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MHD FLOW AND HEAT TRANSFER ANALYSIS OF A THIRD GRADE FLUID IN POST-TREATMENT ANALYSIS OF WIRE COATING

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Abstract: The present investigation has bearing on post-treatment analysis of wire-coating. The coating material characterizes a third grade fluid. The novelty of the present study is to discuss the effect of magnetic field on the flow and heat transfer of a coating material i.e. third grade fluid, when the wire is extruded from a die. The governing equations, of flow and heat transfer are solved by Runge-Kutta method with shooting technique and the effects of pertinent parameters are shown with the help of graphs. Moreover, in order to establish the consistency and accuracy of the numerical method, we have compared the results of the present study with the results obtained by perturbation method in case of without magnetic field. The case of Newtonian fluid has been derived as a particular case. The retarding effect of magnetic field on velocity field is exclusive, distinct and significant for both Newtonian and non-Newtonian coating materials, which may be an essential design requirement in wire coating.
Keywords: MHD flow, wire-coating, third grade fluid, Runge-Kutta method, shooting technique, magnetic field

1. INTRODUCTION

Wire coating process is an industrial process to coat a wire for insulation, mechanical strength and environmental safety. The plasticized polyvinyl chlorides (PVC), low density polyethylene (LDPE), high density polyethylene (HDPE), nylon are used as coating material. There are several methods used for wire coating. Out of them coaxial extrusion process is frequently used.

The co-axial extrusion process is an operation in which either the polymer is extruded on axially moving wire or the wire is dragged inside a die filled with molten polymer. The hydrodynamic model by Akter and Hashmi [1, 2] is used to enhance the efficiency of co-axial extrusion process. In this process of coating, the velocity of continuum and the melt polymer develop high pressure in a specific region which in turn produces strong bonding and also offers fast coating. Therefore, many researchers Tadmor and Gagos [3], Mitsoulis [4], Roy and Dutt [5], Caswell and Tanner [6] contributed to this field of study. Recently, Siddiqui et al. [7, 8] analysed wire coating using third grade fluid and fourth order fluid. Moreover, Basu [9] presented a theoretical analysis of non-isothermal process in wire coating co-extrusion of dies.

In wire coating, the quality of material and wire drawing velocity are important within the die. After leaving the die, the temperature of the coating material is also important.

Many non-Newtonian fluids are used in wire coating. In particular, visco-elastic fluids are of great interest. The present study is related to a visco-elastic fluid i.e., third grade fluid. Recently, a visco-elastic fluid model known as Phan-Thien-Tanner (PTT) model is widly used for wire coating. Many authors have contributed to enrich the field of heat transfer of post-treatment analysis of wire coating. Kasajima and Ito [10] have worked on post-treatment of polymer extrudate in wire coating. Symmonset al. [11] have studied plasto-hydrodynamics dieless wire drawing. Wagner and Mitsoulis [12] have studied the effect of die design on the analysis of wire coating. Akter and Hashmi [2] have studied problems on wire drawing and coating using different fluid models representing suitably many industrial fluids. Sajjidet al. [13] studied wire coating process using MHD oldroyd8-constant fluid. Siddiqui et al. [14] have considered third grade fluid in their study. Siddiqui et al. [15] have examined the effect of thin film flow of a third grade fluid on an inclined plane. Aksoy and Pakdemirli [16] have analyzed approximate analytical solution for flow of a third grade fluid through a parallel plate channel filled with a porous medium.

Shah et al. [17] have considered third grade fluid as coating material in wire coating analysis in the absence of magnetic field and used perturbation method for analytical solution.

The objective of the present study is to consider the flow and heat-transfer analysis of third-grade fluid in the presence of uniform transverse magnetic field producing a resistive force called Lorentz force. The coupled non-linear differential equations with appropriate boundary conditions are solved numerically by Runge-Kutta method empowered by shooting technique. An attempt has been made to solve the governing equations by perturbation method also. Only zeroth order equation has been solved analytically for a particular value of magnetic parameter. The results of both the methods have been compared and are found to be in good agreement. Moreover, the case of without magnetic field Shah et al. [17] has been discussed as a particular case.

