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MODELING OF THE FLANK OF A TOOTH OF WORM GEAR MILLING CUTTERS FOR MAKING GROOVED SHAFTS

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Abstract: The paper presents the computer model of replacement (approximation) of a curved (theoretical) profile of the flank of a tooth of worm gear milling cutter for making grooved shafts with a circle using computers, with the aim of creating quality products for the production and conditions for the achievement of minimum production costs. Also, we compared replacement errors of theoretical profile of the flank of a tooth using the circle, obtained as calculated (output) value from present computer models with the results obtained using Frafeld tables. Based on the analysis carried out for the example of the calculation of a milling cutter for making a grooved shaft with channels it can be concluded that given theoretical computer model of replacement of theoretical profile of the flank of a tooth of the milling cutter using the circle described in the paper in relation to the calculation made using Frafeld tables provides a minor error replacement (approximation).
Keywords: Approximation, Calculating model, Profile of the flank of a tooth, grooved shafts

1. INTRODUCTION

Modern production today cannot be imagined without the use of tools and accessories. In order to produce more economically, better quality tools[1] and accessories are required. The production of such resources necessarily requires prior in-depth study, calculation and design for each specific case. In this field, there is a great potential to improve the production in the whole [1,4,5].

During the design of tools certain specifics that are related to the production process and technology of tools, i.e. "technologicality" that represents the support for the quality of products design for manufacturing and requirements for achieving minimum production costs, must be met [2,3,5].

The achievement of the required and the sufficient level of "technologicality" is achieved, among other things, by simplifying the shape and using standardized elements. For this purpose during the construction of the tooth of the worm gear milling cutters for making grooved shafts, curvilinear (theoretical) profile of the flank of atoo this usually replaced by lines suitable for production (arcs of the circle), which also leads to defects in the item[6,7].

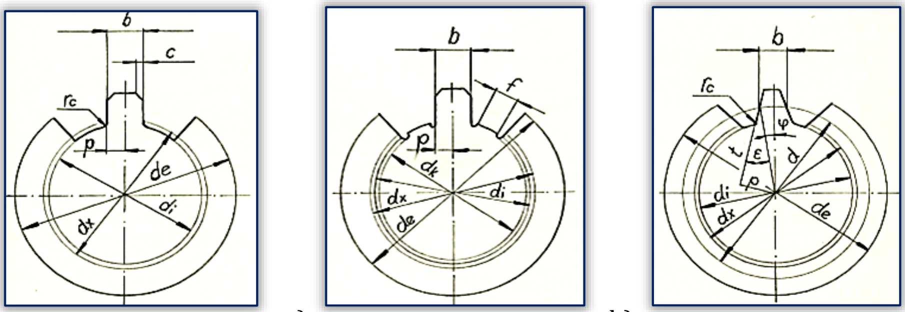
For these reasons the paper will consider an approximation of the curved profile of the flank of a tooth of a worm gear milling cutter for making grooved shafts with the circle using computers [8].

2. PROFILING OF THE SIDE OF THE TOOTH OF A WORM GEAR MILLING CUTTER FOR MAKING GROOVED SHAFTS

Worm gear milling cutters for making grooved shafts are cutters intended for processing grooves with a rectilinear profile (Figure 1). There are the ordinary straight and flank grooves (Figure 1 a) which are used for centering groove joints at the outer diameter, the straight and flank grooves with the channels (Figure 1 b) centering at the inner diameter and the sharp ones (Figure 1 c).

Basic characteristics of a grooved shaft are external (d_e) and inner (d_i) diameters, as well as width. For a sharp grooved shaft thickness (b) is variable, and in the drawing, its value is set in the middle (usually dividing) diameter.

[1]The paper will consider cutting tools, worm gear milling cutters to produce grooved shafts



a) b) c)

Figure 1. The grooves with the straight and flank profile

If the sides of the groove are in contact with the coupler element it is necessary to strictly maintain the straight line of the profile at a certain height. The lower limit of the active straight part of the groove is characterized by a diameter d_x. Channels on the straight and flank grooves are made during the exit of a brushed circle, if the sides are sharpened, or to enlarge the amount of rectilinear profile (impairment d_x).

Channel parameters are characterized by the value f of cylindrical sleeve and a diameter d_k. For processing such channels groove shafts are supplied by the special "outlets".

