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NATURAL FREQUENCY SHIFT OF DAMAGED CIRCULAR PLATE CLAMPED ALL AROUND

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ABSTRACT: The paper highlights the natural frequency changes that occur in circular plates clamped all around, when damage appears. The natural frequencies for undamaged circular plates were computed by the analytic approach and they were compared with the natural frequencies obtained by numerical methods. Afterwards, damage is created and stepwise moved along the whole length of the plate radius. Three different wide angles are assumed for the damage. The results obtained from the finite element analysis are plotted and commented. As an important conclusion, the shape of the curves does not change by increasing the damage angle and only the frequency ratio will be increased.
Keywords: circular plate, natural frequency, damage, finite element analysis

1. INTRODUCTION

Plates are flat structures characterized by thickness h, which is small compared to the other in-plane dimensions [1]. In the case of a circular plate, the only in-plane dimension is the radius R. Circular plates have a large field of application in engineering, with several papers written in the relevant literature dealing with the topic of plate vibrations. Several analytical solutions have been obtained in recent decades by researchers who have focused their research on the topic of natural frequencies of the circular plates [2-6]. The following methods can be classified as global ones: the Ritz method and the differential quadrature method. Among the class of local methods we mention the finite element method and the difference method. Local methods are less accurate than global ones, although, they impart a greater degree of flexibility in handling complex geometries and boundary conditions [7]. This paper presents the changes in the natural frequency for damaged circular plates clamped all around when the damage moves along the whole radius of the plate.

2. ANALYTICAL APPROACH

The classical differential equation of motion for transverse displacement w of a plate is given by [4]:

$$D\nabla^4 w + \rho\frac{\partial^2 w}{\partial t^2} = 0 \tag{1}$$

where:

$$D = \frac{Eh^3}{12(1-\nu^2)} \tag{2}$$

is the flexural rigidity with: E [N/m²] - the Young's modulus, h [m] - the plate thickness, ν - Poisson's ratio,

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} \tag{3}$$

is the Laplacian operator expressed in polar coordinates with: r [m] and θ [rad], ρ [kg/m³] is the mass density, t [s] is the time. Note that $\nabla^4 = \left(\nabla^2\right)^2$.
Taking into account that for free vibrations, the motion is expressed as:

$$w = W\cos\omega t \tag{4}$$

where: ω [rad/s] is the circular frequency; W is the function only of the position coordinates.
By substituting the equation (4) into equation (1) we obtained:

$$\left(\nabla^4 - k^4\right)W = \left(\nabla^2 - k^2\right)\left(\nabla^2 + k^2\right)W = 0 \tag{5}$$

With the dimensionless wave number k defined as;

$$k^4 = \frac{\rho\omega^2}{D} \tag{6}$$

The complete solution to equation (5) can be obtained by superimposing the solutions of the equations:

$$\begin{cases} \nabla^2 W_1 - k^2 W_1 = 0 \\ \nabla^2 W_2 + k^2 W_2 = 0 \end{cases} \tag{7}$$

These solutions, when the Fourier components in θ are assumed, become:

$$W(r,\theta) = \sum_{n=0}^{\infty} W_n(r)\cos n\theta + \sum_{n=1}^{\infty} W_n^*(r)\sin n\theta \tag{8}$$

Substituting the equation (8) into equation (7) and taking in consideration the Bessel functions, the general solution in polar coordinates becomes:

$$\begin{cases} W(r,\theta) = \sum\limits_{n=0}^{\infty}\left[A_n J_n(kr) + B_n Y_n(kr) + C_n I_n(kr) + D_n K_n(kr)\right]\cos n\theta + \\ + \sum\limits_{n=1}^{\infty}\left[A_n^* J_n(kr) + B_n^* Y_n(kr) + C_n^* I_n(kr) + D_n^* K_n(kr)\right]\sin n\theta \end{cases} \tag{9}$$

where, J_n, Y_n are the Bessel functions of the first and second kind; I_n, K_n are the modified Bessel functions of the first and second kind; A_n, B_n, C_n, D_n, A_n^*, B_n^*, C_n^*, D_n^*, are the coefficients obtained from boundary conditions.
For a circular plate without an internal hole, with the origin of the polar coordinate system in the center of the plate, in order to avoid infinite deflections and stresses at r=0, the terms $Y_n(kr)$ and $K_n(kr)$ must be discarded and the equation (9) will be written:

$$W_n(r,\theta) = \left[A_n J_n(kr) + C_n I_n(kr)\right]\cos n\theta \tag{10}$$

where, $n = 0\ldots\infty$ represents the number of nodal diameters.
For a circular plate clamped all around [8-10], made of structural steel, with the external radius R=0.4 m and the thickness h=2 mm, having the Young's modulus $E = 2\cdot 10^{11}$ N/m², the mass density ρ = 7850 kg/m³ and the Poisson's coefficient ν = 0.3, the boundary conditions are:

$$\begin{cases} W(R) = 0 \\ \dfrac{\partial W(R)}{\partial r} = 0 \end{cases} \tag{11}$$