2. FORMULATION OF THE PROBLEM

Consider the flow of the polymer extrudate given in Figure 1(a), denoted by the solid line. To analyze the flow behavior of a polymer used in wire coating, it is convenient to divide the flow transversely into many short sections as shown by broken lines in Figure 1(a) with the assumption that each section has almost the same shape, we analyze only one section because each section can be assumed to be approximately of the shape shown in Figure1(b) and readily analyzable.

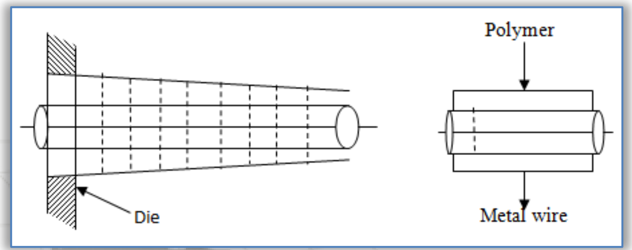

Figure 1(a). Polymer extrudate in wire coating

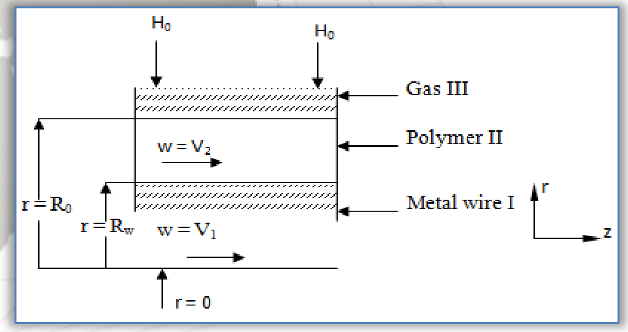

Figure 1(b). Drag flow in wire coating

The wire of radius R_w and at temperature θ_1 is dragged in the z direction through an incompressible third grade polymer (II) with a velocity V_1 and the gas (III) surrounding the polymer (II) at the surface of coated wire of radius R_0 is at temperature θ_2 and flowing with a velocity V_2. An uniform magnetic field of strength H_0 is applied normally in the direction of flow of fluid. The fluid is acted upon by a constant pressure gradient $\frac{dp}{dz}$ in the axial direction. The flow is considered steady, laminar, unidirectional and axisymmetric.

The problem has been studied with the following assumptions,

(i) wire-velocity, V_1> fluid velocity, V_2

(ii) non-Newtonian parameter, B<1

(iii) The flow is incompressible due to the high viscosity of the polymer.

(iv) Polymer II holds the third grade fluid model for shear rate.

In Figure1(b)., the metal wire I, the polymer II and gas III are in contact with each other and consider no slippage occurs along the contacting surfaces of the wire, polymer and the gas.

With the above frame of reference and assumptions the fluid velocity $(\vec{q})$, extra stress tensor (S) and the temperature field (θ) are considered as

$$\vec{q}=\left[0,0,w(r)\right],\ S=S(r),\theta=\theta(r) \tag{1}$$

Boundary conditions are:

$$\left.\begin{aligned} w=V_1,\ \theta=\theta_1\ \ at\ \ r=R_w, \\ w=V_2,\ \theta=\theta_2\ \ at\ r=R_0 \end{aligned}\right\} \tag{2}$$

For third grade fluid, the extra stress tensor S is defined as

$$S=\mu A_1+\alpha_1 A_2+\alpha_2 A_1^2+\beta_1 A_3+\beta_2\left(A_1A_2+A_2A_1\right)+\beta_3\left(t_rA_3\right)A_1 \tag{3}$$

In which μ is the coefficient of viscosity of the fluid, $\alpha_1, \alpha_2, \beta_1, \beta_2, \beta_3$ are the material constants and A_1, A_2, A_3 are the kinematic tensors.

