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## INFLUENCE OF SURFACE ROUGHNESS THROUGH A SERIES OF FLOW FACTORS ON THE PERFORMANCE OF A LONGITUDINALLY ROUGH FINITE SLIDER BEARING

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**ABSTRACT:** Efforts have been made to analyse the influence of roughness parameters on the pressure and load carrying capacity in a rough finite plane slider bearing for longitudinally rough surfaces by taking account of the influence of surface roughness through a series of flow factors which has been introduced by Patir (1978). The associated stochastically averaged Reynolds' type equation is solved with appropriate boundary conditions. Expressions are obtained for pressure and load carrying capacity numerically. The results are presented graphically. In addition it is easily seen that the increment in the measure of longitudinal roughness causes the decrease in load carrying capacity of the bearing.

**Keywords:** average Reynolds' equation, finite slider bearing, flow factor, longitudinal roughness

### 1. INTRODUCTION

The study of roughness effect is very important in a bearing system. It is well known that the bearing surfaces after having some run-in and wear develop roughness. There are many studies dealing with their investigations were confined to slider and journal bearings various film shapes by Pinkus (1961) and Hamrock (1994). The effect of surface roughness was discussed by many investigators viz. Davies (1963), Burton (1963), Michell (1950) and Tonder (1972). It has gained an increasing attention after the introduction of stochastic concept and Stochastic Reynolds' equation by Christensen (1969,1972), Christensen and Tonder (1971,1972) governing the mean pressure in bearings having transverse and longitudinal roughness. Christensen and Tonder's approach formed the base of the analysis to study the effect of surface roughness in a number of investigations by Prakash and Tiwari (1982), Guha (1993), Gupta and Deheri (1996), Andharia et al.(1997). The surface roughness effects on the dynamic characteristics of slider bearing with finite width were theoretically studied by Chiang, Hsiu-Lu, et al.(2005) and observed that the steady load-carrying capacity, dynamic stiffness and damping coefficient were increased as the effects of transverse roughness increased while the influences of the isotropic and longitudinal roughness had a reverse tendency. The effect of surface roughness on the performance of hydrodynamic slider bearings was studied by Andharia et al.(2001). The effect of longitudinal roughness on magnetic fluid based squeeze film was studied by Andharia and Deheri (2010). The effect of longitudinal surface roughness on the behaviour of slider bearing with squeeze film formed by a magnetic fluid was analysed by Deheri et al.(2004). The effect of surface roughness on the performance of a magnetic fluid based parallel plate porous slider bearing was observed by Patel and Deheri (2011). Slip velocity and roughness effect on magnetic fluid based infinitely long bearings was analysed by Patel et al. (2014). Patir and Cheng (1978) modified the averaged Reynolds' equation for rough surfaces. They defined pressure and shear flow factors,

which were obtained independently by numerical flow simulation using randomly generated or measured surface roughness profiles.

In this paper, we analysed the influence of surface roughness parameters and flow factor- which is strongly dependent on the surface pattern parameter ( $\gamma$ ) on the longitudinally rough slider bearing.

**2.ANALYSIS**

Patir and Cheng (1978, 1978) developed “Averaged Reynolds’ equation” which took account of the surface topography. The estimation of the average film thickness (Mean gap) was described by,

$$\bar{h}_T = \int_{-h}^{\infty} (h + \delta) f(\delta) d\delta = E(h_T) \quad (1)$$

where,

$$h_T = h + \delta \quad (2)$$

The mean pressure in a rough slider bearing is governed by the averaged Reynolds’ equation (Patir(1978)) is given by,

$$\frac{\partial}{\partial x} \left[ \varphi_x \frac{h^3}{12\mu} \frac{\partial \bar{p}}{\partial x} \right] + \frac{\partial}{\partial y} \left[ \varphi_y \frac{h^3}{12\mu} \frac{\partial \bar{p}}{\partial y} \right] = \frac{U}{2} \frac{\partial \bar{h}_T}{\partial x} + \frac{U\sigma}{2} \frac{\partial \varphi_s}{\partial x} \quad (3)$$

Assuming that the flow of lubricant is steady and in X-direction only and  $U_1=U$ ,  $U_2=0$ . Moreover for longitudinally rough surface ( $\gamma > 1$ , the variations in roughness heights in X-direction is negligible (Figure2), so the effect of  $\varphi_s$  may be treated as negligible (Patir (1978)).

