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COMBINED EFFECT OF SLIP VELOCITY AND TRANSVERSE SURFACE ROUGHNESS ON THE PERFORMANCE OF A SQUEEZE FILM FOR A CIRCULAR CYLINDER NEAR A PLANE

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ABSTRACT: An attempt has been made to study and analyze the effect of surface roughness on the performance of a rough circular cylinder near a plane considering slip velocity. The slip model of Beavers and Joseph has been deployed to study the effect of velocity slip while the stochastic model of Christensen and Tonder has been used to estimate the effect of transverse surface roughness. The associated stochastically averaged Reynolds type equation is solved with appropriate suitable boundary conditions to obtain the pressure distribution leading to the calculation of load carrying capacity. Equally important is the role of standard deviation associated with roughness in augmenting the performance characteristics. However, lower values of slip may be preferred for enhancing the performance characteristics of the bearing system.

Keywords: circular cylinder, squeeze film, roughness, slip velocity, magnetic fluid, load carrying capacity

1. INTRODUCTION

Now a days the squeeze film lubrication finds a good application in Bio Tribology in general and studying the performance of synovial joints, in particular.

The effect of surface roughness has drawn considerable attentions during the last decade because of its effect on hydrodynamic lubrication as in practice, most of the bearing surfaces are rough to certain extent. The random character of surface roughness has been very well recognized.

Stochastic model of Tzeng and Saibel [1967] was developed by Christensen and Tonder [[1969a, 1969b, 1970] to study the effect of surface roughness. Pajapati [1992] dealt with the combined effect of surface roughness and deformation on the squeeze film behavior between rotating porous circular plates with a concentric circular pocket. The introduction of the pocket reduced the load carrying capacity of the bearing. Bhat and Deheri [1993] studied the effect of magnetic fluid on the action of a curved squeeze film existing between two circular disks. The effects due to magnetization were independent of the curvature of the upper disk. Lin et al. [2013] discussed the squeeze film characteristics of parallel circular disks lubricated by a ferrofluid with non-Newtonian couple stresses. Comparing with the Newtonian non-ferrofluid case, the non-Newtonian ferrofluid lubricated squeeze film provided a higher load carrying capacity and lengthened the response time. Dowson et al. [1976] studied the lubrication of lightly loaded cylinders in combined rolling sliding and normal motion. The problem was related to the lubrication of rigid cylindrical solids by an isoviscous lubricant. The influence of the cavitation boundary condition was considered and it was shown that sinusoidal ‘normal’ motion superimposed upon ‘entraining’ action could lead to a substantial increase in the nett load carrying capacity. Bearman and Zdravkovich [1978] investigated the flow around a circular cylinder placed at various heights above a plane boundary, experimentally. For gaps greater than 0.3 the Strouhal number was found to be remarkably constant. An algorithm was developed to simulate the normal approach, contact and rebound of lubricated cylinders by Chandra and Rogers [1983]. The simulated results for the constant load case were compared with the results of Wada and Tsukijihara [1981] and a formula developed by Cameron [1972]. For sinusoidal load

the results were compared with experimental data. Zdravkovich [1985] studied the effect of forces on a circular cylinder near a plane wall. It was found that the lift coefficient was governed by the gap to diameter ratio while the drag coefficient was dominated by the ratio of gap to thickness of the boundary layer. Kawamura et al. [1986] dealt with the computation of high Reynolds number flow around a circular cylinder with surface roughness. Navier-Stokes equations were integrated for incompressible high-Reynolds number flow using finite-difference method. Sharp reduction of drag coefficient was observed around Reynolds number $Re=2 \times 10^4$. The surface roughness had a profound impact and the effect depended on pattern of the roughness. Trahan et al. [1987] investigated experimentally the limits of lubrication theory for a disk approaching a parallel plane wall. Trahan et al. [1999] analyzed experimentally the velocity of a circular disk moving edgewise in quasi-steady Stokes flow towards a plane boundary. Price et al. [2002] employed flow visualization, particle image velocimetry and hot-film anemometry to study the fluid flow around a circular cylinder near to a plane wall. For the largest gap ratios considered, there was no separation of the wall boundary layer, either upstream or downstream of the cylinder. Dipankar and Sengupta [2005] reported the results for flow past a stationary circular cylinder in the vicinity of a solid wall. Navier-Stokes equations were solved in stream-function vorticity formulation using an improved overset grid method while using a very high spectral accuracy compact scheme to represent the convection process. Chu et al. [2006] studied elastohydrodynamic lubrication of circular contacts of pure squeeze motion with non-Newtonian lubricants. In this study a numerical method for general applications with non-Newtonian fluid was developed to investigate the pure squeeze motion in an isothermal elastohydrodynamic lubricated spherical conjunction under constant load conditions. Chu et al. [2014] studied the elastohydrodynamic lubrication of circular contacts of pure squeeze motion with coated layer. An elastic sphere approaching a lubricated flat surface with an elastic coated layer and substrate was explored under constant load condition. The finite difference method and the Gauss-Seidel iteration method were used to solve the transient modified Reynolds equation, the elastic deformation equations of ball, coating layer and substrate, load balance equation, and lubricant rheology equations simultaneously. The transient pressure profiles, film shapes and elastic deformation during the pure squeeze process under various operating conditions in the elastohydrodynamic lubrication regime, were discussed. Patel and Deheri [2003] analyzed the magnetic fluid based squeeze film behavior between rotating porous circular plates with a concentric circular pocket. The adverse effect of surface roughness was observed to be reduced due to the ferrofluid lubrication, with a suitable choice of pocket size. Patel et al. [2011] investigated the effect of surface roughness on the performance of a ferrofluid-based squeeze film between circular plates with porous matrix of variable thickness. A significant observation was that with a proper selection of thickness ratio parameter, a magnetic fluid based squeeze film bearing with variable porous matrix thickness could be made to perform better than that of a conventional porous bearing with a uniform porous matrix thickness, at least in the case of negatively skewed roughness.