By solving the system (11) and taking into consideration the recursion relationships:

$$\begin{cases} \lambda J_n{}'(\lambda) = nJ_n(\lambda) - \lambda J_{n+1}(\lambda) \\ \lambda I_n{}'(\lambda) = nI_n(\lambda) + \lambda I_{n+1}(\lambda) \end{cases} \tag{12}$$

with: λ=kR, the characteristic equation which permits us to calculate the values of λ² is:

$$J_n(\lambda)I_{n+1}(\lambda) + I_n(\lambda)J_{n+1}(\lambda) = 0 \tag{13}$$

The achieved values of λ² for n=0…5 number of nodal diameters and s=0…2 number of nodal circles are presented in Table 1.
The natural frequencies of the considered circular plate clamped all around are given by the relationship (14) and the obtained values are shown in Table 2.

$$f_{n,s} = \frac{\lambda^2}{2\pi R^2}\sqrt{\frac{Eh^2}{12\rho(1-\nu^2)}} \tag{14}$$

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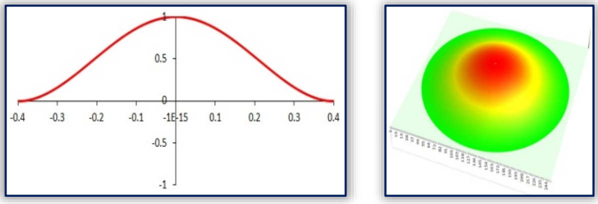
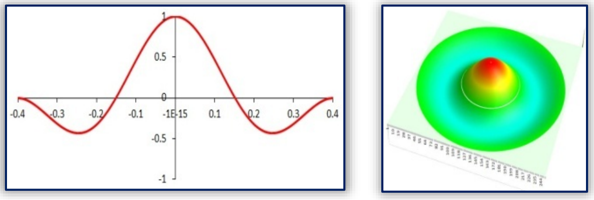
The first ten vibration modes of the circular plate clamped all around are presented in the Figure 1.a…j for $\theta = 0$ on the left side and for $\theta = 0\ldots2\pi$ on the right side.

Table 1: Values of λ^2 for a clamped circular plate

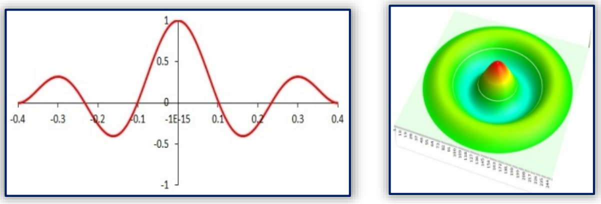
Nodal diameters: n	Nodal circles: s 0	1	2
0	10.2158262298673	39.7711482364571	89.1041439739694
1	21.2603976946146	60.8286718200205	120.079236582622
2	34.8770354203197	84.5826495514749	153.815084910931
3	51.0300354837762	111.021411877498	190.303779572114
4	69.6658306967891	140.107903826781	229.518563910683
5	90.7389863174461	171.802983556901	271.428181055347