Equation (3) turns out to be,

$$\frac{d}{dx} \left[ \varphi_x \frac{h^3}{12\mu} \frac{d\bar{p}}{dx} \right] = \frac{U}{2} \frac{dh_T}{dx} \quad (4)$$

For a rough plane slider bearing as shown in Figure3, one considers

$$h = h_m + m(l - x) \quad (5)$$

$\delta$  is assumed to be stochastic in nature and is governed by the probability density function  $f(\delta)$ ,  $-c < \delta < c$ , where  $c$  is maximum deviation from the mean film thickness. Then the variance  $\alpha$ , the standard deviation  $\sigma$  and the skewness parameter  $\epsilon$  which is the symmetry of the random variable  $\delta$  are described by Deheri et al.(2004) in terms of the expected values as :

$$E(R) = \int_{-c}^c R f(\delta) d\delta \quad (6)$$

$$E(\delta) = \alpha \quad (7)$$

$$E[(\delta - \alpha)^2] = \sigma^2 \quad (8)$$

and

$$E[(\delta - \alpha)^3] = \epsilon \quad (9)$$

It is to be noted that while  $\alpha$  and  $\epsilon$  can assume both positive and negative values,  $\sigma$  is always positive.

Chiang, Hsiu-Lu, et al. (2005) presented the approximation to  $f(\delta)$  as,

$$f(\delta) = \begin{cases} \frac{32}{35c} \left(1 - \frac{\delta^2}{c^2}\right)^3 & -c \leq \delta \leq c \\ 0 & \text{elsewhere} \end{cases} \quad (10)$$