Determination of coordinates of profile of milling cutters

Consider coupling of the milling cutter and elements (Figure 2). Profile element is described in the coordinate system XOY, whose center (O) is located in the center of the element. The coordinate system of the milling cutter will be set as follows. Axis YO will coincide with the axis Y, and the axis XO with primary straight line of the milling cutter.

Profiling of an element point arises when the vertical on the profile of the milling cutter and the element at the moment of contact coincide. Geometric position of points of contact is called the coupling line (contact line – tangent), and a vertical point of intersection with the basic straight line is the pole of coupling (P).

The position 1 corresponds to the moment of profiling the element point which lies on the main cylinder. At some basic moment the starting position of point of an A element is defined by the polar coordinates r and φ and μ angle between the radius vector r and tangent line t in the observed point of an A element. During a turn from the starting position 2 to position 3 the contact point A moves for the angle ψ and coincides with a milling cutter molded point AO, which during that time moves over the straight line to position 3. Starting from the final position of the element and the milling cutter it is necessary to determine the coordinates of AO point at the initial moment. If we take into consideration that the point AO moved to distance A'A" (over the basic circle basic straight line moves without slipping), then the coordinates of the points of the milling cutter are determined as follows:

$$X_O = r \cdot \sin\sigma - R \cdot \psi, \; Y_O = r \cdot \cos\sigma - R \qquad (1)$$

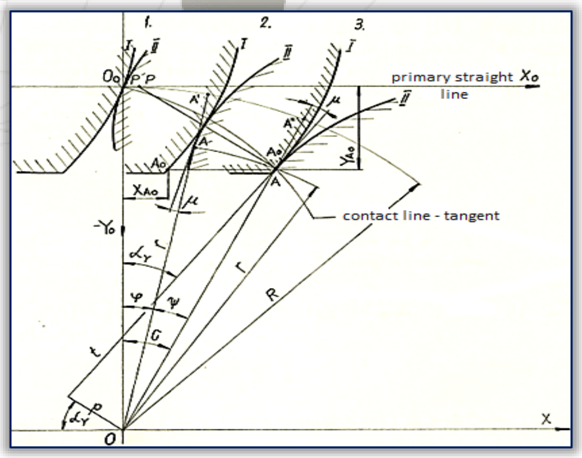


Figure 2. Scheme of profiling of the saw tooth of the element. I – profile tools; II – the profile element

where it is found that: σ – the angle of rotation of the observed point from the axis Y, ψ – the angle of rotation of the observed point from the basic position

The angle σ is determined as the difference between α-μ, where α – is the angle of the milling cutter profile:

$$\alpha = \arccos\frac{t}{R}. \qquad (2)$$

The length of tangent t in the observed point and the distance to it (p) shall be determined as:

$$t = r \cdot \cos\mu, \; p = r \cdot \sin\mu. \qquad (3)$$

In this way, by taking on the element a set of points and determining for each of them r, φ and μ it is possible to calculate the coordinates of the milling cutter profiles:

$$t = r \cdot \cos\mu,\ \alpha = \pm \arccos\frac{t}{R}, \sigma = \alpha - \mu,\ \psi = \sigma - \varphi,$$

$$X_O = r \cdot \sin\sigma - R \cdot \psi,\ Y_O = r \cdot \cos\sigma - R. \quad (4)$$

The angle μ has a "+" sign if the tangent in the observed point is rotated in a clockwise direction, i.e. in radius r direction (Figure 2). The angle αto the right side of the hollow between teeth is positive and negative to the left.

In the profiling of the element point which lies on the main cylinder (r = R), relations (4), are simplified:

$$\alpha = \mu,\ \sigma = 0, \psi = \varphi, \quad (5)$$

$$X_O = R \cdot \psi,\ Y_O = 0.$$

Equations (4) can be used to solve the so-called reverse task – determining the element profile based on the current milling cutter profile. The necessity of solving this task appears in the following cases:

≡ the milling cutter may have parts that are not profiled and that the designer empirically determined. These sections somewhat distort the theoretical element profile

