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MATHEMATICAL MODEL FOR OPTIMAL FUEL SUPPLY OF INTERNATIONAL TRANSPORT ACTIVITY

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ABSTRACT: Determination of optimal petrol station selection and the optimal quantity of the amount of refilled fuel during long international transport loops is not defined by a central decision making process at forwarding companies, but it depends on the individual decision of the camion driver. Therefore, the total cost of the burned fuel is not optimal. The aim of this study is to elaborate a method for planning the optimal fuel supply for international transport activities.
Keywords: transport loop, planning, optimal fuel supply

1. STRUCTURE OF INTERNATIONAL TRANSPORT LOOPS

The aims of the transport loop planning are to ensure a more efficient operation and to realize higher profit, which require the integration of more transport tasks into one transport loop as shown in figure 1. It means that the vehicle departs from the parking lot of the company to the first dispatch station where the products to be transported are loaded in. Then the vehicle goes to the first discharge station, where the products to be transported are loaded out and then proceeds to the next dispatch station. The number of dispatch and discharge stations can be i and a discharge station can be a dispatch station simultaneously [1,2]. After the last discharge station the vehicle returns to the parking lot of the company.

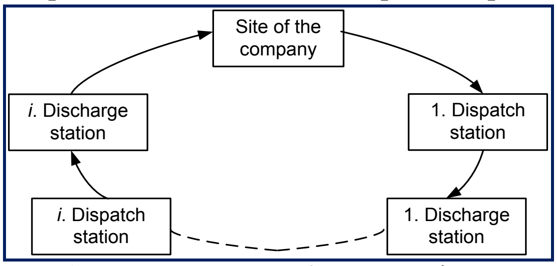

Figure 1. Structure of transport loops

2. CALCULATION OF TOTAL PRIME COST

At first we have to define the total prime cost of a transport loop to find the possibilities of cost reduction. It is suggested to define the sections of the total loop. A section is a way between a dispatch station and a discharge station. The cost components of the sections are different due to the different volume of transported goods, the fuel consumption depends on topography, etc.

Total prime cost of the α–th transport loop ($C_{P\alpha}$)can be calculated [3,4,5]:

$$C_{P\alpha} = C_{L\alpha} + C_{UL\alpha} + C_{WT\alpha} + C_{A\alpha} + C_{W\alpha} + C_{M\alpha} \tag{1}$$

where: $C_{L\alpha}$ - a cost of transport way with useful load; $C_{UL\alpha}$ - cost of transport way without useful load; $C_{WT\alpha}$ - cost of waiting time; $C_{A\alpha}$ - total additional costs (fee of motorway usage, parking fee, …); $C_{W\alpha}$ - wage cost of drivers; $C_{M\alpha}$ - maintenance cost of own vehicles; α - identifier of the loop.

Because the aim of the optimization is the minimization of the cost of fuel consumption, we only examined the $C_{L\alpha}$ and $C_{UL\alpha}$ cost components, the other components can be neglected.

3. DETERMINATION OF IDEAL PETROL STATION AND OPTIMAL AMOUNT OF REFILLED FUEL

Determination of optimal petrol station selection and the optimal amount of refilled fuel during a long transport loop is not defined by a central decision making process of the forwarding companies, it depends on the individual decision of the camion driver. Therefore, the total cost of the burned fuel is not optimal.

The aim of the study is to elaborate a mathematical model that allows the elimination of losts arising from the mentioned problems. Therefore the task is to define the location of the optimal petrol station from the preferred (t_i) stations if it is required in case of a transport task from point F_{i+1} (dispatch station) to point L_{i+1}(discharge station), and to define the volume of fuel to be refilled (Q_T).

All of the petrol stations and dispatch- and discharge stations can be defined by (x,y) coordinates.

At the beginning of the transport task the starting point is known (which could be the site of the transport company (site/parking lot) or the i-th discharge station (L_i)), the dispatch station (F_{i+1}) and the destination discharge station (L_{i+1}) (Figure 2).