$$A_1 = L^T + L \tag{4}$$

$$A_n = A_{n-1}L^T + LA_{n-1} + \frac{DA_{n-1}}{Dt},\ n = 2,3, \tag{5}$$

where the superscript T denotes the transpose of the matrix.
The basic equations governing the flow of an incompressible conducting fluid are:

$$\nabla.\vec{q} = 0 \tag{6}$$

$$\rho\frac{D\vec{q}}{Dt} = -\nabla.p + \vec{F} + \vec{J} \times \vec{H} \tag{7}$$

$$\rho C_p \frac{D\theta}{Dt} = k\,\nabla^2\theta + \phi \tag{8}$$

where $\vec{q}$ is the velocity vector, ρ is the density, P *is* the pressure, $\vec{F}$ is the viscous force per unit volume, $\frac{D}{Dt}$ denotes the substantive derivative, θ is the fluid temperature, k *is* the thermal conductivity, C_p is the specific heat at constant pressure, ϕ is the dissipation function, J is the current density and H is the magnetic field strength.
In the equation of motion (7) a body force $\vec{J} \times \vec{H}$ per unit volume of electromagnetic origin appears due to the interaction of the current and the magnetic field. The electrostatic force due to charge density is considered to be negligible. A uniform magnetic field of strength of H_0 is assumed to be applied in the positive radial direction normal to the wire i.e. along z-axis. Hence the retarding force per unit volume acting along z-axis is given by

$$\vec{J} \times \vec{H} = \left(0, 0, -\sigma H_0^2 w\right) \tag{9}$$

where σ is the electrical conductivity.
Using the velocity field, the continuity equation (6) is satisfied identically and the nonzero components of extra tensor S from equation (3) are given by

$$S_{rr} = (2\alpha_1 + \alpha_2)\left(\frac{dw}{dr}\right)^2 \tag{10}$$

$$S_{zz} = \alpha_2\left(\frac{dw}{dr}\right)^2 \tag{11}$$

$$S_{rz} = \mu\frac{dw}{dr} + 2(\beta_2 + \beta_3)\left(\frac{dw}{dr}\right)^3 \tag{12}$$

In view of velocity field and Equations (9)–(12), the momentum (7) gives

$$\frac{-\partial p}{\partial r} = (2\alpha_1 + \alpha_2)\frac{1}{r}\frac{d}{dr}\left\{r\left(\frac{dw}{dr}\right)^2\right\} \tag{13}$$

$$\frac{\partial p}{\partial \theta} = 0 \tag{14}$$

$$\frac{\partial P}{\partial z} = \frac{\mu}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right) + \frac{2(\beta_2 + \beta_3)}{r}\frac{d}{dr}\left(r\left(\frac{dw}{dr}\right)^3\right) - \sigma H_0^2 w \tag{15}$$

After leaving the die, there is only drag flow, so the pressure gradient in the axial direction is to be zero. Hence, equation (15) becomes

$$\frac{2(\beta_2 + \beta_3)}{r}.\frac{d}{dr}\left(r\left(\frac{dw}{dr}\right)^3\right) + \frac{\mu}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right) - \sigma H_0^2 w = 0 \tag{16}$$

and the energy equation (8) becomes

$$k\left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr}\right)\theta + \mu\left(\frac{dw}{dr}\right)^2 + 2(\beta_2 + \beta_3)\left(\frac{dw}{dr}\right)^4 = 0 \tag{17}$$

Introducing the dimensionless parameters

$$\left.\begin{aligned} &r^* = \frac{r}{R_w}, \quad w^* = \frac{w}{V_1}, \quad \theta^* = \frac{\theta-\theta_1}{\theta_2-\theta_1}, \; B = \beta_2+\beta_3 \\ &\frac{R_0}{R_w} = \delta > 1, \; \frac{V_2}{V_1} = U < 1, M^2 = \frac{\sigma H_0^2 R_w^2}{\mu} \end{aligned}\right\} \tag{18}$$