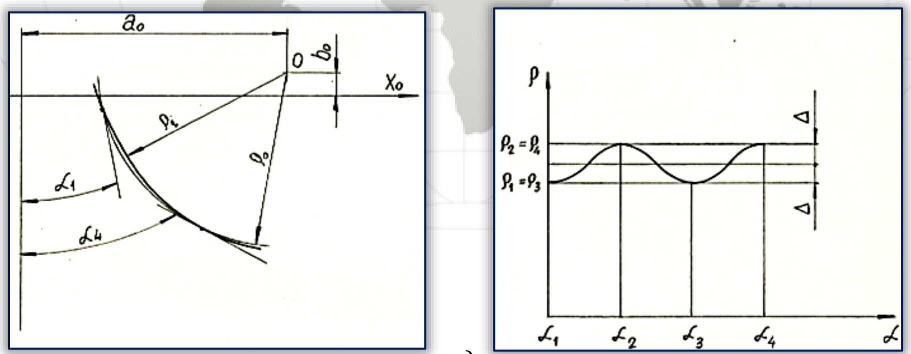
≡ curvilinear profile of flank of a tooth of the milling cutter is usually replaced by lines suitable for making (arc of circle), which also leads to defects in the item.

Approximation of flank of a tooth of the milling cutter profile by arc of circle

The profile of the flank of a tooth of a grooved milling cutter, as a rule, is replaced by circles. Consider the approximation of the profile of the flank of a tooth of the grooved milling cutter with a circle with the use of computers.

According to Chebyshev's theory, at the best approximation of the deviations Δ reach their maximum on the part of approximation n+2 times, where, n – is a curve by which approximation is done. Since the circle– is a second-order equation, error replacement reaches its peaks four times, consistently changing its character.

Figure 3 a shows the curve of the profile of a grooved milling cutter and the circle that replaces it.



a) b)

Figure 3. Approximation of profile of grooved milling cutter by circle
a) replacement scheme; b) determination of fault replacement

The distance from the center of the circle to any profile point is expressed as:

$$\rho_{i(\alpha)} = \rho_i = \sqrt{(X_{Oi} - a_O)^2 + (Y_{Oi} - b_O)^2}, \quad (6)$$

and the error is expressed as:

$$\Delta = \rho_i - \rho_O, \quad (7)$$

where ρ_o– is the radius of the circle that approximates profile, ρ_o radius should be chosen so that the error replacement is the smallest.

This requirement is fulfilled by the following relations:

$$\rho_1 = \rho_3,\ \rho_2 = \rho_4,\ \frac{d\rho}{d\alpha} = 0 \quad (8)$$

By fulfilling the requirements (8) we get

$$\rho_O = \frac{\rho_1 + \rho_4}{2},\ \Delta = \left|\frac{\rho_1 - \rho_4}{2}\right|. \quad (9)$$

If the error replacement, measured at the vertical on the milling cutters profile exceeds allowed

$$3\Delta < T_b, \quad (10)$$

the replacement by arcs of two circles so called two-radius profile presentation is performed.

In order to use equations (6), (7) and (8) we will change equations (4), reducing them to a parametric equation of the form:

$$X_O = f(\alpha),\ Y_O = f(\alpha). \tag{11}$$

Bearing in mind that:

$$\cos\alpha = \frac{r\cdot\cos\mu}{R}, p = r\cdot\sin\mu,\ -\mu = \varphi \tag{12}$$

it follows:

$$\begin{gathered} X_O(\alpha) = R\cdot\cos\alpha\cdot\sin\alpha - p\cdot\cos\alpha - R\cdot\alpha, \\ Y_O(\alpha) = R\cdot\cos^2\alpha + p\cdot\sin\alpha - R. \end{gathered} \tag{13}$$

By differentiating the equations (13) by α, the third equation of the system (8) we get in the form of:

$$\theta(\alpha) = (R\cdot\alpha + a_O)\cdot tg\alpha + b_O = 0. \tag{14}$$

Using the expression (6), the first two equations of the system (8) take the form:

$$\begin{gathered} (X_{O1} - a_O)^2 + (Y_{O1} - b_O)^2 = (X_{O3} - a_O)^2 + (Y_{O3} - b_O)^2, \\ (X_{O2} - a_O)^2 + (Y_{O2} - b_O)^2 = (X_{O4} - a_O)^2 + (Y_{O4} - b_O)^2 \end{gathered} \tag{15}$$

where X_{oi} and Y_{oi} are determined by the formula (13).