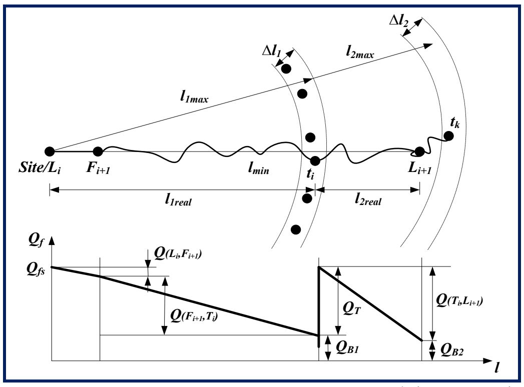

Figure 2. Determination of possible petrol stations, monitoring of the changes in the fuel level of the camion during the transport way

The task is to determine whether the vehicle is able to complete the transport task with the available amount of fuel (level of fuel is known at starting position, Q_{fs}) or the vehicle have to refill fuel between the dispatch- and discharge stations. If refilling is required, the ideal fuel station (t_i) and the amount of fuel to be refilled should be defined (Q_T).

Most of the transport companies – due to the huge amount of fuel consumption – can buy fuel on a discounted price at a contracted fuel selling company. Transport companies prefer those fuel suppliers, which have a worldwide patrol station network.

The contracted fuel selling company provides the GPS coordinates of the available petrol stations and the actual fuel price at the individual stations. Accordingly the x,y coordinates of the petrol stations can always be defined. Digital maps convolute the road network to short, strait sections [7]. This allows the determination of the length of the shortest way between any two chosen points (between dispatch/discharge station and petrol station, between the dispatch and the discharge station, etc.). This can be achieved by the use of an A* algorithm [6], or with the aid of roadmap sheets (data sheets) [8]. Furthermore topological information is also provided, which is also valuable since fuel consumption is different on flat roads than uphill or downhill conditions.

In the following sections thee possible petrol stations will be chosen from every possible petrol stations, which can contribute to the resolution of the task. This significantly decreases the number of variables in the final model.

The total cost of a transport loop is the sum of costs of sections:

$$C_\alpha = \sum_\beta s_{\alpha\beta} \cdot c_{\alpha\beta} \,[\mathrm{euro}] \tag{2}$$

where: $l_{\alpha\beta}$ - the distance of β -th section of α - th transport loop [km]; $c_{\alpha\beta}$ - specific cost of the β -th section of α - th transport loop $\left[\frac{euro}{km}\right]$; β - section identifier, α - transport loop identifier.

The fuel consumption of a vehicle depends on the consumption of vehicle without useful load (characteristics of the engine), the weight of the transported useful load and the topography. The specific cost can be calculated by equation 2.

$$c_{\alpha\beta} = p_{\alpha\beta}\left(f_f + f_f \cdot \varepsilon^T_{\alpha\beta} + \varepsilon^L \cdot q_{\alpha\beta}\right)\left[\frac{\mathrm{euro}}{\mathrm{km}}\right] \tag{3}$$

where: $p_{\alpha\beta}$ - price of fuel $\left[\frac{\mathrm{euro}}{\mathrm{liter}}\right]$; f_f - specific fuel consumption in case of empty vehicle $\left[\frac{\mathrm{liter}}{\mathrm{km}}\right]$; $\varepsilon^T_{\alpha\beta}$ - correction factor for fuel consumption depending on topography, varies between 0 (flatland), 0.3 (downy), 0.6 (mountain); ε^L - correction factor for different loading conditions (every additional tons of useful load results 0.5 liter extra fuel consumption) $\left[\frac{liter}{ton \cdot km}\right]$; $q_{\alpha\beta}$ - transported useful load [ton].

The fuel level of the vehicle is known at starting point(Q_{fs}), the fuel consumption can be calculated continuously between dispatch- and discharge stations (Figure 2).