The equations (16), (17) and boundary condition (2) after dropping the asterisks, become:

$$r\frac{d^2w}{dr^2}+\frac{dw}{dr}+2B\left[3r\frac{d^2w}{dr^2}\left(\frac{dw}{dr}\right)^2+\left(\frac{dw}{dr}\right)^3\right]-M^2wr=0 \tag{19}$$

$$\frac{d^2\theta}{dr^2}+\frac{1}{r}\frac{d\theta}{dr}+Br\left(\frac{dw}{dr}\right)^2+2Br\;B\left(\frac{dw}{dr}\right)^4=0 \tag{20}$$

$$\begin{gathered} w(1)=1, \quad \theta(1)=0 \\ w(\delta)=U\,, \; \theta(\delta)=1 \end{gathered} \tag{21}$$

where the Brinkman number (Br) and non-Newtonian parameter (B) are, respectively

$$Br=\frac{\mu V_1^2}{k(\theta_2-\theta_1)}, \quad B^*=\frac{B}{\mu\left(\frac{R_w^2}{V_1^2}\right)} \tag{22}$$

3. METHOD OF SOLUTION

3.1. Numerical method

The Runge-Kutta method with shooting technique has been used to solve equations (19) and (20) with boundary conditions (21). The physical and computation domains are finite. For computational purpose we have taken $\delta = 2$.

3.2. Perturbation Solution

To approximate the solutions of Equation (19) and (20) subject to the boundary conditions given in Equation (21), B is assumed to be a small perturbation parameter, the approximate velocity profile and temperature profiles are chosen as

$$w(r,B)=w_0(r)+Bw_1(r)+B^2\;w_2(r)+\ldots \tag{23}$$

$$\theta(r,B)=\theta_0(r)+B\,\theta_1(r)+B^2\;\theta_2(r)+\ldots \tag{24}$$

Substituting Equation (23) and (24) into equation (19) to (21) and collecting the order of B we get Zeroth order:

$$r\frac{d^2w_0}{dr^2}+\frac{dw_0}{dr}-M^2w_0r=0 \tag{25}$$

$$\frac{d^2\theta_0}{dr^2}+\frac{1}{r}\frac{d\theta_0}{dr}+Br\left(\frac{dw_0}{dr}\right)^2=0 \tag{26}$$

$$\left.\begin{aligned} w_0(1)&=1 \quad , \; \theta_0(1)=0 \\ w_0(2)&=U \quad , \; \theta_0(2)=1 \end{aligned}\right\}, \tag{27}$$

First order:

$$r\frac{d^2w_1}{dr^2}+\frac{dw_1}{dr}+2\left(\frac{dw_0}{dr}\right)^3+6r\left(\frac{dw_0}{dr}\right)^2\frac{d^2w_0}{dr^2}-M^2w_1r=0 \tag{28}$$

$$\frac{d^2\theta_1}{dr^2}+\frac{1}{r}\frac{d\theta_1}{dr}+2Br\left(\frac{dw_0}{dr}\right)^4+2Br\frac{dw_0}{dr}\frac{dw_1}{dr}=0 \tag{29}$$

With boundary conditions:

$$\left.\begin{aligned} w_1(1)&=0 \quad , \; \theta_1(1)=0 \\ w_1(2)&=0 \quad , \; \theta_1(2)=0 \end{aligned}\right\} \tag{30}$$

Solution of zeroth order perturbation for M = 0 and M = 1 are:

$$w_0(r)=1+\frac{\ln r}{\ln 2}(U-1) \tag{31}$$

$$w_0(r) = A_1 I_0(r) + A_2 K_0(r) \quad (32)$$

where, $A_1 = 0.5161U - 0.1396$, $A_2 = 2.795 - 1.552U$ and $I_0(r)$ and $K_0(r)$ are solutions of modified Bessel function of first kind of order zero

Due to complexity of the solution of first order equation we have not mentioned those here. It is enough to verify our result with Shah et al. [17] by considering only the zeroth order solution for particular values of M =0 and M = 1.

4. RESULTS AND DISCUSSION

From the practical point of view, it is interesting to know the effects of non-Newtonian parameter, B, which exhibits the property of the coating material, Brickman number, Br, a relative measure of viscous heating to conductive heating of fluid, velocity ratio, $U<1$ and magnetic parameter, M.

In the process of wire coating, the quality of material and wire drawing velocity are important within the die. Therefore, the effect of non-Newtonian parameter which characterizes the fluid property of the melt polymer (third grade fluid) is important. Moreover, after leaving the die, temperature and transverse sectioning is also very important. Hence, heat transfer analysis has been carried out showing the effects of pertinent parameters involved in the equations.