Beavers and Joseph [1967] constructed a simple theory to replace the effect on the boundary layer, with a slip velocity proportional to the exterior velocity gradient. The result obtained from this theory was found to be in good agreement with the experimental results. Wu [1972] analyzed the effect of velocity slip on the squeeze film lubrication between porous rectangular plates. The presence of slip velocity decreased the load carrying capacity and response time. Sparrow et al. [1972] discussed the effect of velocity slip on porous-walled squeeze films. Substantially faster response could be attained by the use of porous materials with accentuate velocity slip. Prakash and Vij [1976] presented the effect of slip velocity on the squeeze film between rotating porous annular disks. The effect of slip was to reduce the load carrying capacity and the response time of the squeeze film. Patel [1980] investigated the effect of velocity slip on the hydromagnetic squeeze film between porous circular disks. The load carrying capacity was found to be decreased when the slip parameter increased. Thakkar et al. [2008] analyzed the effect of velocity slip on the behavior of a squeeze film between rotating rough porous circular plates having a concentric circular pocket. Here, it was shown that the negatively skewed roughness resulted in an improved performance when the slip velocity was at minimum level. Patel and Deheri [2011] studied the effect of surface roughness on the performance of a magnetic fluid based parallel plate porous slider bearing with slip velocity. It was established that the slip velocity caused reduced load carrying capacity but the friction remained unaltered. Patel and

Deheri [2013] dealt with the effect of slip velocity on the performance of a short bearing lubricated with a magnetic fluid. It was established that for large values of aspect ratio, the effect of slip was increasingly significant.

Patel and Deheri [2014] analyzed the effect of slip velocity and roughness on the performance of a Jenkins model based magnetic squeeze film in curved rough circular plates. It was noticed that the Jenkins model moved ahead of Neuringer-Rosensweig model for reducing the adverse effect of roughness. Patel and Deheri [2014] presented the effect of slip velocity and surface roughness on the behavior of Jenkins model based magnetic squeeze film in curved rough circular plates. This study suggested that for any type of improvement in the performance characteristics, the slip parameter was required to be reduced even if variance (-ve) occurred and suitable magnetic strength was in force.

Here, it has been proposed to discuss the effect of slip velocity on the squeeze film performance for a circular cylinder near a plane considering surface roughness effect.

2. ANALYSIS

The geometry and configuration of the bearing system is presented below.

It is well-known that for an isoviscous incompressible fluid the Reynolds equation can be written as

$$\frac{\partial}{\partial x} \left(h^3 \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial z} \left(h^3 \frac{\partial p}{\partial z} \right) = -12 \eta V \tag{1}$$

where V is the squeeze velocity $-\frac{dh}{dt}$ and η is the viscosity of the lubricant.

The length of the cylinder is assumed to be large relative to the radius of the cylinder, so that the side leakage can be neglected. The governing equation then turns out to be

$$\frac{d}{dx} \left(h^3 \frac{dp}{dx} \right) = -12 \eta V \tag{2}$$

The film thickness h is a function of x which is given by $h = h_0 + R - \sqrt{R^2 - x^2}$ which approximately equals $h_0 + \frac{1}{2} \frac{x^2}{R}$ in view of the series expansion.