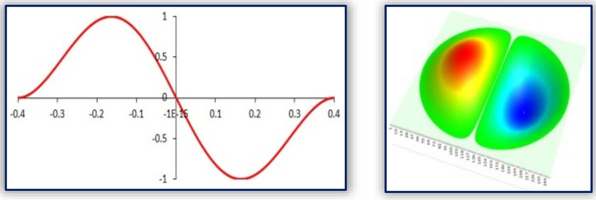
a) n=0, s=0



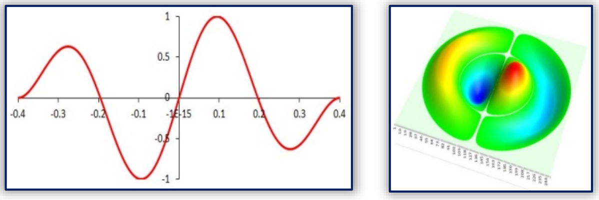
b) n=0, s=1



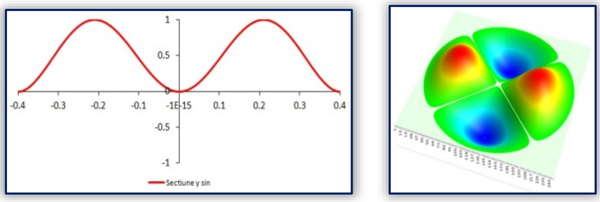
c) n=0, s=2



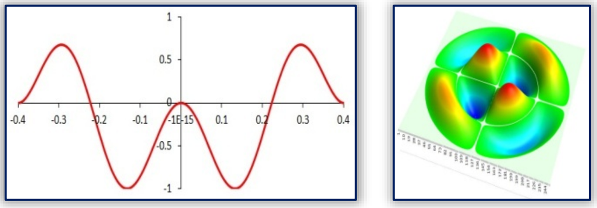
d) n=1, s=0



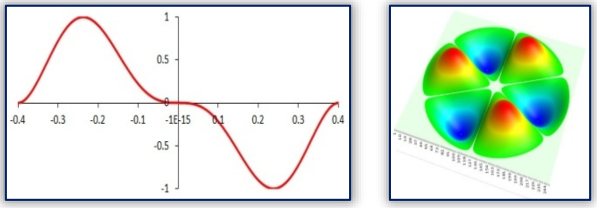
e) n=1, s=1



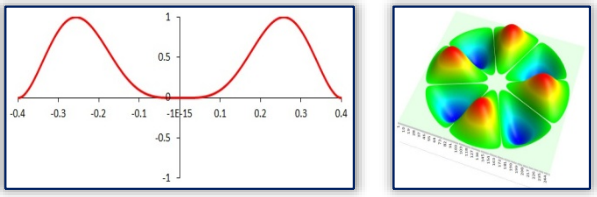
f) n=2, s=0



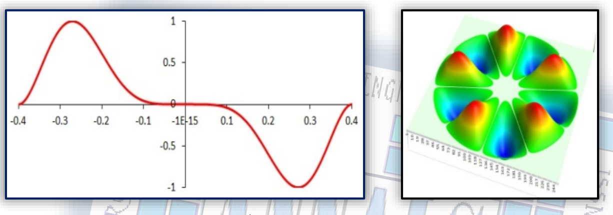
g) n=2, s=1



h) n=3, s=0



i) n=4, s=0



j) n=5, s=0

Figure 1: Normalized mode shapes of the circular plate clamped all around

Table 2: The first ten natural frequencies for the circular plate clamped all around

Vibration mode	Natural frequencies [Hz] Analytic	FEM	Deviation [%]
1 (n=0, s=0)	30.94348031156	31.05860460783	-0.37205
2 (n=1, s=0)	64.39720906331	64.63301199483	-0.36617
3 (n=2, s=0)	105.64166172864	106.0212362851	-0.35930
4 (n=0, s=1)	120.46580616504	120.8967248204	-0.35771
5 (n=3, s=0)	154.56869202354	155.1118140769	-0.35138
6 (n=1, s=1)	184.24851465641	184.8894738680	-0.34788
7 (n=4, s=0)	211.01604628434	211.7386302085	-0.34243
8 (n=2, s=1)	256.19871483753	257.0621701266	-0.33070
9 (n=0, s=2)	269.89420754591	270.7994651486	-0.33541
10 (n=5, s=0)	274.84610382230	275.7600656933	-0.33254

