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SLIDING MODE CONTROL FOR DEPTH OF ANESTHESIA

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Abstract: This paper proposes a sliding mode control for depth of anesthesia. A robustly tuned controller brings the patient to a desired level of hypnosis. This is based on the standard pharmacokinetic/pharmaco-dynamic patient model that is described by two compartments. First, the drug enters the body in the central compartment, where the drug metabolism and elimination occurs. This compartment is connected with the peripheral compartment, with distinct equilibrating rates. The control system contains a sliding mode controller, an observer and two additional proportional integral control channels for the patient and for the reduced patient model. The main problem in the proposed control structure is how to preserve system stability in the presence of the exogenous disturbance. Introduction of proportional integral control channels is necessary to guarantee asymptotic stability of the control system and its robustness to the disturbances. The effectiveness of the proposed control is demonstrated by simulation on an example.
Keywords: anesthesia, hypnosis, control system, patient model, sliding mode

1. INTRODUCTION

Anesthesia represents a medical method, which with the use of anesthetics provides a sense of loss, and one can only involve loss of consciousness or just a loss of feeling pain or simultaneous loss of feeling pain and consciousness. It is used during surgery, diagnostic methods and treatment of complex conditions, which can be classified under the term "intensive care".

Today, thanks to the parallel development of surgery and anesthesia possible and most complex operations, which were previously only subject to the imagination. During surgery, advanced technology is used to monitor the patient's vital functions. Are monitored: heart rate, blood pressure, blood oxygen saturation, brain activity, the amount of carbon dioxide in exhaled air ... These parameters are dynamic (changing from minute to minute) and an anesthesiologist based on these parameters assessed and accurately titrate the amount of anesthetic and other drugs necessary for the depth of anesthesia and the stability of the patient during the operation.

Anesthetics are drugs, which act on the nervous system caused by anesthesia. For anesthetics is important to have minimal adverse effects on other organs, and after stopping, the function of the nervous system returns to normal. The paper discusses the application of propofol [1], which is intervenski anesthetic used for the induction and maintenance of hypnosis (loss of consciousness). Hypnosis, or depth of unconsciousness, is one of the goals of general anesthesia.

Measuring the level of hypnosis is achieved by monitoring the electroencephalogram records (EEG), which is determined by the depth of anesthesia. [2] As every drug, antistetik be with his useful therapeutic effect lead to serious consequences. The most common cause is disbilans between loaded and the necessary amount of anesthetic due to poor quality equipment for titration [3]. If ingested insufficient amount of anesthetic, the patient can remain conscious during surgery which can cause trauma and anxiety [4]. On the other hand if ingested excessive amounts of anesthetics can have long term effects on the patient, or at worst lethal outcome [5].

2. PATIENT MODEL FOR DEPTH OF ANESHESIA CONTROL

Like many drugs, the pharmacologic effects of anesthesia are traditionally modeled in two stages. The pharmaco-kinectics (PK) explain the distribution and metabolism of the drug, in other words, what the body does to the drug. The pharmacodynamics (PD) shows how the plasma drug concentration affects

to clinician effects; in other words, what the drug does to the body. [6] Figure 1 shows the components of the PK/PD model described in more detail below.

Compartment models are built on the same basic concepts as physiologic models, but with gross simplifications. The one compartment model contains a single volume and a single clearance. For anesthetic drugs, this model is not a completely satisfactory description of the plasma concentration versus time data. For these drugs one compartment [7] or two compartment [8,9] are often added to the PK model.

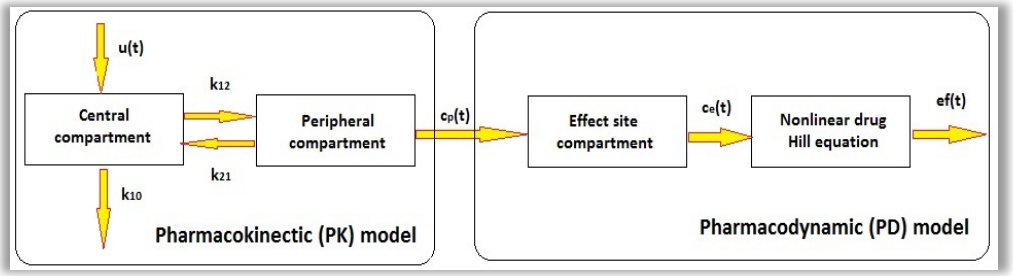

Figure 1. Patient model

The PK model is assumed to be described by two compartments represented in Figure 1. First, the drug enters the body in the central compartment (blood, liver and brain), where the drug metabolism and elimination occurs. This compartment is connected with the peripheral compartment, with distinct equilibrating rates. The PK model, that relates the drug infusion $u(t)$ [ml/h] with the drug plasma concentration $c_p(t)$ (μg/ml), can be described in the state-space form [10]:

