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WAVE PROPAGATION THROUGH ULTRASONIC HORNS – SIMULATION AND TESTING

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ABSTRACT: The present paper is a short state of art about the ultrasonic waves propagation through horns. There are presented the main aspects about the mathematical modelling of the propagation using the Webster's equation and its solution. It is highlight that the solution in space domain is difficult to be found by analytical method. In context there are shown the numerical methods that can be used in waves propagation modelling. At the end a set up used by authors is presented.
Keywords: ultrasonic horns, transfer matrix method, finite element method

1. INTRODUCTION

Any ultrasonic system used in ultrasound applications is made of five main components: ultrasonic generator, transducer, ultrasonic horn (sonotrode), tool, and manufactured piece. The role of the ultrasonic horn (UH) is to provide, with high efficiency, acoustic energy developed by a magneto-strictive transducer to a tool. Practically, UH is a solid device that amplifies, in a mechanical way, the generated signal by the transducer. At the end of these devices there are attached tools for high-power technical applications as: ultrasonic welding, ultrasonic machining, ultrasonic cutting, ultrasonic drilling, etc. As geometry, the UH have a continuous cross-section variation along the longitudinal axis from a large diameter to a small diameter (Figure 1).

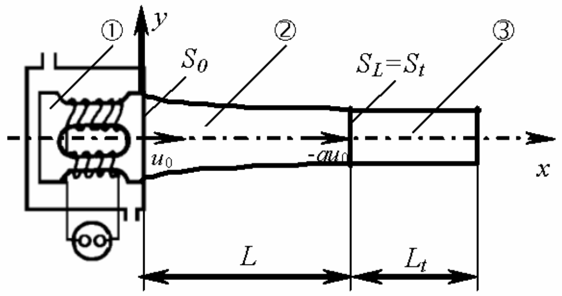


Figure 1: Ultrasonic system: 1 - transducer; 2 – ultrasonic horn; 3 – cutting tool

The ultrasonic horns design it is described in many technical books and papers. The main design principles can be found in [5] [6], [7], [9]. Based on all description and taking into consideration the practical aspects of industrial applications one can say that the use of these devices can be considered as part of the non-conventional technologies. Taking into consideration the mathematical functions of cross-section variation in practice there are used the following horns type:

a) with stepped variable cross-section;
b) with linear cross-section variation;
c) with exponential cross-section variation;
d) with different cross-section function variations, as catenoidal, parabolic, Gaussian, hyperbolic, and/or their combinations.

The initial signal induced in the UH has an amplitude u_0, given by the magnetostrive transducer, and at the small end, at the distance L, the value is $u_L=-qu_0$, where q is the magnification coefficient.

The wave propagation of the signal through UH is based on the Webster's horn equation:

$$\frac{\partial^2 u(x,t)}{\partial x^2}+\frac{\partial u(x,t)}{\partial x}\frac{\partial}{\partial x}\left(\ln S_x\right)=\frac{1}{c^2}\frac{\partial^2 u(x,t)}{\partial t^2}, \tag{1}$$

where, $u(x,t)$ is the ultrasonic signal and $S_x = S(x)$ is the cross section area at a distance x from the reference, and c is the sound velocity in the horn's material (Figure 1).
The solution of the equation (1) it is obtained considering the separation of variables method. According with this method, the solution of the equation (1) is considered as a product between two different variables: one time dependent and the other one space dependent:

$$u(x,t)=u(x)\,u(t)=u_x u_t\,. \tag{2}$$

Considering solution (2), the propagation equation (1) becomes:

$$\frac{u''_x}{u_x}+\frac{d}{dx}\left(S_x\right)\frac{u'_x}{u_x}=\frac{1}{c^2}\frac{\ddot{u}_t}{u_t}, \tag{3}$$

with the following notations: $u''_x = d^2u_x/dx^2$, $u'_x = du_x/dx$, and $\ddot{u}_t = d^2u_t/dt^2$.
The congruency of the left the right sides of the equation (3) is achieved and only if the two functions are equal with a common separation constant β. It is considered as constant the value $\beta=-\omega^2$, where ω is the ultrasound frequency [rad/s], and thus, the individual functions $u(x)=u_x$ and $u(t)=u_t$ must satisfy ordinary differential equations:

$$\begin{cases} \ddot{u}_t+\omega^2 u_t=0; \\ u''_x+\dfrac{d}{dx}\left(S_x\right)u'_x+k^2u_x=0, \end{cases} \tag{4}$$

where $k=\omega/c$ is the wave number, $\omega=2\pi f$ is the angular frequency of vibrations, and f is the frequency of ultrasound.
The solution of the equation (4) is:

$$u_t = C_1\cos\omega t + C_2\sin\omega t\,. \tag{5}$$

Considering the initial conditions:

$$\begin{cases} u_t\big|_{t=0}=u_0; \\ \left.\dfrac{du_t}{dt}\right|_{t=0}=0, \end{cases} \tag{6}$$

where u_0 is the initial amplitude of the ultrasonic wave, and one can obtain that the time domain solution (5) becomes:

$$u_t = u_0\cos\omega t\,, \tag{7}$$

and the total solution (2) is:

$$u(x,t)=u(x)\,u(t)=u_x u_t = u_x u_0\cos\omega t\,. \tag{8}$$

Considering the assumption of infinitesimal motions in a beam non-dissipative medium with cross variable section area, for a steady-state mode, the second equation from (4), which represents the plane wave, can be rewritten as:

$$\frac{d}{dx}\left(S_x\frac{du_x}{dx}\right)+k^2S_xu_x=0\,. \tag{9}$$

2. NUMERICAL METHODS USED IN WAVE PROPAGATION ANALYSIS

Longitudinal wave propagation through horns is described by equation (1) that has two components, one time dependent and the other one space dependent (4). The solution for the space dependent equation is difficult to be found due to the term $\frac{\partial}{\partial x}\left(\ln S_x\right)$. As a consequence, the study can be done easier considering some numerical methods.

2.1. Finite-difference method used in wave propagation through horns with variable cross-section variation analysis

Based on this method one can found the variation of amplitude, velocity and acceleration of the waves in a large number of points. Details about this method can be found in many books or papers. In [9] it is analysed the propagation of a longitudinal wave through horns with harmonic cross-section variation. It is considered a horn with a harmonic cross-section variation given by the following equation:

$$S(x)=S_0+\Delta S\cdot\cos(k_p x), \tag{10}$$

where for ΔS there were considered some values equal with initial cross-section S_0 fractions, and k_p is the wave number that corresponds to the cross section periodicity:

$$k_p=\frac{2\pi}{\lambda_p}, \tag{11}$$

where λ_p represents the length of the periodicity.

Based on relationship (10), the wave propagation equation (2) is, in terms of spatial components:

$$u''-\frac{k_p\Delta S\sin(k_p x)}{S_0+\Delta S\cos(k_p x)}u'+k^2u=0. \tag{12}$$

Considering the relations for the first and second derivative and the method of centred finite differences, equation (12) can be rewritten as:

$$\frac{u_{i+1}-2u_i+u_{i-1}}{h^2}-\frac{k_p\Delta S\sin(k_p x_i)}{S_0+\Delta S\cos(k_p x_i)}\frac{u_{i+1}-u_{i-1}}{2h}+k^2u_i=0. \tag{13}$$

with the associated initial conditions:

$$\begin{cases} u_0=u_1, \\ \dfrac{u_1-u_{-1}}{2\Delta x}=0. \end{cases} \tag{14}$$

2.2.The transfer matrix method

The main advantage of the method is consists in the fact that its components do not depends on both elastic properties of the propagation media and wave type. Elastic constant are introduced by the boundary conditions at the level of boundary layers which are dependent on interface type between layers [1], [2], [3], [4], [8].

The advantages of the method are useful in finding the velocities of the longitudinal waves in solid media as: small cylindrical samples, the method permits to realise a study of dispersion for a given material sample. The experimental set-up can be sized in such way as the phase velocity can be found in a large range of frequencies, the method permits to generalise the attenuation influence. This method offers important information when is applied to binary periodically systems with extrapolation in the area of acoustic-optical properties of solid media, the method is useful to be applied for special materials as ceramics used in extreme conditions and with special mechanical properties.