Thus  $\bar{h}_T$  can be approximated as,

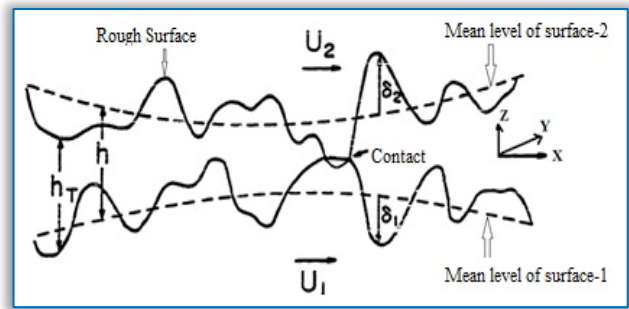


Figure 1. Surface roughness and film geometry

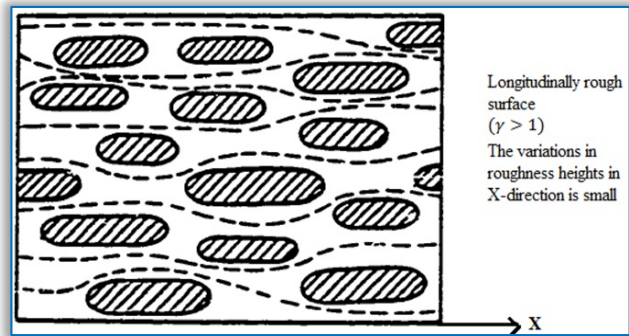


Figure 2. Longitudinally rough surface

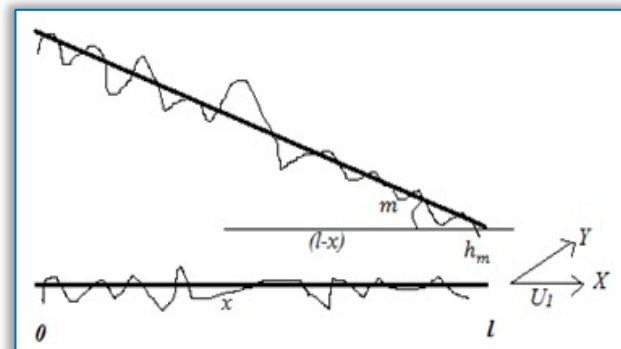


Figure 3. Bearing geometry

$$\bar{h}_T = \frac{13}{8} h \approx h$$

Now as per the average process as discussed by Andharia et al.(2001),equation(4) reduces to,

$$\frac{d}{dx} \left[ \varphi_x \frac{m(h)^{-1} d(\bar{p})}{12\mu dx} \right] = \frac{U}{2} \frac{d}{dx} [n(h)^{-1}] \tag{11}$$

where  $\bar{p}$  is expected value of the mean pressure level  $\bar{p}$  and

$$m(h) = h^{-3} [1 - 3\alpha h^{-1} + 6h^{-2}(\sigma^2 + \alpha^2) - 20h^{-3}(\epsilon + 3\sigma^2\alpha + \alpha^3)] \tag{12}$$

while

$$n(h) = h^{-1} [1 - \alpha h^{-1} + h^{-2}(\sigma^2 + \alpha^2) - h^{-3}(\epsilon + 3\sigma^2\alpha + \alpha^3)] \tag{13}$$

The non-dimensional form of equation(11) is found to be,

$$\frac{d}{dX} \left[ \varphi_X M(h^*)^{-1} \frac{d\bar{P}}{dX} \right] = 6 \frac{d}{dX} [N(h^*)^{-1}] \tag{14}$$

where,

$$h^* = \frac{h}{h_m}, X = \frac{x}{l}, m^* = \frac{ml}{h_m}, \bar{P} = \frac{h_m^2 \bar{p}}{\mu U l} \tag{15}$$

$$M(h^*) = h^{*-3} [1 - 3\alpha^* h^{*-1} + 6h^{*-2}(\sigma^{*2} + \alpha^{*2}) - 20h^{*-3}(\epsilon^* + 3\sigma^{*2}\alpha^* + \alpha^{*3})]$$

and

$$N(h^*) = h^{*-1} [1 - \alpha^* h^{*-1} + h^{*-2}(\sigma^{*2} + \alpha^{*2}) - h^{*-3}(\epsilon^* + 3\sigma^{*2}\alpha^* + \alpha^{*3})]$$

Patir(1978)established the experimental relation for  $\varphi_x$  which is as under,

$$\varphi_x = 1 + C H^{-r} \quad (\text{for } \gamma > 1) \tag{16}$$

$$\varphi_x = 1 + C (h^* H_m)^{-r} \quad (\text{for } \gamma > 1) \tag{17}$$

where

$$H = \frac{h}{\sigma}, H_m = \frac{h_m}{\sigma} \tag{18}$$

and the constants C and r are given as functions of  $\gamma$ (Patir (1978)) in Table-1.

Many of the investigators had observed that if  $H = \frac{h}{\sigma}$  is very large ( $H \gg \gg 6$ ) the smooth film theory is applicable and so the roughness effects

are not that important. If  $H > 3$ , the roughness effect is significant. If  $H < 3$ , the roughness effect increases further, which is called partial lubrication regime due to the presence of rough surface contacts. If  $H < 0.5$ , this assumption may not be justified because a very large portion of the nominal area remains in contact.