$$\begin{bmatrix} \dot{c}_1(t) \\ \dot{c}_2(t) \end{bmatrix} = \begin{bmatrix} -(k_{10}+k_{12}) & k_{12} \\ k_{21} & -k_{21} \end{bmatrix} \begin{bmatrix} c_1(t) \\ c_2(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{V_1} \\ 0 \\ 0 \end{bmatrix} u(t) \tag{1}$$

$$c_p(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} c_1(t) \\ c_2(t) \end{bmatrix}$$

where:

- $c_i(t)$ (μg), with $i = 1,2$ is the concentration in the compartment i,
- k_{ij} (s^{-1}), with $i, j = 1,2$ is the equilibrium constant from the i-th to the j-th compartment
- V_1 is the volume of the central compartment
- k_{10} describes the reversible elimination or biotransformation of the drug.

The PD model is a linear single compartment version (the “effect" compartment) with nonlinear output mapping [6]. It was augmented by a first order system:

$$\dot{c}_e(t) = k_d\, c_p(t) - k_d\, c_e(t) \tag{2}$$

where:

- $c_e(t)$ is the effect site concentration,
- k_d is the rate constant between plasma and effect site.

The PK model (1) and the PD model (2), may be described as

$$\begin{bmatrix} \dot{c}_1(t) \\ \dot{c}_2(t) \\ \dot{c}_e(t) \end{bmatrix} = \begin{bmatrix} -(k_{10}+k_{12}) & k_{12} & 0 \\ k_{21} & -k_{12} & 0 \\ k_d & 0 & -k_d \end{bmatrix} \begin{bmatrix} c_1(t) \\ c_2(t) \\ c_e(t) \end{bmatrix} + \begin{bmatrix} 1/V_1 \\ 0 \\ 0 \\ 0 \end{bmatrix} u(t)$$

$$c_e(t) = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_1(t) \\ c_2(t) \\ c_e(t) \end{bmatrix}$$

The actual anesthetic effect $ef(t)$ is described by mapping through the saturating Hill equation [11, 12]:

$$ef(t) = \frac{c_e^{\gamma}(t)}{EC_{50}^{\gamma} + c_e^{\gamma}(t)} \tag{3}$$

where EC_{50}^{γ} is the 50% effect concentration and γ is the cooperatively coefficient. This normalized measure of effect E then ranges from $E = 1$ (awake) to $E = 0$ (deep coma). In the maintenance phase the system should be operating near $c_e(t) \approx EC_{50}$ where (3) is essentially linear. The output mapping can be further simplified to be linear:

$$ef(t) \approx \frac{\gamma}{2EC_{50}} c_e(t) \tag{4}$$

The transfer function of the patient model (PM) from $u(t)$ to $ef(t)$ is

$$W_{PM}(s) = \frac{ef(s)}{u(s)} = \frac{k_d}{2EC_{50}V_1} \frac{(s+k_{21})}{(s+p_1)(s+p_2)(s+k_d)} \tag{5}$$

where p_1 and p_2 are defined accordingly from k_{10}, k_{12} and k_{21} as

$$p_1 = \frac{1}{2}(k_{10} + k_{12} + k_{21} + \sqrt{(k_{10} + k_{12} + k_{21})^2 - 4k_{21}k_{12}}$$

$$p_1 = \frac{1}{2}(k_{10} + k_{12} + k_{21} - \sqrt{(k_{10} + k_{12} + k_{21})^2 - 4k_{21}k_{12}}$$