In the transfer matrix method the wave propagation along the horn is characterized by a wave transfer matrix If we consider a progressive wave of spectral amplitude $A=A(f)$ and frequency f moving from an input end to the output end of the horn and a regressive wave of spectral amplitude $B=B(f)$ moving in the opposite direction, then the total output wave is given by:

$$\begin{pmatrix} A_{out} \\ B_{out} \end{pmatrix}=T\begin{pmatrix} A_{in} \\ B_{in} \end{pmatrix}. \tag{15}$$

Then the transfer matrix from input to output could be expressed as the right-product of the propagation and discontinuity matrices along the path of propagation:

$$T_{in,out}=P_N D_{N-1,N} P_{N-1} \ldots P_3 D_{2,3} P_2 D_{1,2} P_1. \tag{16}$$

The propagation in the opposite direction is characterized by the matrix:

$$T_{out,in}=P_1 D_{2,1} P_2 \ldots P_{N-1} D_{N,N-1} P_N. \tag{17}$$

The propagation from input to output and back again is characterized by the total propagation matrix:

$$T_{total} = T_{out,in} R T_{in,out} \cdot \tag{18}$$

2.3.Computer simulation using coupled-oscillators model

In many cases, for 1D propagation, the horn can be modelled as a linear arrangement of coupled masses (lumped system). The horn of length L is split in a number of n bodies each with mass m_i, of the same length Δx_i and connected with springs of elastic constants $k_{i,i+1}$. If an initial perturbation is applied to one end, the displacement $u_i(t)$ of the oscillators can be obtained by numerically integrating of the equations of motion:

$$m_i \ddot{u}_i = k_{i-1,i}(u_{i-1} - u_i) + k_{i,i+1}(u_{i+1} - u_i). \tag{19}$$

where:

$$\Delta x = \frac{L}{n-1},\ m_i = \rho S_i \Delta x,\ k^*_{i,i+1} = m_j \left(\frac{c}{\Delta x}\right)^2. \tag{20}$$

2.4.Computer simulation using finite element method

Good results for 3D simulation are given by the Finite Element Method (FEM) [8]. The horn can be modelled with tetrahedral volume elements as shown in figure 2. Based on FEM model one can find the eigenmodes illustrated in figure 3.