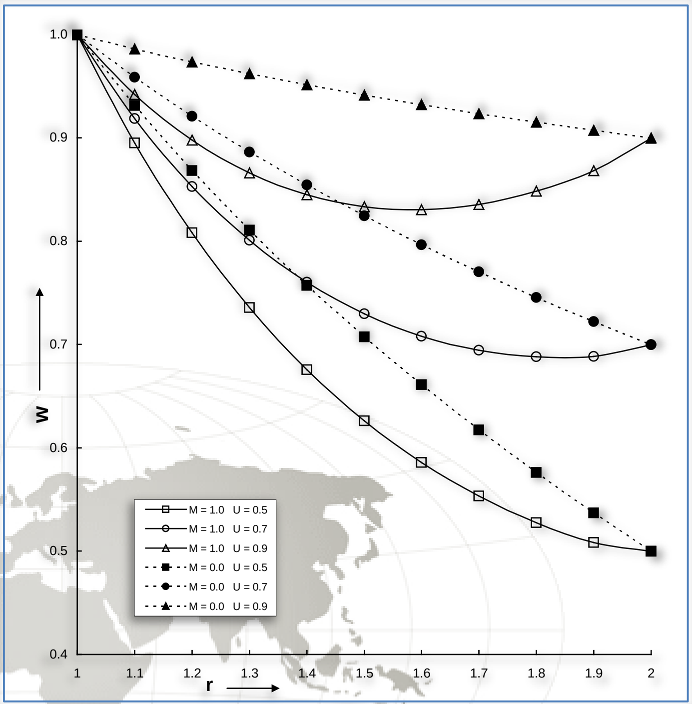

Figure 2. Velocity distribution (Perturbation solution)

Figure 2 presents the velocity distribution basing upon the solution obtained by perturbation method. We have considered the zeroth order perturbation solution which corresponds to Newtonian fluid. It is seen that magnetic parameter reduces the velocity at all points. Moreover, an increase in velocity ratio increases the velocity for both presence as well as absence of magnetic field.

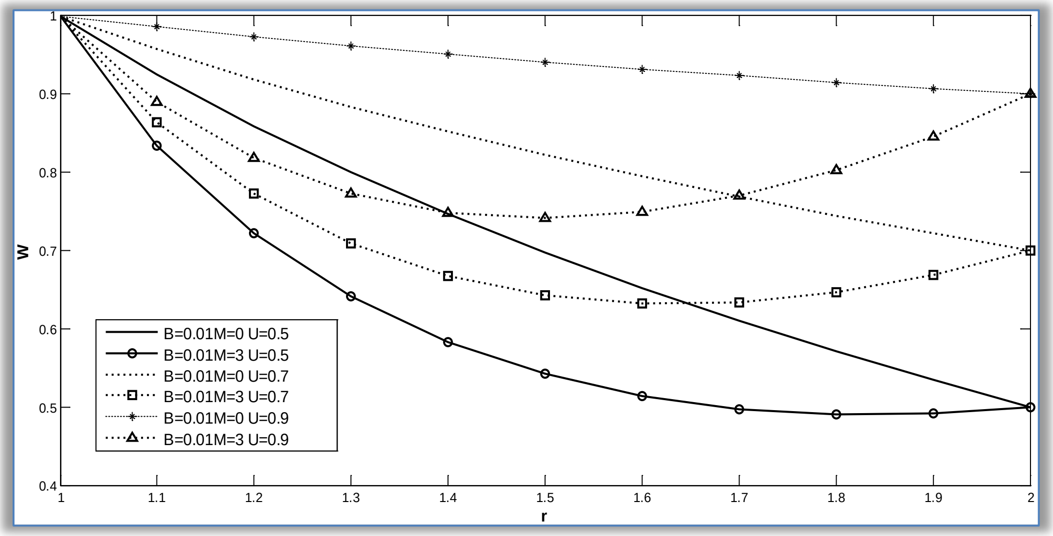

Figure 3. Velocity distribution for different value of M and U

From Figure 3, it is seen that presence of magnetic field is responsible for non-linear variation of velocity within the flow domain when non-Newtonian parameter is set to a fixed value. Further, the resistive force of electromagnetic origin reduces the velocity whereas the velocity ratio parameter increases it at all points irrespective of presence or absence of magnetic field. Thus, it is concluded that to enhance or reduce the velocity of fluid used in wire coating as the post treatment, it is required to control the parameters M and U as per the design requirement.