Between one and twelve years of age, these parameters are estimated as [6]

$$V_1 = 458.4 * weight, \quad k_{10} = 0.1527 * weight^{-0.3}, \quad k_{12} = 0.114, \quad k_{21} = 0.055$$

where $weight$ is measured in [kg]. We use

$$weight = 10, \quad T_d = 20s, \quad k_d = 40, \quad EC50 = 3, \quad k_d = 2.$$

Now, the PM transfer function is:

$$W_{PM}(s) = \frac{0.00145 \quad (s+0.055)}{(s+0095)(s+0.66)(s+2)} \tag{6}$$

3. CONTROL SYSTEM FOR DEPTH OF ANESTHESIA

The proposed control system structure is depicted in Figure 2. In the given scheme, reduced patient model without finite zero (RPM) provides estimation of canonical controllable coordinates of the patient model (PM), which are necessary for sliding mode control realization. Namely, under assumption that the outputs of the PM and the RPM are the same at every time moment, then its canonical state coordinates will be identical also. Taking into consideration that the PM parameters are liable to changes, and it is exposed to load action, with the aim to provide condition of equality of the mentioned outputs, new control channels must be introduced. These control channels is based on difference within the PM and the RPM outputs. The first additional control channel (CRPM) is directed to the RPM input, and makes conventional state observer structure [13]. The second control channel (CPM) is a new control which acts at the PM input. This control is not conventional. Its introduction may be necessary to guarantee asymptotic stability of the control system and its robustness to the disturbances.

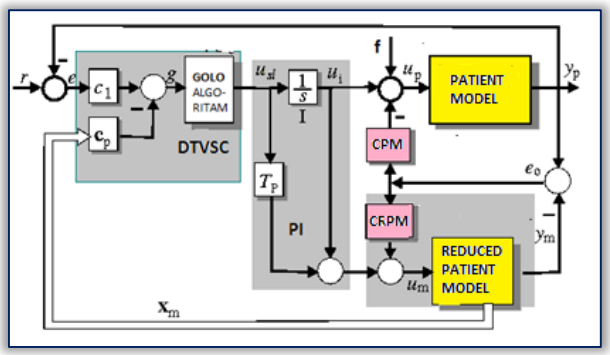

Figure 2. Control system for depth of anesthesia

The proposed system structure is based on the fact that the introduction of a proportional-integral (PI) action between the discrete-time variable structure controller (DTVSC) and the PM without finite zeros

does not violates sliding mode existence conditions established for the system without the introduced PI action [14]. The main problem in the proposed control structure is how to preserve system stability in the presence of the exogenous disturbance f(t).
The PM transfer function is a third order plant with stable finite stable zero:

$$W_{PM}(s) = \frac{0.00145 \quad (s + 0.055)}{s^3 + 2.6695s^2 + 1.34527s + 0.01254} \tag{7}$$

We introduce a model of reduced plant without finite stable zero of the form:

$$W_{PM}(s) = \frac{0.00145}{s^3 + 2.6695s^2 + 1.34527s + 0.01254}. \tag{8}$$

Continuous model reduced plant (8) in state space is:

$$\begin{bmatrix} \overline{x_1(t)} \\ \overline{x_2(t)} \\ \overline{x_3(t)} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -0.01254 & -1.34527 & -2.6695 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(y) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0.00145 \end{bmatrix} u_p(t) \tag{9}$$

According to the theorem on selection, was chosen sampling time $T = 0.1ms$. By applying the δ transformation for the selected sampling time, a discrete-time model of the system (9) becomes:

$$\delta x(k) = A_\delta x(k) + b_\delta u_p(k), \tag{10}$$

where:

$$A_\delta = \begin{bmatrix} 0 & 1 & 0.002 \\ 0 & 0 & 0.9996 \\ -0.012539 & -1.345269 & -2.66949 \end{bmatrix}, \quad b_\delta = \begin{bmatrix} 0 \\ 0.002 \\ 0.001449 \end{bmatrix}.$$

Let the sliding hyperplane defined by [15]:

$$g(k) = c_\delta x(k), \tag{11}$$

where

$$c_\delta = [c_1 \quad c_2 \quad c_3] P^{-1}, \tag{12}$$

$$c_i = \frac{1}{(i-1)!} \left. \frac{d^{i-1} \prod_{j=1}^{n} (\delta - \delta_i)}{d\delta^{i-1}} \right|_{\delta=0}, \quad \delta_i = \frac{e^{-\alpha_i T} - 1}{T}, \quad \alpha_i > 0, \quad i = 1,2,3$$

$$P = \left[b_\delta \quad b_\delta A_\delta \quad b_\delta A_\delta^2 \right] \begin{bmatrix} \overline{a_1} & \overline{a_2} & 1 \\ \overline{a_2} & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$\overline{a_i}$ - coefficients of the characteristic polynomial $det(zI - A_\delta) = z^n + \overline{a_2} z^2 + \overline{a_1} z + \overline{a_0}$.