By solving the system of equations (15) using the method of determinants we get coordinates of the center of the circle:

$$\begin{gathered} a_O = \frac{1}{2} = \frac{(X_{O1}{}^2 + Y_{O1}{}^2 - X_{O3}{}^2 - Y_{O3}{}^2)\cdot(Y_{O2} - Y_{O4}) - (X_{O2}{}^2 + Y_{O2}{}^2 - X_{O4}{}^2 - Y_{O4}{}^2)\cdot(Y_{O1} - Y_{O3})}{(X_{O1} - X_{O3})\cdot(Y_{O2} - Y_{O4}) - (X_{O2} - X_{O4})\cdot(Y_{O1} - Y_{O3})} \\ b_O = \frac{1}{2} = \frac{(X_{O2}{}^2 + Y_{O2}{}^2 - X_{O4}{}^2 - Y_{O4}{}^2)\cdot(X_{O1} - X_{O3}) - (X_{O1}{}^2 + Y_{O1}{}^2 - X_{O3}{}^2 - Y_{O3}{}^2)\cdot(X_{O2} - X_{O4})}{(X_{O1} - X_{O3})\cdot(Y_{O2} - Y_{O4}) - (X_{O2} - X_{O4})\cdot(Y_{O1} - Y_{O3})} \end{gathered} \tag{16}$$

The values of α_1 and α_4 determine the lines of the profile and they are of constant size. The equation (14) gives two roots α_2 and α_4 (Figure 4).

Figure 5 shows a block diagram of the calculation of the circle that is closest to the methodology described above. It is necessary to bear in mind that the solution to the equations (14) and (15) is performed by the method of gradual approximation. Valuesα_1 and α_4 (block 2) are found as the lines of approximate curve, and they correspond to the height of the groove from the radius of the lower line of the straight part, Rx to the radius of the outer diameter. The angles α_2 and α_3 are changeable and their original values are close to the final ones. The coordinates of the four points of the profile and the coordinates of the center of the circle are being calculated. If the value a_o is different from the first one (in the first cycle a' is assumed to be zero) for the smaller value, 0.00001 (block 4), the solution is found. In the case of non-compliance with the conditions values$\alpha_2$and α_3 are taken as a starting point to determine their exact value by the formula (14).

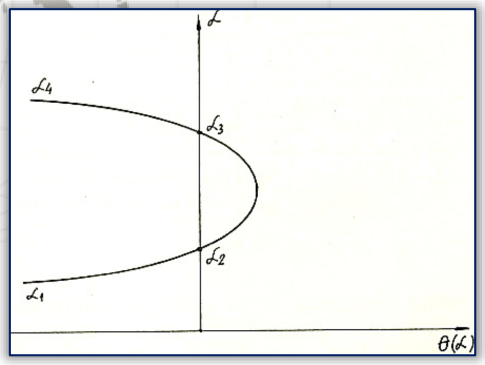


Figure 4. The graph of θ (α)

Finding roots of the equation (14) is carried out by the method of Newton. Applying some starting values $\alpha_k = \alpha_{ko}$, αk is calculated by the formula:

$$\alpha_{j+1} = \alpha_k - \frac{\theta(\alpha_k)}{\theta'(\alpha_k)}, \tag{17}$$

or:

$$\alpha_{j+1} = \alpha_k - \frac{(R\cdot\alpha_{k+a_O})\cdot tg\alpha_{k+b_O}}{R\cdot tg\alpha_k + (R\cdot\alpha_{k+a_O})\cdot(1 + tg^2\alpha_k)}, \tag{18}$$

as long as α_{k+1} (in the algorithm marked as α_k) and α_j (in the algorithm marked as α_{ko}) are not equal among themselves with accuracy up to 0.000001. If two roots are required, the procedures in the block 10-14 are repeated twice. After getting the new values, a new value for X_{oi}, Y_{oi}, a_o and b_o is calculated. Calculation by shown scheme is repeated until the fulfillment of conditions at block 4.