There is a constraints relating to the emergency amount of fuel capacity (Q_{B1}, Q_{B2}) which has to be available. Q_{B1} emergency amount covers the extra consumption resulting from diversion, traffic jam, missed way etc. between points L_i and t_i, Q_{B2} emergency amount covers the consumption of the vehicle after the L_{i+1}-th discharge station to find the next petrol station.

The maximal distance (l_{1max}), which can be completed by a vehicle, is calculated as follows (Fig. 2):

$$s_{1max}^{\alpha\beta} = \frac{Q_{fs} - Q_{B1}}{f_f + f_f \cdot \varepsilon_{\alpha\beta}^{T} + \varepsilon^{L} \cdot q_{\alpha\beta}} \tag{4}$$

Distance (l_{1max}-$\Delta 1_1$) defines the coordinates of possible t_i petrol stations. The ideal station should be chosen from these stations. The value of $\Delta 1_1$ can be defined, which means the distance of beginning of search of petrol station before the exhaustion of fuel (searching zone).

4. NOTATION OF THE KNOWN DATA

A specific road plan includes some stations that must be crossed to dispatch and/or discharge the load. Their order is predetermined and used for the identification of each dispatch and discharge station. N notes the number of dispatch and discharge stations including the departure and final destination stations as well (which can be the same in case of transport loops, but in this case they are noted with different indexes)[9]. Each dispatch and discharge station is noted by a natural number between 1 and N. From this point on the dispatch and discharge stations will not be differentiated (both of them are called interchanges). Dispatch and discharge stations will differ in the mass of the load during the road-section departing from the given interchange (0 in case of discharge, and higher than 0 in case of dispatch).

Several definitions are:

- ✓ **Road-section**: the set of roads between the given dispatch or discharge station and the next interchange according to the planned transport route is called a road-section. Usually there are more alternative directions to the next interchange, which will be defined at the petrol stations in this model. Road-sections will be noted by i indexes. Therefore, the road-section β of loop α can be easily identified by the interchanges of the loop and the identities of the assigned road-sections (road-sections will be noted by k index). This identification is more appropriate in case of computerized solutions.
- ✓ **Set of fuel stations, probable road-section**: T_i stands for the set of fuel stations between departure station i and destination station $i + 1$. In this case $\tau_i = |T_i|$ notes the number of petrol stations on road-section i. A specific road of the road-section belonging to the fuel stations is called probable road-section. Probable road-sections are noted by i, k indexes. A probable road-section must include only one petrol station. If there is more than one, the model can be changed to a new model in which there is only one petrol station on each road-section (see also Remark 4).
- ✓ **Notation of petrol stations**: $t_{ik} \in T_i$ notes petrol station k (petrol station that belongs to probable road-section k) on road-section i.
- ✓ **Maximum number of petrol stations on a given road-section**: M ($M = \max_i \tau_i$) notes the maximum number of the petrol stations on a road-section. If road-section i includes fewer than M petrol stations, then fictive probable road-sections are to be defined between $\tau_i + 1$ and M. The length of the probable road-section should be high enough to ensure that it will not be chosen during the optimization process. Road 0 (with 0 index) is a road-section without petrol refill, which means the vehicle does not stop on this probable road-section.
- ✓ **Topography.** There will be two topography-related variables assigned to the probable road-sections in this model: the first one is the topography factor assigned to the section from the departure station to the petrol station; the second one is assigned to the section from the petrol station to the destination.
  - ≡ ε_{ik}^{TL} notes the topography factor in relation $i, i + 1$ of the section to the petrol station k.
  - ≡ ε_{ik}^{TM} notes the topography factor in relation $i, i + 1$ of the section from petrol station k to the destination.
- ✓ **Correction factor for different loading conditions:** ε^{L} notes the correction factor for different loading conditions (See also (3)).
- ✓ **Unit price matrix.** p_{ik} notes the unit price of the fuel at the petrol station t_{ik} ($P = [p_{ik}]_{N\times M+1}$). $p_{i0} = 0$ ($i = 1; \dots; N$).