If we choose $\alpha_1 = 2$ and $\alpha_2 = 2$, the elements of the vector c_δ (12) become:

$$c_\delta = [-0.2008 \quad -0.4008 \quad -0.09996].$$

For the discrete-time model of reduced plant (10) can be synthesized control in the form:

$$u_{sl}(k) = -c_\delta A_\delta x(k) - \min\left(\frac{|g(k)|}{T}, \alpha + \beta |g(k)|\right) \operatorname{sgn}(g(k)). \tag{13}$$

where: α, β - real numbers such that $0 \le \beta T < 1$ and $\alpha > 0$.

Let $\alpha = \beta = 50$. Control (13) is:

$$u(k) = -0.2008 x_2(k) - 0.20048 x_3(k) + \min(2500 |g(k)|, 50 + 20 |g(k)|)$$

and the sliding hyperplane (11) is:

$$g(k) = -0.2008 e(k) + 0.40008 x_3(k) + 0.09996 x_1(k)$$

The CRPM is usually determined in accordance to obtain observer dynamics faster that the plant dynamics. Further, for the avoidance of the observer unsensitivity to the slow varying disturbances, the CRPM must have a PI structure. Based on the given assumption only CPM structure and parameters

must be determined. It may be recommended, if the stability and desired dynamics are preserved, to choose CRPM and CPM identical:

$$W_{CPM}(z) = W_{CRPM}(z) = 200\frac{2z-1}{z}$$

Simulation results are presented in the form of a diagram step response of the plant (Figure 3 and Figure 4). The diagrams illustrate the constant infusion rate required at an exponentially decreasing control signal to maintain a steady state blood concentration of propofol.

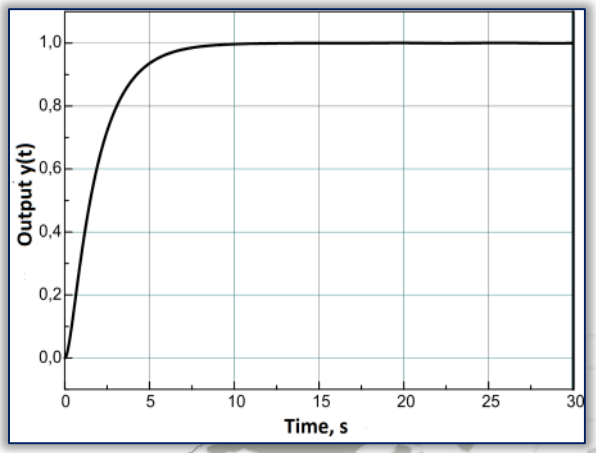

Figure 3. Blood concentration of propofol

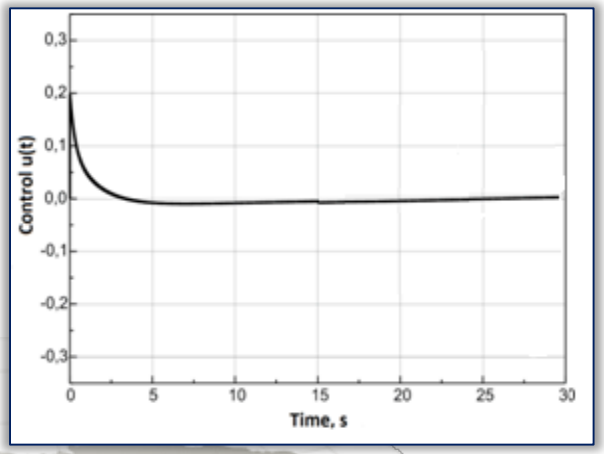

Figure 4. Infusion rate

4. CONCLUSIONS

We provided a two dimensional PK/PD model of the effect of propofol infusion on depth of hypnosis during the anesthesia maintenance phase of a surgery. For sliding mode control system suggests a combination of variable structure control law with flexible working regimes of linear control law and proportional-integral type for a high quality control system robust to changing the effect of external disturbances. Finally, computer simulation results are shown the proposed syste

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