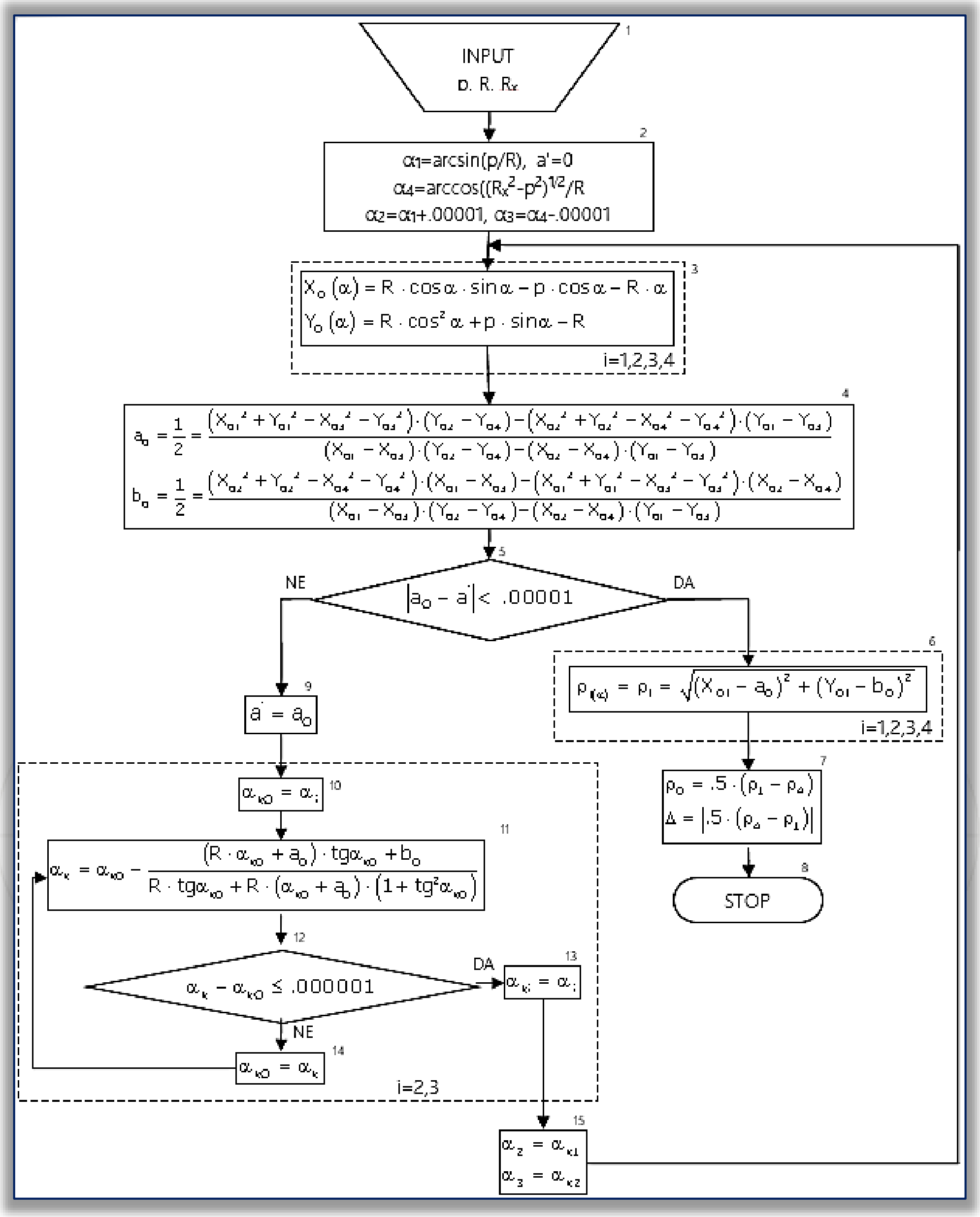
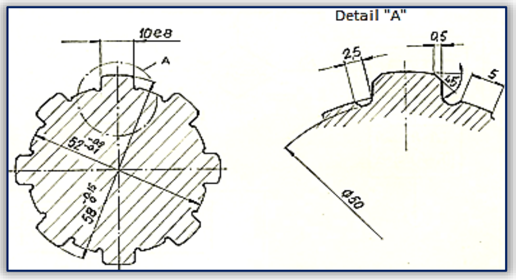


Figure 5. Flowchart (algorithm) of the calculation of the circle

3. EMPIRICAL TESTING OF COMPUTER MODELS

Example of the grooved shaft

To calculate and replace (approximate) theoretical profile of a flank of a tooth of worm gear milling cutter for making grooved shafts with a circle for the grooved shaft (with the following information) shown in Figure 6[2].



