with hexagonal shape, seen from above. With further increase of Ra cells divides, oscillates and becomes turbulent.

Analysis of R-B convection is applied under the assumption of small temperature differences between the walls, and that the assumption of Oberbeck-Boussinesq approximation applies. This means that all properties of the fluid are constant except density, which is represented as a linear function of temperature:

$$\rho = \rho_2\left[1 - \beta\left(T - T_2\right)\right]$$

where $\rho_2$is fluid density at the lower plate, T is fluid temperature.

In this paper, fluid flow has been analyzed. Rayleigh number is considered to be under critical value, while wave number being very close to critical value. Numeric simulation is obtained using two-dimensional Navier-Stokes equations in the form of vorticity stream-functions of flow, with constant temperature at the lower and spatial modulation of temperature at the upper wall. Inclination is considered to be small.

2. MATHEMATICAL MODEL

Deriving of equations and description of the numerical process of resolving of the equations that describe the non-isothermal flow of viscous, compressible fluid is as follows:

$$\frac{\partial \vec{v}}{\partial t} + \left(\vec{v}\cdot\nabla\right)\vec{v} = \vec{F} + \frac{1}{\rho}\nabla p + \nu\Delta\vec{v} + \nu\nabla\left(\nabla\cdot\vec{v}\right)$$

$$div\left(\rho\vec{v}\right) = \nabla\cdot\left(\rho\vec{v}\right) = \left(\nabla\rho\right)\cdot\vec{v} + \rho\left(\nabla\cdot\vec{v}\right) = 0$$

$$\frac{\partial T}{\partial t} + \left(\vec{v}\cdot\nabla\right)T = \frac{\lambda}{\rho c_p}\Delta T + \frac{\dot{Q}}{\rho c_p} + \frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2$$

The above equations represent the law on maintaining momentum, matter and thermal energy. We want to solve these equations in the form of stream function – vorticity. Vector of the fluid velocity can be represented as a rotor ofthe vector stream function, while the vorticity vector can be represented as a rotor of the velocity vector of the fluid, respectively,

$$\vec{v} = rot\vec{\psi} = \nabla\times\vec{\psi}$$

$$\vec{\omega} = rot\,\vec{v} = \nabla\times\vec{v}$$

If we want to determine the rotor impulse equation, that is, if the momentum vector equation is multiplied with nabla (Hamilton) operator, we obtain the transport equation of vorticity

$$\frac{\partial}{\partial t}\left(\nabla\times\vec{v}\right) + \left(\vec{v}\cdot\nabla\right)\left(\nabla\times\vec{v}\right) = \nabla\times\vec{F} + \frac{1}{\rho}\nabla\times\nabla p + \nu\Delta\left(\nabla\times\vec{v}\right)$$

The member in parentheses represent the definition of a vorticity vector, while another member on the right side is equal to zero, because the gradient rotor of any scalar function is by definition equal to zero.

$$\frac{\partial\vec{\omega}}{\partial t} + \left(\vec{v}\cdot\nabla\right)\vec{\omega} = \nabla\times\vec{F} + \nu\Delta\vec{\omega}$$

In the impulse equation, the force per unit mass for the case of fluid flow between two parallel plates inclined at an angle relative to the horizontal plane can be written as

$$\vec{F} = \vec{g} = g\sin\gamma\vec{i} + g\cos\gamma\vec{j}$$

Taking into account the linear density dependence on temperature only in the member that represents the force in the impulse equation

$$\vec{F} = \frac{\rho}{\rho_0}\vec{g} = \frac{\rho_0\left[1-\beta\left(T-T_0\right)\right]}{\rho_0}\left(g\sin\gamma\vec{i} + g\cos\gamma\vec{j}\right)$$

Oberbeck-Businesc approximation of this equation is obtained.

$$\frac{\partial\vec{\omega}}{\partial t} + \left(\vec{v}\cdot\nabla\right)\vec{\omega} = \nabla\times\left[1-\beta\left(T-T_0\right)\right]\left(g\sin\gamma\vec{i} + g\cos\gamma\vec{j}\right) + \nu\Delta\vec{\omega}$$