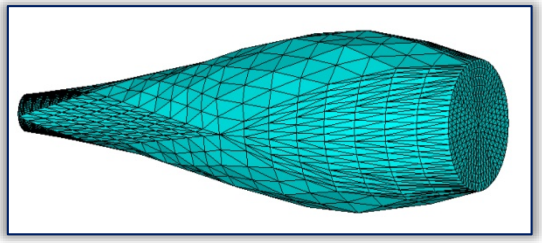


Figure 2: Ultrasonic horn modelled with finite elements

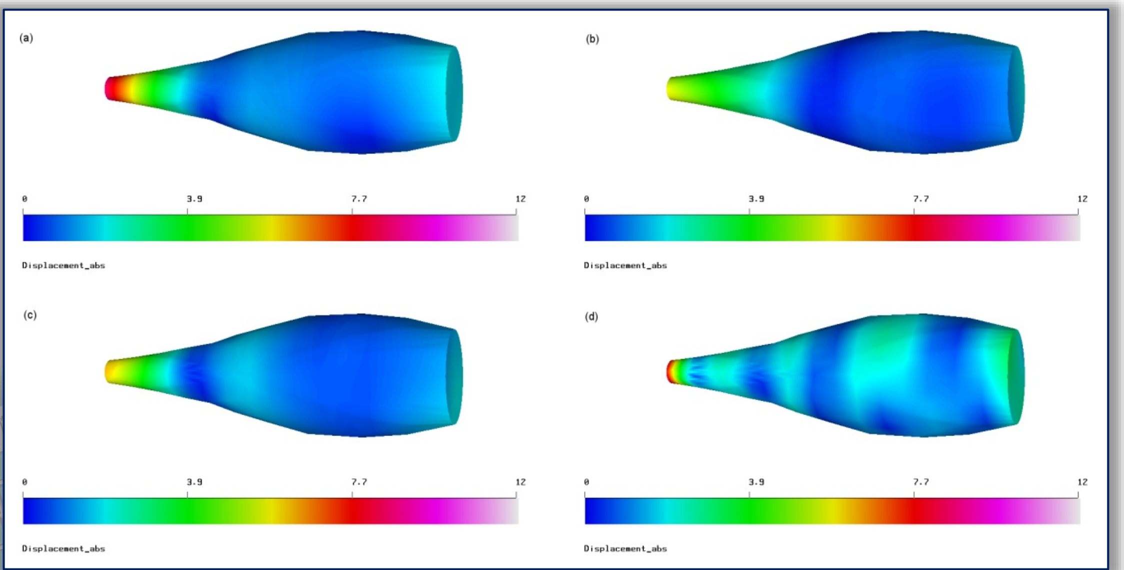


Figure 3: Longitudinal eigenmodes in the USH based on FEM:
(a) 12091.67Hz, (b) 27841.47Hz, (c) 44802.37Hz, (d) 62225.28 Hz.

3. HORNS TESTING

The block scheme ultrasonic horn testing is presented in figure 4 [9]. The testing system is made of an ultrasound generator, a transducer (ceramic or magneto-strictive), the ultrasonic horn, the acquisition and processing system.

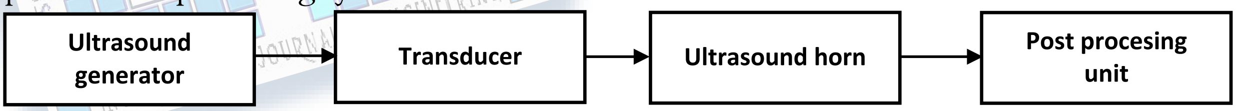


Figure 4: Bloc scheme of ultrasound horns testing

In figure 5 it is presented the set-up used in different research work by the authors. The data acquisition system was made of a Brüel & Kjær Pulse platform 12 (type 3039 with control module 7539), two accelerometers 4517-002 and dedicated soft for time and frequency domains analysis.

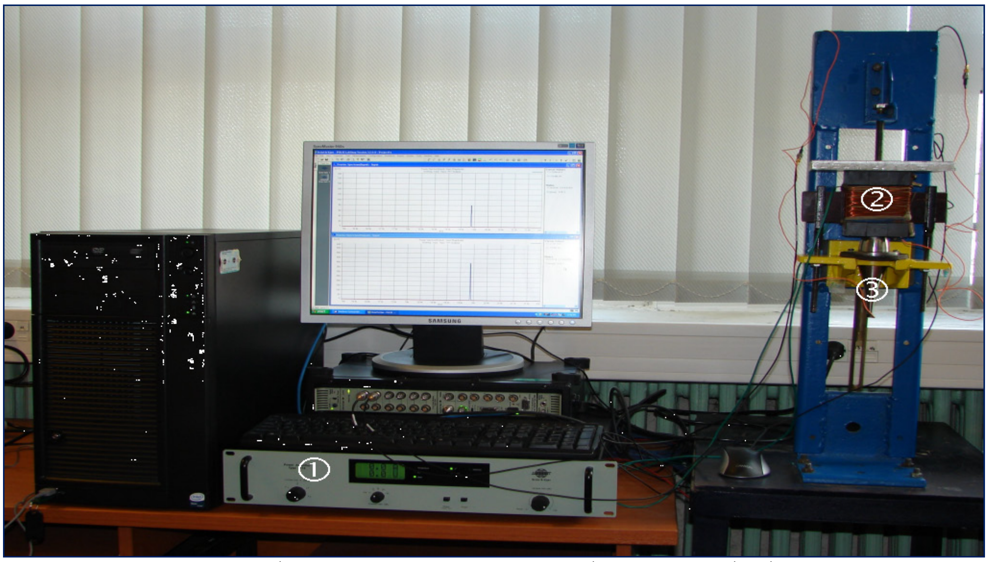
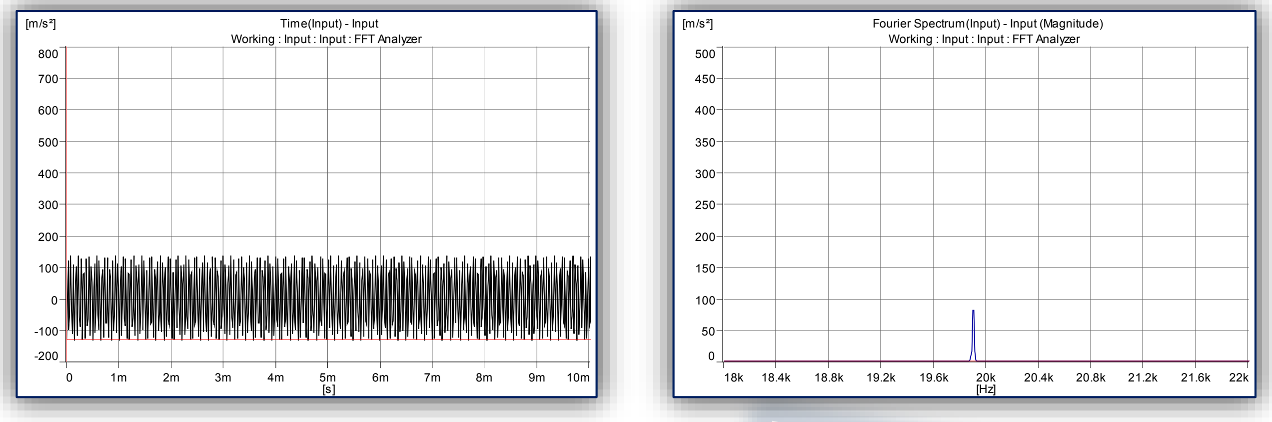