Following Christensen and Tonder (1970), the thickness $h(x)$ of the lubricant film is considered to be

$$h(x) = \bar{h}(x) + h_s \tag{3}$$

Here, \bar{h} is the mean film thickness and h_s is the deviation from the mean film thickness characterizing the random roughness of the bearing surfaces. h_s is assumed to be stochastic in nature and governed by the probability density function $F(h_s)$, which is defined by

$$F(h_s) = \begin{cases} \frac{32}{35c} \left(1 - \frac{h_s^2}{c^2} \right)^3 & ; -c \leq h_s \leq c \\ 0 & ; \text{otherwise} \end{cases} \tag{4}$$

where c is the maximum deviation from the mean film thickness. The mean α , standard deviation σ and the parameter ϵ , which is the measure of symmetry of the random variable h_s , are defined by the following relationships:

$$\alpha = E(h_s); \sigma^2 = E[(h_s - \alpha)^2]; \epsilon = E[(h_s - \alpha)^3] \tag{5}$$

where E denotes the expected value defined by

$$E(R) = \int_{-c}^c R F(h_s) dh_s \tag{6}$$

The details regarding roughness aspects can be had from Christensen and Tonder [1969(a),1969(b), 1970]. In order to describe the steady flow of magnetic fluids in the presence of slowly changing magnetic fields a mathematical model was proposed by Neuringer-Rosensweig in 1964. This model and related aspects has been discussed in detail by Bhat [2003]. Following the model outlined in Bhat[2003], equation (2) assumes the following form when a magnetic fluid is taken as the lubricant;

$$\frac{d}{dx} \left(h^3 \left(\frac{d}{dx} \left(p - \frac{\mu_0 \bar{\mu} H^2}{2} \right) \right) \right) = -12 \eta V \tag{7}$$

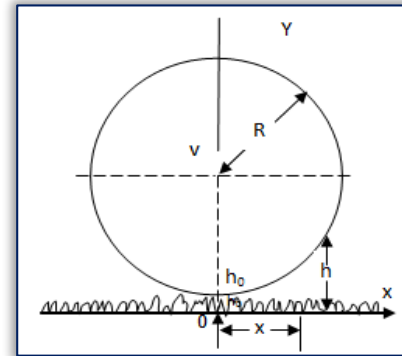


Figure 1. A cylinder near a plane

where μ_0 is the magnetic permeability, $\bar{\mu}$ is the susceptibility of the magnetic particles and H is the magnitude of the applied external magnetic field. In view of Bhat[2003] the magnitude of magnetic field is described as

$$H^2 = \frac{kx^2}{h} (R - x) \tag{8}$$

where k is a suitably chosen constant to suit the dimension for manufacturing a required strength of the magnetic field. The discussions regarding the inclination angle of the magnetic field is discussed in Prajapati[1995], Bhat[2003] and Bhat and Deheri[1991].

Tzeng and Seibel[1967] recognized the random character of roughness and proposed a stochastic method to evaluate the effect of surface roughness. This method was modified and developed by Christensen and Tonder[1969(a), 1969(b), 1970] who characterized the surface roughness by a random variable with non-zero mean, variance and skewness. Resorting to the stochastic model of Christensen and Tonder and the method outlined in Gupta and Deheri[1996], the equation (7) turns to

$$\frac{d}{dx} \left((g(h)) \frac{d}{dx} \left(p - \frac{\mu_0 \bar{\mu} H^2}{2} \right) \right) = -12 \eta V \tag{9}$$

where $g(h) = h^3 + 4(\sigma^2 + \alpha^2)h + 2\alpha^2h + 2\alpha^3 + 3\sigma^2\alpha + \varepsilon$

The role of standard deviation is known to be predominant for this type of bearing system and hence neglecting the other two parameters, one obtains

$$g(h) \cong h^3 + 4\sigma^2h$$

Lastly, making use of (Beavers and Joseph, 1967) slip model one arrives at the stochastically averaged Reynolds' type equation, governing the film pressure, in dimensionless form as

$$\frac{d}{dx} \left(\left(\frac{4 + \bar{S}}{2 + \bar{S}} \right) (h^3 + 4\sigma^2h) \left(\frac{d}{dx} \left(P - \frac{\mu^*}{2} \right) \right) \right) = -12 \tag{10}$$

wherein the dimensionless quantities are $\bar{S} = sh$, $\mu^* = \frac{-k\mu_0\bar{\mu}h^3}{\eta V}$, $P = \frac{ph^3}{\eta V R}$, $\bar{\sigma} = \frac{\sigma}{h}$