Subject to the following boundary conditions:

$$\bar{P} = 0, \text{ at } X = 0 \text{ and } 1 \tag{19}$$

$$\frac{d\bar{P}}{dX} = 0, \text{ at which the mean gap is maximum, say } Q^* \text{ (Constant)}$$

Equation (14) leads to,

$$\bar{P}(X) = \int_0^X \frac{1}{\varphi_X} \frac{1}{M(h^*)^{-1}} [6 N(h^*)^{-1} - Q^*] dX \tag{20}$$

where,

$$Q^* = \int_0^1 \frac{6 M(h^*)}{\varphi_X N(h^*)} dX / \int_0^1 \frac{M(h^*)}{\varphi_X} dX \tag{21}$$

The dimensionless load carrying capacity per unit width is given by,

$$W^* = \frac{w \cdot h_m^2}{\mu U l^2} = \int_0^1 \bar{P} dX \tag{22}$$

### 3. RESULTS AND DISCUSSION

Figure 4–6 dealing with the effect of variance on the load carrying capacity established that the variance (+ve) decreases the load carrying capacity while the variance (-ve) causes increased the load carrying capacity.

It is interesting to see that the load carrying capacity enhances due to an increasing standard deviation (Figure 7-9) which does not happen in the case of transverse roughness pattern.

The trends of load carrying capacity with respect to skewness run almost similar to that of variance (Figure 10-12). However load decreases as  $\gamma$  increases.