For us it is interesting to see how the rotor of the vector force acting on the fluid small parts is determined

$$\nabla \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ F_x & F_y & 0 \end{vmatrix} = \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \right) \vec{k} = \frac{\partial}{\partial x}\left[1-\beta(T-T_0)\right] g \cos\gamma \vec{k} - \frac{\partial}{\partial y}\left[1-\beta(T-T_0)\right] g \sin\gamma \vec{k} =$$

$$= -\beta g \cos\gamma \frac{\partial}{\partial x}(T-T_0)\vec{k} + \beta g \sin\gamma \frac{\partial}{\partial y}(T-T_0)\vec{k}$$

By substituting this last equation for vorticity transport equation, we get the following expression for it

$$\frac{\partial \vec{\omega}}{\partial t} + (\vec{v}\cdot\nabla)\vec{\omega} = \left[\sin\gamma \frac{\partial}{\partial y}(T-T_0) - \cos\gamma \frac{\partial}{\partial x}(T-T_0)\right]\beta g \vec{k} + \nu\Delta\vec{\omega}$$

In this equation, γ represents an angle of inclination of the plate relative to the horizontal plane, measured in a positive direction, i.e. in the direction counter clockwise from the positive part of the x-axis. If both plates in vertical position are at an angle $\gamma=\pi/2$, the previous equation reduces to the form

$$\frac{\partial \vec{\omega}}{\partial t} + (\vec{v}\cdot\nabla)\vec{\omega} = \nu\,\Delta\vec{\omega} + \sin\gamma \frac{\partial}{\partial y}(T-T_0)\beta g \vec{k}$$

If the plates that are parallel to each other are in a horizontal position, the convection-diffusion equation of transport of vorticity becomes

$$\frac{\partial \vec{\omega}}{\partial t} + (\vec{v}\cdot\nabla)\vec{\omega} = \nu\,\Delta\vec{\omega} - \cos\gamma \frac{\partial}{\partial x}(T-T_0)\beta g \vec{k}$$

The system of equations to be solved now becomes

$$\frac{\partial \vec{\omega}}{\partial t} + (\vec{v}\cdot\nabla)\vec{\omega} = \nu\Delta\vec{\omega} + \left[\sin\gamma \frac{\partial}{\partial y}(T-T_0) - \cos\gamma \frac{\partial}{\partial x}(T-T_0)\right]\beta g \vec{k}$$

$$div\vec{v} = \nabla\cdot\vec{v} = 0$$

$$\frac{\partial T}{\partial t} + (\vec{v}\cdot\nabla)T = \alpha\,\Delta T + \dot{S}$$

We cannot use the second equation, or the equation of continuity, i.e. the law of maintenance of mass (substance), in the formulation of stream function - vorticity, because this equation is identically equal to zero, as

$$div\vec{v} = \nabla\cdot\vec{v} = \nabla\cdot rot\vec{\psi} = \nabla\cdot(\nabla\times\vec{\psi}) = 0\,.$$

3. NUMERICAL SIMULATION

Numerical solution is achieved by translation of physical to dimensionless computational domain where domain of the process is changing as well. This form is suitable for use of Fourier-Chebyshev approximation.

Results of the simulation shows that convection appears in the form of usual two-dimensional vorticity cells. For heated fluid where $R_a > R_{ac}$, stable patterns appears in the range of wave numbers closely grouped around critical wave number q_c. Within a boundary of stable range, cell dimensions do not correspond to the critical value of wave number q, but corresponds to the space and time distributions of boundary condition. Stable range is limited by instabilities from the both sides. Those instabilities strive to change the vorticity cells wave number, but not their shape.

Since obtained cells have wave number that does not match the stability range, instability occurs shifting their dimensions towards critical values. As the Ra rises, a vorticity cell becomes unstable and the structure of convection converges by very complex space and time distribution pattern.