- ✓ **Distance matrices** – similarly to topography factors – are divided into two parts: the first one is assigned to the length of the section from the departure station to the petrol station, and the second one is assigned to the length of the section from the petrol station to the destination.
  - ≡ l_{ik} notes the length of the road-section from the departure station i (with destination station $i+1$) to the petrol station k ($L = [l_{ik}]_{N\times M+1}$).
  - ≡ m_{ik} notes the length of the road-section between departure station ito destination stationi $+ 1$ from the petrol station k to the destination station ($M = [m_{ik}]_{N\times M+1}$).
  - ≡ The definition of the total length of road-section i is also necessary: $s_{ik} = l_{ik} + m_{ik}$ ($S = [s_{ik}]_{N\times M+1}$).
- ✓ **Vector of the amount of load.** q_i notes the amount of load in the relation $i,i+1$. It is 0 in case of zero load. This way the loaded and idle transfer loop can be handled uniformly. ($q = [q_i]_{N-1}$)
- ✓ **Specific fuel consumption.** f_f notes specific fuel consumption ($\left[\frac{\text{liter}}{\text{km}}\right]$) see also at (3).
- ✓ **Capacity of fuel tank.** Q_{max} notes the maximum capacity of the fuel tank.

5. REMARKS & STATEMENT

1. In case of L,M, S, P matrices, due to computational reasons, indexes starts from 0 and ends at M.
2. Because of practical reasons in case of road 0 the total length assigned to l is m = 0 ($l_{i0} = s_{i0}$, $m_{i0} = 0$).
3. When it is possible to refill fuel on the highway as well, which is road 0, then this road must be defined again, but in this case with l and m indexes assigned to the petrol station. This probable road-section appears twice, but the section with index different than 0 is a probable road-section where fuel is refilled. In case of there are more than one petrol stations on the highway, the procedure described by Statement 1 should be executed.
4. There are two possible ways to proceed from one intersection to the next one:
   a) With free choice, that means it is allowed to choose freely from the probable road-sections.
   b) Mandatory proceeding, which means that if we arrive from probable road-section k then the next intersection also must be approached on road-section k. The significance of it lies in the following Statement.

Statement 1. The original problem can always be converted into a task, in which there is only one petrol station between two intersections. The convolution of a probable road-section is necessary when there are more petrol stations on the same probable road-section. In the original problem, each intersection allows free choice.

6. UNKNOWN QUANTITIES OF THE MODEL

The following matrix contains the first group of the variables. The elements of it define the amount of loaded fuel at each road-section. It is noted by:

$$x_{ik} \geq 0,\ X = [x_{ik}]_{N\times M} \geq 0 \tag{5}$$

the amount of loaded fuel at petrol station k that is between departure station i and destination station $i+1$. In case of $k = 0$ no fuel loading takes place on the given road-section, so that the amount is a fictive quantity $x_{i0} = 1$. This does not cause any problems, since this 0 variable will not appear in the cost of the fuel consumption. This leads to the following:

$$\sum_{k=0}^{\tau_i} \mathrm{sgn}(x_{ik}) = 1, \tag{6}$$

as a consequence one of the road-sections has to be chosen.

Then

$$\sum_{k=1}^{M} \mathrm{sgn}(x_{ik}) \cdot s_{ik} = \sum_{k=1}^{M} \mathrm{sgn}(x_{ik}) \cdot (l_{ik} + m_{ik}) \tag{7}$$

means the length of the road-section between iand $i+1$, if we refill at petrol station k. Based on these and taking into account the bypasses necessary for refueling, the total length of the drive is:

$$s = \sum_{i=1}^{n}\sum_{k=1}^{M} \mathrm{sgn}(x_{ik}) \cdot s_{ik} \tag{8}$$

$$\mathbf{Q} = [Q_i]_N \tag{9}$$

where $Q_i$notes the amount of fuel in the tank of the vehicle at intersectioni.
Then the amount of fuel after the refueling intersection can be defined as:

$$Q_{i+1} = Q_i + \sum_{k=0}^{M} \operatorname{sgn}(x_{ik}) \cdot \left[-(l_{ik} + m_{ik}) \cdot \left(f_f + f_f \cdot \varepsilon_{ik}^{TM} + \varepsilon^{L} \cdot q_i\right)\right] + x_{ik}, \tag{10}$$

which means to divide from the available amount of fuel at the previous intersection the consumed amount of fuel (depending on the load and mass) and adding the amount of refilled fuel.
An important criterion is that the amount of fuel cannot drop under a minimal limiting value. The tank of the vehicle must be filledwith this minimal amount of fuel when arriving to the petrol station:

$$Q_i - \sum_{k=0}^{M} \operatorname{sgn}(x_{ik}) \cdot l_{ik} \cdot \left(f_f + f_f \cdot \varepsilon_{ik}^{TL} + \varepsilon^{L} \cdot q_i\right) \geq Q_{B2}. \tag{11}$$

The amount of refilled fuel (x_{ik}) cannot exceed the capacity of the tank:

$$x_{ik} \leq Q_{max} - Q_i + l_{ik} \cdot \left(f_f + f_f \cdot \varepsilon_{ik}^{TL} + \varepsilon^{L} \cdot q_i\right). \tag{12}$$

O notes the set of the indexes of mandatory intersections. Then the following criterion belongs to the mandatory intersections:

$$\operatorname{sgn}(x_{ik}) = \operatorname{sgn}\left(x_{i+1,k}\right), i \in O, k = 0, \dots, M. \tag{13}$$

C_{t_i} notes the cost of loaded fuel at the relation $i, i+1$ (0 in case there were no refueling), furthermore

$$C_T = \sum_{i=1}^{N-1} C_{t_i}. \tag{14}$$

The cost of refueling at the given fuel station is:

$$C_{t_i} = \sum_{k=0}^{M} p_{ik} \cdot x_{ik}. \tag{15}$$

7. THE MODEL IS THE FOLLOWING

Set of criteria:

$$\begin{gathered}
x_{ik} \geq 0, i = 1, \dots, N-1; k = 0, \dots, \tau_i \\
\sum_{k=0}^{M} \operatorname{sgn}(x_{ik}) = 1.\ i = 1, \dots, N-1 \\
Q_1 = Q_{fs} \\
Q_i - \sum_{k=0}^{M} \operatorname{sgn}(x_{ik}) \cdot l_{ik} \cdot \left(f_f + f_f \cdot \varepsilon_{ik}^{TL} + \varepsilon^{D} \cdot q_i\right) \geq Q_{B2}, i = 1, \dots, N-1 \\
x_{ik} \leq Q_{max} - Q_i + l_{ik} \cdot \left(f_f + f_f \cdot \varepsilon_{ik}^{TL} + \varepsilon^{L} \cdot q_i\right) \\
\operatorname{sgn}(x_{ik}) = \operatorname{sgn}\left(x_{i+1,k}\right), i \in O, k = 0, \dots, M. \\
Q_N \geq Q_{B2} \\
Q_{i+1} = Q_i + \operatorname{sgn}(x_{i0}) \cdot \left[-(l_{i0} + m_{i0}) \cdot \left(f_f + f_f \cdot \varepsilon_{i0}^{TM} + \varepsilon^{L} \cdot q_i\right)\right] + \textstyle\sum_{k=1}^{M} \operatorname{sgn}(x_{ik}) \cdot \\
\left[-(l_{ik} + m_{ik}) \cdot \left(f_f + f_f \cdot \varepsilon_{ik}^{TM} + \varepsilon^{L} \cdot q_i\right)\right] + x_{ik},\ i = 1, \dots, N-1.
\end{gathered} \tag{16}$$