The non-dimensional boundary conditions associated with the bearing system are

$$\frac{dP}{d\bar{x}} = 0 \text{ at } \bar{x} = 0 \tag{11}$$

where $\bar{x} = \frac{x}{R}$

Solving equation (10) with the aid of boundary conditions (11) one gets the expression for non-dimensional pressure distribution:

$$\bar{P} = \frac{\mu^*}{6} (\bar{x}^2(1 - \bar{x})) - \frac{3}{\left(\frac{4+\bar{S}}{2+\bar{S}}\right)\bar{\sigma}^2} \log \left[\frac{\bar{h}}{\sqrt{\bar{h}^2 + 4\bar{\sigma}^2}} \right] \tag{12}$$

Then, the load carrying capacity of the bearing system in non-dimensional form is found to be

$$\bar{W} = \frac{\mu^* \pi}{\sqrt{\frac{h_0}{R} \left(\frac{4+\bar{S}}{2+\bar{S}}\right)}} \left[\log 4\bar{\sigma}^2 \frac{1}{4\bar{\sigma}^2} \frac{1}{12\bar{\sigma}^2} \left(\frac{C_0}{h_0}\right)^2 \left(\frac{h_0}{R}\right) \frac{1}{80} \left(\frac{C_0}{h_0}\right)^4 \left(\frac{h_0}{R}\right)^2 \frac{1}{\bar{\sigma}^2} \right] \tag{13}$$

where $\bar{W} = \frac{h_0^3 w}{\eta V R L C_0}$, L being the length of the bearing.

3. RESULTS AND DISCUSSION

It is easily observed that the dimensionless pressure distribution is obtained from equation (12) while equation (13) determines the load carrying capacity for the bearing.

It is noticed from equation(12) that the non-dimensional pressure distribution enhances by $\frac{\mu^*}{6} \bar{x}^2(1 - \bar{x})$, while the dimensionless load carrying capacity gets increased by $\frac{\mu^* \pi}{\sqrt{\frac{h_0}{R}}}$ as can be seen

from equation (12), in comparison with the conventional lubricant based bearing system. This is not surprising because the magnetization increases the viscosity of lubricant. One can notice that the expression in equation (12) is linear with respect to μ^* . Thus, an increase in μ^* would lead to increased load carrying capacity. The fact that the load carrying capacity increases sharply, is displayed in Figures 2-3. The variation of load carrying capacity with respect to the standard deviation presented in the Figures 4-6 ensures that the load carrying capacity reduces due to the standard deviation σ . However, the effect of h_0/R on the variation of load carrying capacity with respect to $\bar{\sigma}$ remains nominal.

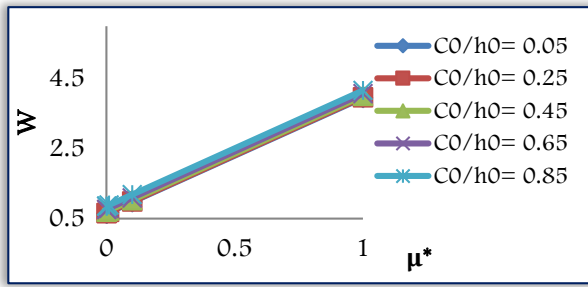


Figure 2. Variation of Load carrying capacity with respect to μ^* and C_0/h_0 .

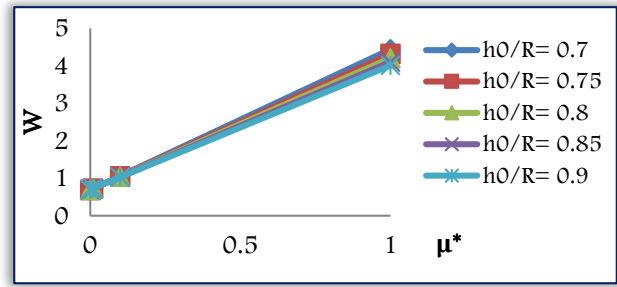


Figure 3. Variation of Load carrying capacity with respect to μ^* and h_0/R .

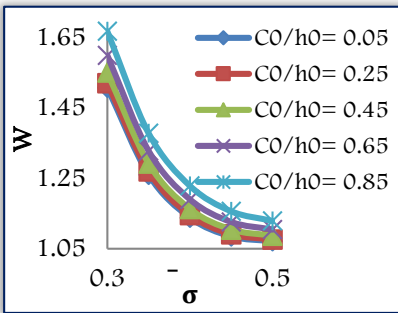


Figure 4. Variation of Load carrying capacity with respect to $\bar{\sigma}$ and C_0/h_0 .