**Table-1.** Relation between,C, r and H

$\gamma$	C	r	H
3	0.225	1.5	$H > 0.5$
6	0.520	1.5	$H > 0.5$
9	0.870	1.5	$H > 0.5$

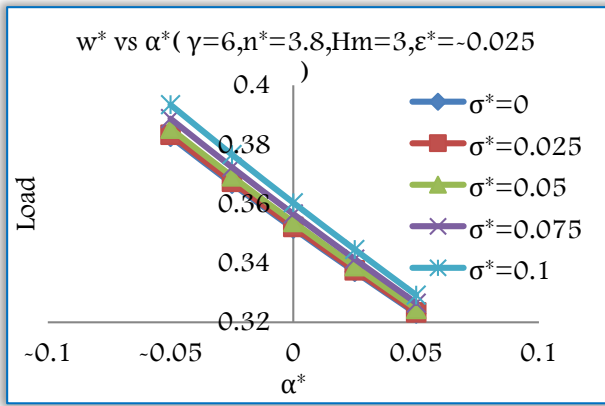


Figure 4. Variation of load carrying capacity with respect to  $\alpha^*$

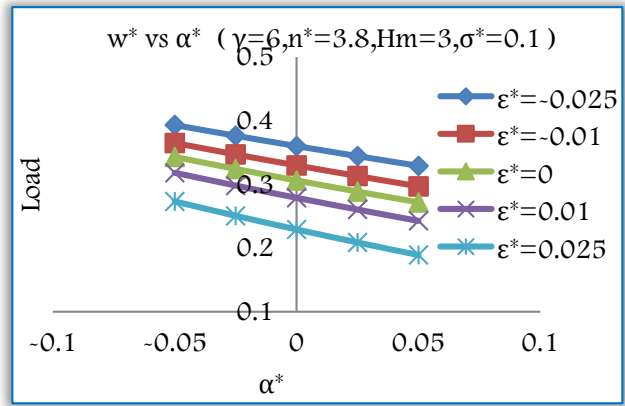


Figure 5. Variation of load carrying capacity with respect to  $\alpha^*$

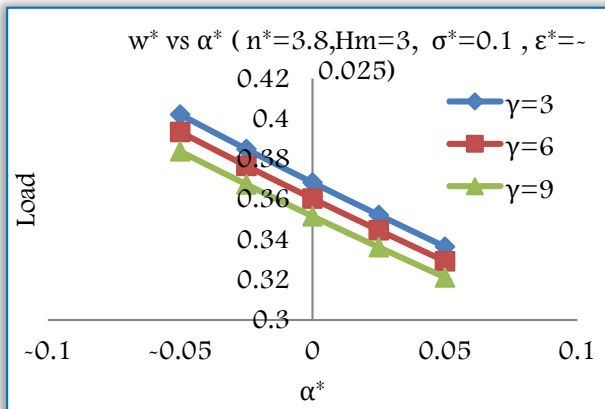


Figure 6. Variation of load carrying capacity with respect to  $\alpha^*$

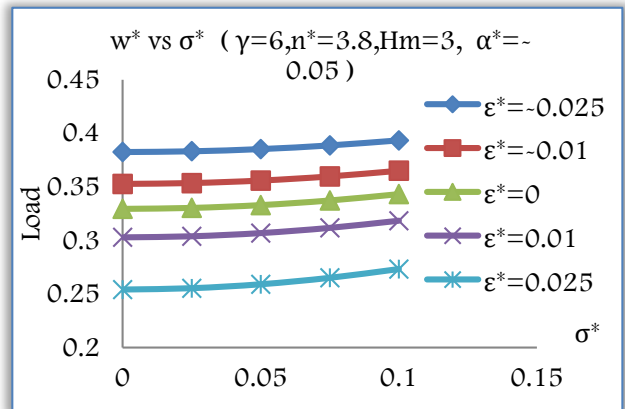


Figure 7. Variation of load carrying capacity with respect to  $\sigma^*$

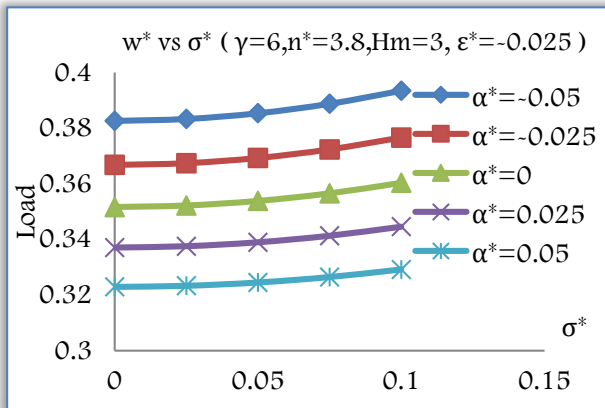


Figure 8. Variation of load carrying capacity with respect to  $\sigma^*$

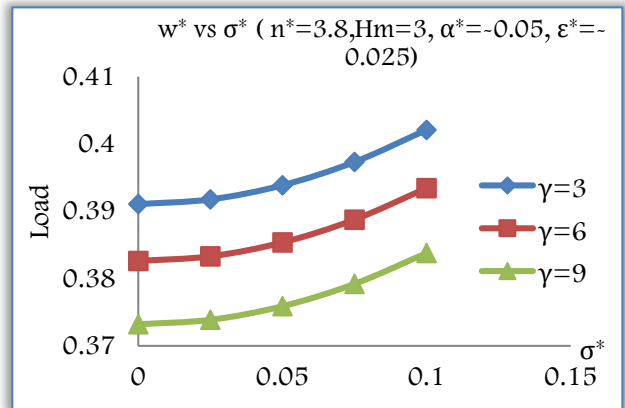


Figure 9. Variation of load carrying capacity with respect to  $\sigma^*$

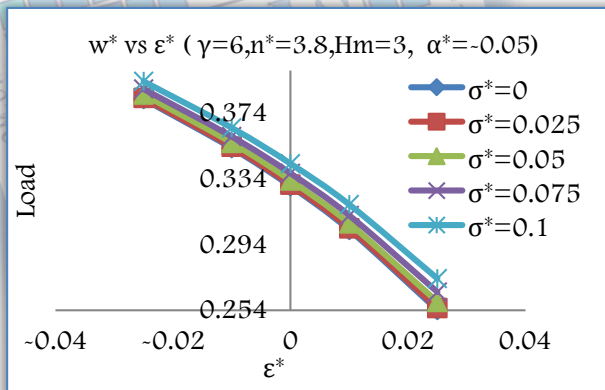


Figure 10. Variation of load carrying capacity with respect to  $\epsilon^*$

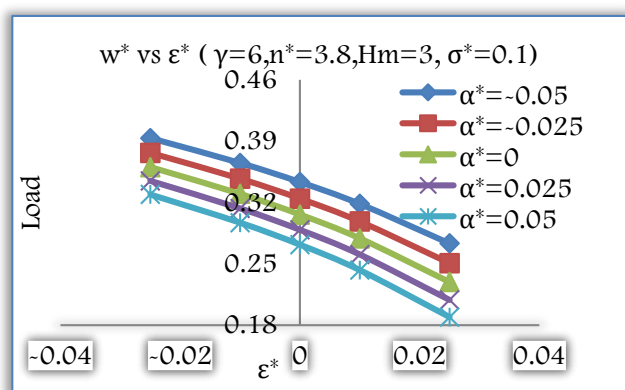


Figure 11. Variation of load carrying capacity with respect to  $\epsilon^*$

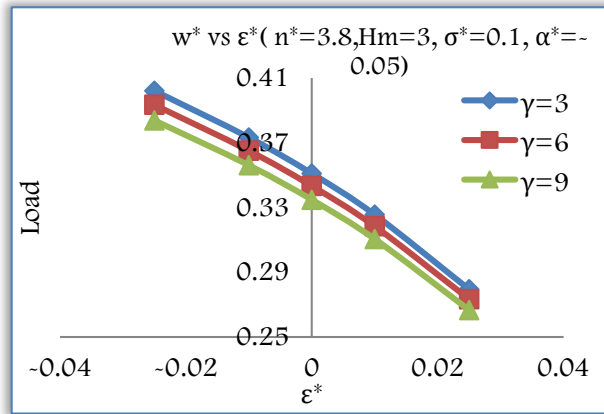


Figure 12. Variation of load carrying capacity with respect to  $\epsilon^*$

#### Nomenclature:

$\bar{h}_T$	Average film thickness (Mean gap) (m)
$f(\delta)$	Frequency density function of combined roughness amplitude $\delta$ ( $m^{-1}$ )
$m$	Inclination of slider bearing
$l$	Length of slider bearing (m)
$w$	Load carrying capacity (N)
$W^*$	Load carrying capacity (Dimensionless)
$h_T$	Local film thickness (m)
$p$	Local pressure ( $N/m^2$ )