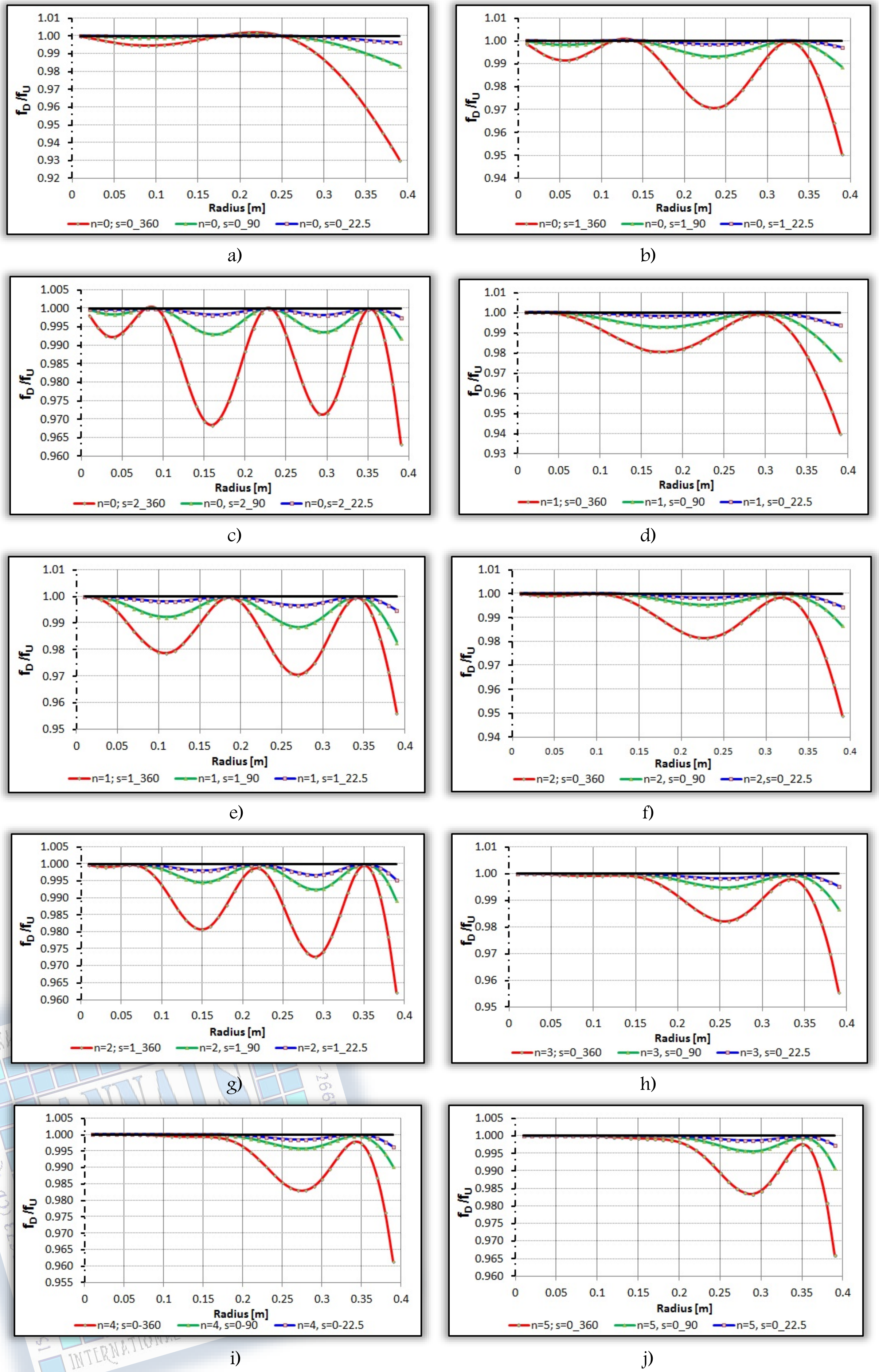


Figure 2: Natural frequency shift for damages with angular opening of: 22.5 deg; 90 deg and 360 deg.

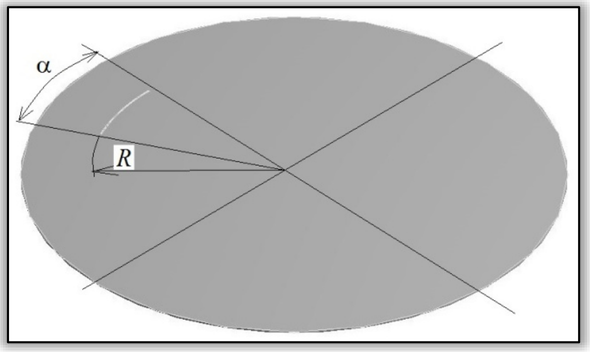


Figure 3. Circular plate with damage

3. NUMERICAL INVESTIGATION

The numerical investigation was made by the finite element method, first for undamaged circular plates clamped all around, and then for damaged circular plates. The first ten natural frequencies for undamaged circular plates clamped all around, with mechanical characteristics and physical properties specified in the previous chapter are presented in table 2, which also contains the deviations between analytic and numeric results.

Afterwards, the damage is removed along the whole radius of the circular plate. For each position of the damage with a step of 10 mm from the radius, a finite element method analysis was performed. The damage taken into account is of a circular shape with a wide angle of: $\alpha = 22.5$ deg, $\alpha = 90$ deg and $\alpha = 360$ deg (figure 3). The depth of damage is of $h/2=1$ mm.

The ratios between the natural frequencies for the damaged and undamaged circular plate, for the first ten vibration modes and for each damage case described above, are shown as curves in the figure 2.a-j. These curves, nominated as relative frequency shift curves RFSh, can be used to indicate the damage position and extent since for each damage case a specific sequence of frequency changes exist.

4. CONCLUSION

The RFSh curves provide a general overview of the influence on natural frequencies when damage occurs on a circular segment or a complete circle on any poison along the radius. In the cases when the values of the damage angle are modified, the shape of the curve stays the same. The difference will consist in a decrease of the frequencies ratio due to the increase of the angle of damage. When the damage is located at the center of the plates, the frequency change is insignificant at higher vibration modes.

Note

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