On comparing the Figure 2 (analytical solution) with Figure 3 (numerical solution) it is remarked that the results obtained in both the cases are identical. This proves the consistency of numerical method applied here to solve the governing equations.
It is of worth mentioning that the results in case of without magnetic field of the present study, are in good agreement with the results reported by Shah et al. [17] in respect of velocity ratio.
From Figure 4 it is evident that the non-Newtonian parameter characterizing the melt polymer (third grade fluid) increases the velocity at all points irrespective of presence or absence of magnetic field. It is further seen that magnetic field decreases the velocity for both Newtonian and non-Newtonian fluids. Hence, it is concluded that the effect of non-Newtonian parameter is opposite to that of magnetic field. Therefore, to enhance the fluid velocity inside the die, it is required to choose a fluid with high non-Newtonian property subject to low magnetic field strength.

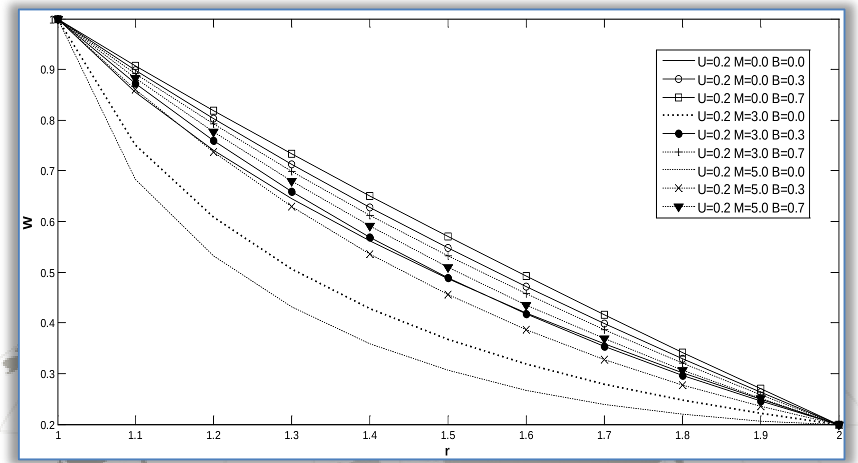

Figure 4. Velocity distribution for different value of M and B

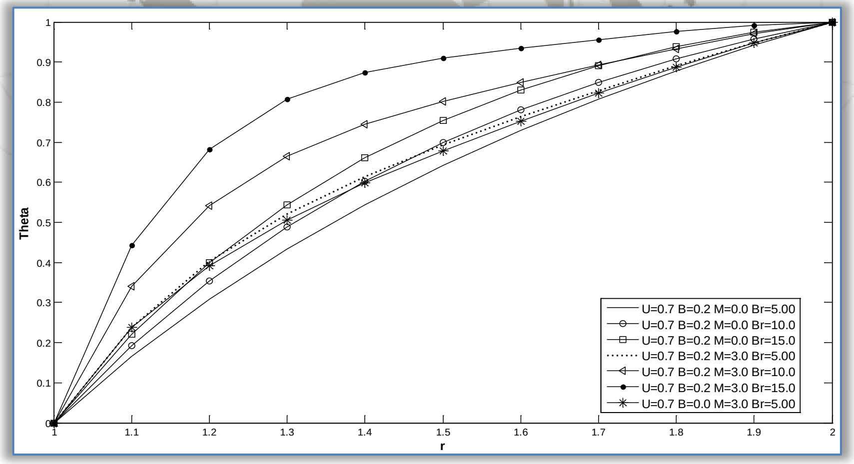

Figure 5. Temperature distribution for different value of M, Br and B

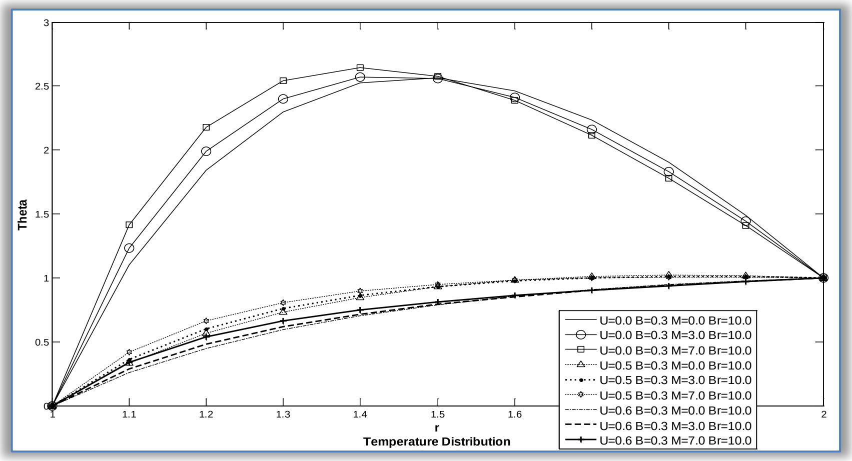

Figure 6. Temperature distribution for different value of U and M

Figure 5 also presents temperature distribution for various values of Brinkman number, Magnetic parameter and non-Newtonian parameter. It is seen that an increase in Brinkman number which is a measure of viscous heating relative to conductive heat transfer leads to increase the temperature both in the presence and absence of magnetic field. This agrees well with Shah et al. [17]. This shows that viscous heating plays a significant role than the conductive heating in enhancing the fluid temperature. Further, it is seen that magnetic parameter and non-Newtonian parameter also increase the temperature for fixed value of velocity ratio and Brinkman number.

Thus, the temperature of the fluid increases due to the resistance offered by the electromagnetic force and the non-Newtonian properties of the fluid.