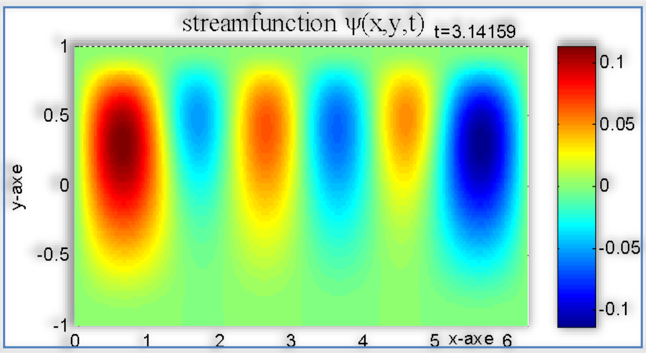

Figure 1. Example of dimensionless stream function field at compulsive R-B convection

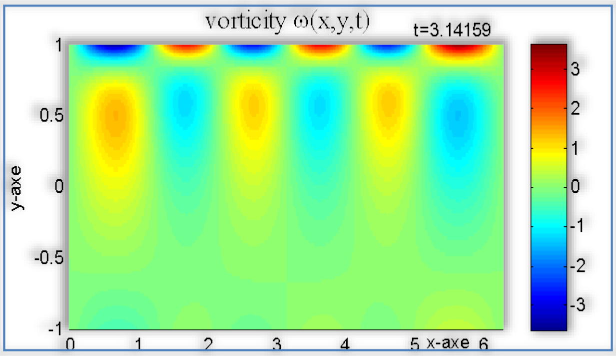

Figure 2. Example of dimensionless vorticity field at compulsive R-B convection

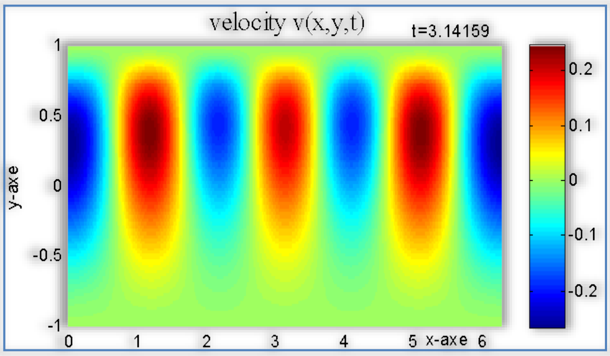

Figure 3. Example of dimensionless velocity field at compulsive R-B convection

4. CONCLUSION

In this paper compulsive Rayleigh-Bernard convection has been examined. Beside temperature gradient, modulation on the upper fluid plate , withamplitude δ_m and two different wave numbers q_{m1} andq_{m2}, has been applied. While at conventional case of Rayleigh-Bernard convection pattern of the vorticity cells becomes unstable or critical value Ra being close to wave number q_c, compulsive convection have wave number value q_{m2} for any Ra. The vorticity cells are affected by different mechanisms of destabilization. In the case of the inclined walls, the destabilization appears almost immediatelly, even for small values of the inclination angle γ.

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REFERENCES

[1] Jovanović, M., Živković, D., Nikodijević, J., (2012). Rayleigh-Benard Convective Instability in the presence of Temperature Variation at the lower wall.Thermalscience, Vol. 16, Suppl. 2, p. S281-S294.
[2] Lappa, M.,(2010).Thermal Convection, Patterns, Evolution and Stability,John Wiley & Sons, Italy.
[3] Jovanović, M.,(2012). Vorticity evolution in perturbed Poiseuille flow.Theoretical and applied mechanics, Vol. 40 (S1), p. 71-86.
[4] Ayed, S. et all, (2013). Experimental study of temperature distribution forTurbulent rayleigh–bénard convection in a rectangular tank. DEMI 2013 Conference Proceedings, p. 665-672.
[5] Versteeg, H. K., Malalasekera, W.,(2005).An introduction to computational fluid dynamics, The finite volume method, Longman Scientific & Technical, England.
[6] Delgado-Buscalioni, R., (2002). Effects of thermal boundary conditions and cavity tilt on suppression of oscillations hydrothermal waves.Physical review E 66, 016301-1–016301-14.
[7] Sharma, A. K., Velusamy, K., Balaji C., (2008). Interaction of turbulent natural convection and surface thermalradiation in inclined square enclosures, International Journal of Heat Mass Transfer, Vol. 44, p. 1153–1170.
[8] Sadoon AYED et al, (2015), Instability of Rayleigh-Bernard convection affected by inclined wall temperature variation, The 12th International Conference on Accomplishments in Electrical and Mechanical Engineering and Information Technology – DEMI 2015, Bosnia & Herzegovina

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