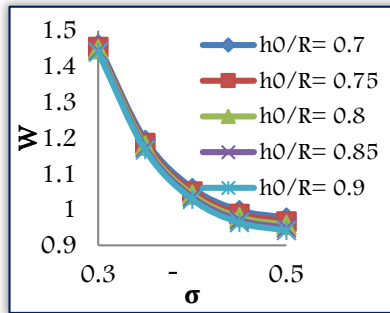


Figure 5. Variation of Load carrying capacity with respect to $\bar{\sigma}$ and h_0/R .

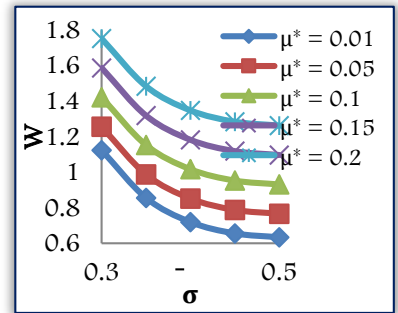


Figure 6. Variation of Load carrying capacity with respect to $\bar{\sigma}$ and μ^* .

Lastly, it is observed from Figures 7-10 that for a better performance, the slip is required to be kept at reduced level.

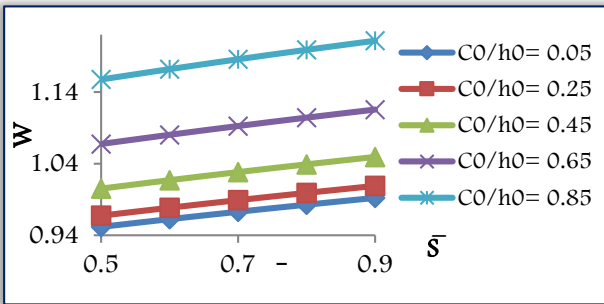


Figure 7. Variation of Load carrying capacity with respect to \bar{s} and C_0/h_0 .

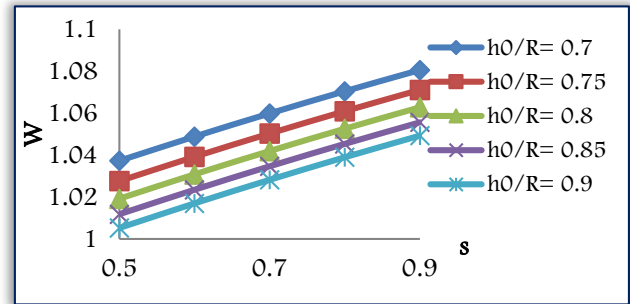


Figure 8. Variation of Load carrying capacity with respect to \bar{s} and h_0/R .

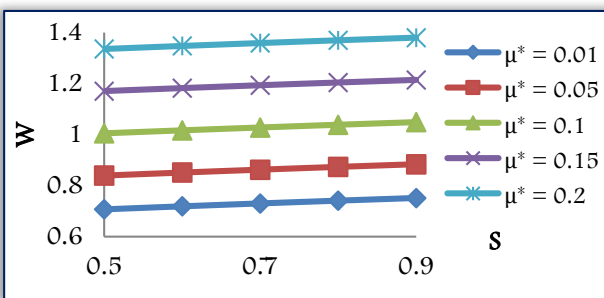


Figure 9. Variation of Load carrying capacity with respect to \bar{s} and μ^* .

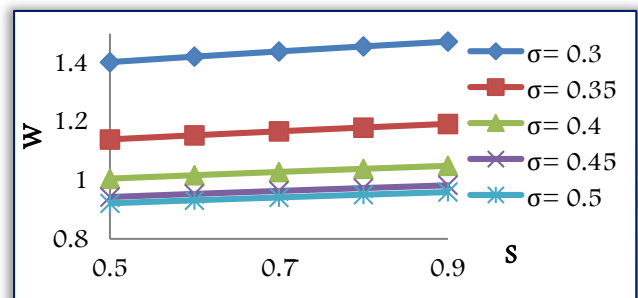


Figure 10. Variation of Load carrying capacity with respect to \bar{s} and $\bar{\sigma}$.

4. CONCLUSION

For an effective performance of the bearing system the slip parameter deserves to be kept at minimum. This study reveals that the roughness aspect must be given due consideration while designing the bearing system, especially from bearing's life period point of view, even if slip is at minimum. Besides, this type of bearing system supports a certain amount of load even in the absence of flow, which fails to happen in the case of conventional lubricant based bearing system.

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