Figure 6 exhibits the temperature distribution for a third-grade fluid with constant Brinkman number (B_r =10), which is the measure of the ratio of viscous heating relative to the conductive heat transfer. Analyzing the profiles for $U \neq 0$, it is seen that an increase in velocity ratio decreases the temperature both in the presence or absence of magnetic field same as Shah et al. [17]. It is observed, that an increase in Magnetic parameter increases temperature at all points. It is further seen that the temperature profile for zero velocity ratio i.e., representing no fluid motion, attains higher values than nonzero velocity ratio.

5. CONCLUSIONS

- ≡ The results of Perturbation method relating to magnetic field on velocity distribution for Newtonian fluid agree with the result of numerical method. Results obtained in the present study coincide with Shah et al. [17] in the absence of magnetic field.
- ≡ Application of magnetic field reduces the fluid velocity for both Newtonian and non-Newtonian fluid whereas velocity ratio parameter and non-Newtonian parameter enhance it irrespective of presence or absence of magnetic field. Thus, it is concluded that magnetic field strength and the non-Newtonian property of the fluid act as controlling parameters on the velocity field i.e., either to reduce or enhance the velocity of fluid used in wire coating as per design requirement.
- ≡ Effect of magnetic field, viscous heating and non-Newtonian property of third grade fluid are favorable to increase the fluid temperature of coating material whereas velocity ratio is counterproductive in enhancing the temperature.

Nomenclature

R_w	Radius of wire	J	Current density
V_1	wire velocity	q	Velocity vector
V_2	Velocity of fluid	μ	Coefficient of viscosity
U	Velocity ratio	δ	Boundary layer thickness
B	Non-Newtonian parameter / perturbation parameter	ρ	Density of the fluid
M	Magnetic parameter	σ	Electrical conductivity
Br	Brinkman Number	S	Extra stress tensor
H_0	Magnetic field strength	θ	Fluid temperature
P	Pressure	θ_1	Wire temperature
k	Thermal conductivity	θ_2	Flow temperature

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