$\bar{p}$	Mean pressure level ( $N/m^2$ )
$\bar{P}$	Mean pressure level (Dimensionless)
$h_m$	Minimum film thickness at the trailing edge of slider bearing (m)
$H_m$	Minimum film thickness – Roughness ratio ( $\frac{h_m}{\sigma}$ )
$h$	Nominal film thickness (m)
$H$	Nominal film thickness – Roughness ratio ( $\frac{h}{\sigma}$ )
$U_1, U_2$	Velocities of surfaces in X-Direction (m/s)
$\sigma$	Composite rms roughness given by Gaussian distribution of heights $\sqrt{\sigma_1^2 + \sigma_2^2}$ (m)
$\rho$	Density of lubricant ( $Kg/m^3$ )
$\varphi_x, \varphi_y$	Pressure flow factors
$\delta = \delta_1 + \delta_2$	Random roughness amplitudes of the two surfaces measured from their mean level (m)
$\varphi_s$	Shear flow factor
$\sigma_1, \sigma_2$	Standard deviations of the surfaces (m)
$\mu$	Viscosity of lubricant ( $Kg/m \cdot s$ )

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The positive effect of standard deviation associated with roughness is displayed in Figures 7-9. It is clearly observed that the rate of increase in the load carrying capacity with respect to the standard deviation is more for large positively skewed roughness.

Thus the trio, negatively skewed roughness, standard deviation and variance (-ve) may result in an enhanced performance irrespective of what  $\gamma$  is.

#### 4. CONCLUSION

This article establishes that the roughness must be accorded top priority while designing this type of bearing systems. It is more crucial from bearing's life period point of view.

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