External diameter:	$d_e = 58^{0}_{-0,15}$ mm
Inner diameter:	$d_e = 52^{-0,2}_{-0,1}$ mm
Channel diameter:	$d_k = 50$ mm
Thickness of the groove:	$b = 10$ e8 mm
Number of grooves	$n = 8$
Material:	Č.0545

Figure 6. Example of the grooved shaft with channels

[2] Programming curvilinear approximation (theory) profile of tooth worm being a hob for making grooved shaft with a circle with the use of computers was made in the programming language TURBOBASIC

Comparison of the obtained results

The following values are calculated using Frafeld tables

X-coordinate of the circle center (a_0)

- Ordinate of the center of the circle (b_0)
- The radius of the circle(ρ_0)
- Replacement error of theoretical profile of the flank of a tooth using circle (Δ)

Table 1 shows the comparative values for the variables that are obtained using the present computer models and Frafeld tables, for the example of calculation of the worm gear milling cutter for making grooved shaft.

Table 1.Comparative values of obtained values for the example of calculating of the worm milling cutter for making grooved shaft

R. br.	Name of the calculated value	Mark	Value obtained by computer model [mm]	Value obtained by Frafeld tables [mm]
1	X-coordinate of the circle center	a_0	18,0933	12,7349
2	Ordinate of the center of the circle	b_0	2,8986	2,7379
3	The radius of the circle	ρ_0	13,4007	13,0259
4	Replacement error of theoretical profile of the flank of a tooth using circle	Δ	$5{,}0166 \cdot 10^{-3}$	$7{,}0425 \cdot 10^{-3}$

Analysis of results

From these results, it is most important to compare the error replacement of theoretical profile of the flank of a tooth using circle. Difference between the errors obtained expressed in percent:

$$K = \frac{\Delta f - \Delta r}{\Delta f} \cdot 100 = \frac{(7{,}0425 - 5{,}0166) \cdot 10^{-3}}{7{,}0425 \cdot 10^{-3}} \cdot 100 = 28{,}77\% \qquad (19)$$

4. CONCLUSION

Based on the analysis carried out for the example of calculation of a worm gear milling cutter for making a grooved shaft with the channels, it can be concluded that presented theoretical computer model of replacement of theoretical profile of the flank of a tooth of a milling cutter using the circle described in the paper in relation to the calculation made by using Frafeld tables provides a minor error replacement (approximation).

Calculations made using the described algorithm allow not only to increase the accuracy of the tools and elements, but in many cases rather than approximations with the theoretical curve or two arcs of the circle there is a possibility of replacing of the curve with the arc of a circle. This significantly simplifies and shortens the process of making the milling cutter itself and control equipment, as well as the economy of making the worm gear milling cutter.

References

[1] Vukelja, D. (1982). Construction of cutting tools. (Serbian language: Konstrukcija alata za obradu rezanjem), Beograd: Građevinska knjiga

[2] Frafeld, I. A. (1985). Calculation and construction of special cutting tools III (Serbian language: Proračun i konstrukcija specijalnog reznog alata III) (prevod). Pucarevo: MMK „Bratstvo“.

[3] Jašericin, I., Sinicin, B. I., Žpgalko, N. I., at. al. (1979). Basics of designing cutting tools with application (Russian: Osnovi projektovanija režušćih ionstrumentov s primeneniem EVM/P). Minsk. Višejaja škola.

[4] Jašericin, P. I., Eremenko, M. L., Žpgalko, N. I. (1981). Basics of material cutting and cutting tools (Russian: Osnovi rezanija materialov i režuših instrument). Minsk. Višejaja škola

[5] Marić, B. (1985). Calculation and construction cutters for making grooved shaft with the channels (Serbian language: Proračun i konstrukcija pužnih odvalnih glodala za izradu ožljebljenih vratila sa kanalima). Graduation work. Faculty of Mechanical Engineering Zenica

[6] Julikov, M. I., Gorbunov, B. I., Kolesov, N. V. (1987). Design and manufacture of cutting tools (Russian: Projektovanie i proizvodstvo režušego instrumenta. Moskva. Mašinostroenie

[7] Rikov, S. P., Shirtladze, A. G., Hrenov, V. L. (2008). Design of metal cutting tools (Russian: Projektovanije metalorežušego instrumenta). Tver: Tverskoj gosudarstvenij tehnićeskij universitet

[8] Dym, C. L., Ivey, E. S. (2006). Principles of mathematical modeling, 2nd edition, Academic Press, New York.