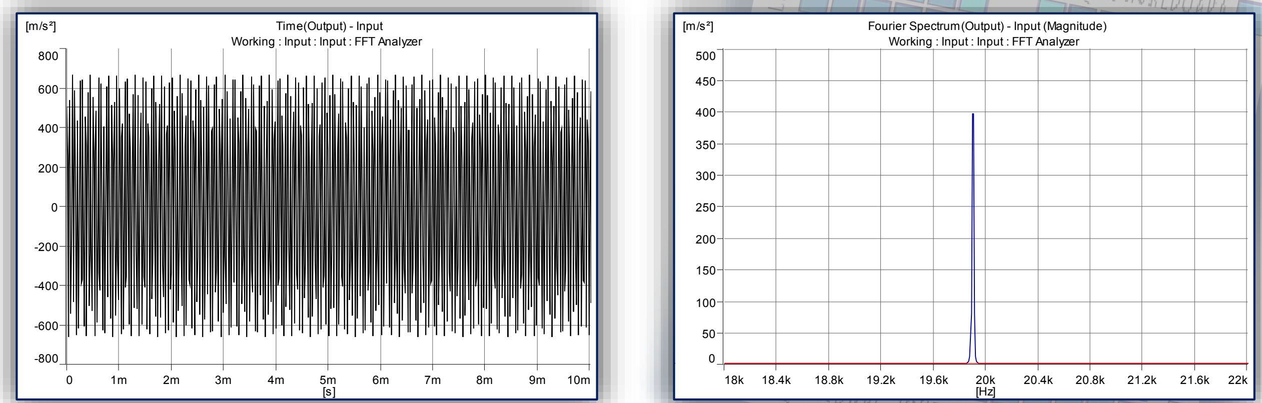
Figure 5: The set-up for experimental testing of the horns:
1 – amplifier; 2 – magneto-strictive transducer; 3 – ultrasonic horn; 4 – Pulse 12 platform

Based on the above presented equipment one can find experimentally the level of the signal magnitude and the value of the signal frequency, both for input and output signals (figures 6 and 7).



a) b)

Figure 6: Input signal: a) time domain representation; b) Fourier spectrum



a) b)

Figure 7: Output signal: a) time domain representation; b) Fourier spectrum

4. CONCLUSIONS

Ultrasonic horns are devices used to increase mechanical an input signal given by a transducer. The magnitude is obtained by the variable cross section and the wave propagation is governed by the equation (1). The solution of the equation has two components: one is a time dependent solution and the other is a space dependent solution. The spatial equation component (9) is difficult to be solved for different section shapes by analytical method thus the solution can be obtained by different numerical methods (§ 2).

Note

This paper is based on the paper presented at The 1st International Conference "Experimental Mechanics in Engineering" - EMECH 2016, organized by Romanian Academy of Technical Sciences, Transilvania University of Brasov and Romanian Society of Theoretical and Applied Mechanics, in Brasov, ROMANIA, between 8 - 9 June 2016, referred here as [10].

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