The objective function
Objective function means the cost of the initial loading of the fuel tank (fix cost), and the cost of the refueling during the drive decreased by the cost of the remaining fuel (variable cost):

$$C = C(X, Q) = C_{fs} + C_T - C_r = C_{fs} + \sum_{i=1}^{N-1} \sum_{k=1}^{M} p_{ik} \cdot x_{ik} - \sum_{k=1}^{M} \operatorname{sgn}\left(x_{N-1,k}\right) \cdot p_{N-1,k} \cdot Q_N. \tag{17}$$

The previous criterion means that the cost of the remaining fuel has to be calculated with the unit price of the last fuel station.
The goal:

$$C = C(X, Q) \rightarrow \min. \tag{18}$$

The task is a mathematical programming task.

8. SUMMARY

The main goal of this study was to elaborate a precise and reliable mathematical model for the determination of optimal fuel refill points and the amount of fuel during the execution of international transportation tasks. The main parameters were given for the determination of the optimal refill conditions. In the first step probable (ideal) petrol stations were chosen, at which refill is advisory. A method was suggested with which the length of the road-sections between two petrol stations or intersections can be determined by the GPS coordinates and digital map analysis of the points. Based on these results it a mathematical model was defined for the original problem, then after eliminating some aggravating issues the final model was given. The model is a mixed integer, non-linear programming model, which can be handled by optimization processes. This was shown by a simple example task. Based on the suggested model and the given process a decision-supporting software will be elaborated, which will produce and transfer to the driver the necessary information for the economic execution of several transport activities.

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References

[1.] Kovács Gy., Cselényi J.: Utilization of historic data evaluation obtained from computer database during the organization of international transport activity, Proceedings of 2nd Conference with International Participation Management of Manufacturing Systems, Presov, 2006. Szeptember 28, pp. 1-8.
[2.] Telek P.: Operation strategies for delivery models of regional logistic networks, Advanced Logistic Systems, Theory and Practice, Vol. 4., HU ISSN 1789-2198, Miskolci Egyetem 2010., pp.: 33-40.
[3.] Kovács Gy., Cselényi J., Schmidt Sz., Izsai Á.: Software conceptions relating to utilization of historic data evaluation of international transport activity and relating to cost calculation of transport loops, MicroCAD 2007. Miskolc, Konferencia-kiadvány, University of Miskolc, Supplementary Volume pp. 47-56.
[4.] Tamás P., Illés B., Bálint R.: Mathematical models for outsourcing of delivery channels of paper industry company, Conversations at Miskolc, University of Miskolc, 2007., Conference proceedings, ISBN:978-963-661-790-5, pp. 230-239.
[5.] Skapinyecz R., Illés B.: Optimalizálási lehetőségek vizsgálata egy E-piactérrel integrált virtuális szállítási vállalatnál, (in Hungarian), Doktoranduszok Fóruma: Gépészmérnöki és Informatikai Kar szekciókiadványa, pp. 90-95.
[6.] Russell S., Norvig P.: Mesterséges intelligencia modern megközelítésben, (in Hungarian), TAMOP - 4.1.2-08/2/A/KMR-2009-0026, Panem Kft., Budapest, 2005.
[7.] Gubán M., Gubán Á.: Egy fuvarozási vállalat szállítmányozási feladatának matematikai modellje és tervezett megoldási algoritmusa, (in Hungarian), Globalitás és vállalkozás, Eszes István (szerk.), Budapest: Budapesti Gazdasági Főiskola, 2001. pp. 226-235.
[8.] Gubán Á., Gubán M., Hua N. S.: Információ, adat, intelligencia, (in Hungarian), Budapest, Gubán Ákos (szerk.) Informatikai témák a gazdaságban [I. kötet]: Saldo Pénzügyi Tanácsadó és Informatikai Rt., 2012. ISBN:978-963-638-437-1 pp. 375-377.
[9.] Gubán Á., Kása R.: Conceptualization of fluid flows of logistificated processes, Advanced Logistic Systems: Theory and Practice 7:(2) 2014, University of Miskolc pp. 27-24.

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