[1.]Germán Valencia GARCÍA, [2.]Ştefan ŢĂLU

TRUE LENGTH AND SLOPE OF AN OBLIQUE LINE IN A 3-D CARTESIAN COORDINATE SYSTEM, BY MEANS OF DIFFERENTIAL RECTANGULAR DISPLACEMENTS

[1.] Universidad del Valle, School of Architecture, Department Projects, Sede Meléndez, Cali, COLOMBIA
[2.] Technical University of Cluj-Napoca, Faculty of Mechanical Engineering, Department of Automotive Engineering & Transportation, The Directorate of Research, Development and Innovation Management, Cluj-Napoca, ROMANIA

Abstract: The determination and projection of an oblique line by means of the Cartesian coordinates (X, Y, Z) of its ends, without the straight line passing by the origin (0,0,0), or defining the same straight line by means of the width, height, and depth of its ends, is a major question to find the true lenght of the straight line as well as the true angle of inclination in relation to the horizontal support plane [H] or isoplane XY. In descriptive geometry it is solved with an auxiliary elevation view, where the straight line and the angle of inclination are projected in true magnitudes. In Analytical Geometry we use the property of the directing cosines of the vector that passes through the Cartesian origin; and in the area of Calculus we use the application of a directional vector, which must be parallel to the straight line that contains the two points given with parametric equations. In short, there is no explicit formula for the straight line in a 3-D Cartesian coordinate system. The fundamental purpose of this study is to implement a methodology to find the true length of the oblique line as well as its slope in relation to the XY isoplane by the joint use of the differential rectangular displacements or delta (Δ) projected on the three Cartesian axes, which allow the design of new simple mathematical formulas based on the Pythagorean theorem and basic trigonometric functions.
Keywords: True length and slope of a 3-D straight line, 3-D Cartesian coordinate system, Rectangular differential displacements, Pythagorean triangle

1. INTRODUCTION

In Descriptive Geometry, it is frequently analyzed and studied the geometric and spatial relations, together with the theory of dihedral projection, to solve projective problems of various geometric objects; the user must apply the appropriate norms and theorems during the process of obtaining the auxiliary orthogonal views, until finding a graphical solution represented in a two-dimensional (2-D) plane or paper space [1].

For the representation of spatial objects, the creation of a broad variety of geometric shapes, the simple acts of abstracting, setting out, and analyzing a particular geometrical problem are part of an enriching experience to increase the level of imagination [2-6]. Undoubtedly, imagination and cogitation in the presence of a geometric problem allow us to establish conjectures or interesting experiments to expand new horizons of knowledge and research, with strong bonds with the sciences related to design, measurement, and construction, such as: Mathematics, Architecture, Engineering, Design, etc. [7-10].

AutoCAD [11-14] and similar software for drawing [15-17], give a high level of imagination on the development of the ability to create new three-dimensional (3-D) shapes and structures, facilitates the complex tasks of drawing auxiliary projections, constructive processes, and elaboration of dynamic virtual models.

On the other hand, the process of dimensioning and of reading Cartesian coordinates is important to corroborate data of linear and angular magnitudes, which can be incorporated in mathematical and geometric formulas. These true magnitudes are used for the input of parametric data in the 3D Studio Max program [18], a powerful program that allows creating objects, to modify them, to map them, and to move them in real time; when incorporating a camera in the 3-D scene, it is possible to obtain renderings in image or video format. This dynamic digital experience allows the user-researcher to check the demonstrations and constructive processes that have been carried out and, consequently, may come to formulate conjectures or new ideas that can be implemented in future investigations.

This study focuses on one of the most representative problems presented by an oblique line in a 3-D Cartesian coordinate system, which consists in determining the true length and slope in relation to the horizontal support plane, by using the differential rectangular displacements of their Cartesian coordinates, and establishing a connection with mathematics and trigonometry by designing novel simple formulas.

2. METHODOLOGICAL PROPOSAL

When determining an oblique line in a three-dimensional Cartesian coordinate system by means of the (X, Y, Z) coordinates of its ends, there can be infinite positions of the line in space in relation to the XY, YZ, and XZ isoplanes. The properties of true lenght and slope of the line in relation to the plane of support XY, have been analyzed for a long time in areas of specialized mathematics and geometry; to date there is no explicit formula of the line in space.

In the area of Calculus, a straight line in general assumes the name of directional vector which must be anchored to two other vectors that start from the Cartesian origin to the given ends of the line, then they are added and multiplied by a scalar λ (parameter of the real numbers) [19-21]. In this way, a system of parametric vector and Cartesian equations of the line is obtained (figure 1).

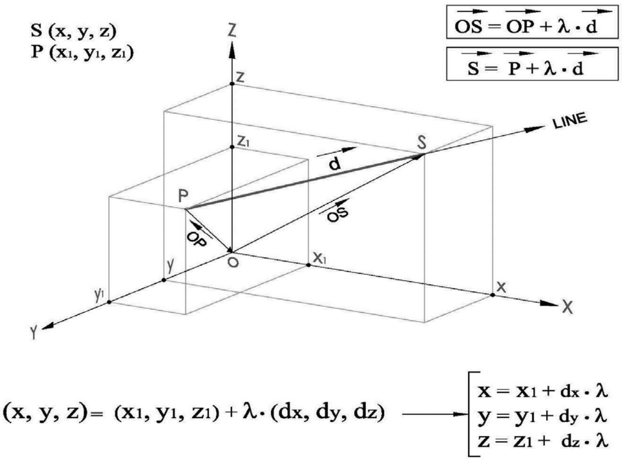

Figure 1. Parametric and Cartesian vector equations of the PS line.

In the area of Analytical Geometry, the straight line is considered as a vector A, which starts from the origin (0,0,0) and must form three angles in relation to the Cartesian axes [22]; this concept is similar to the diagonal of a parallelepiped, where the property: "The sum of the directing square cosines equals unity", is fulfilled (figure 2).

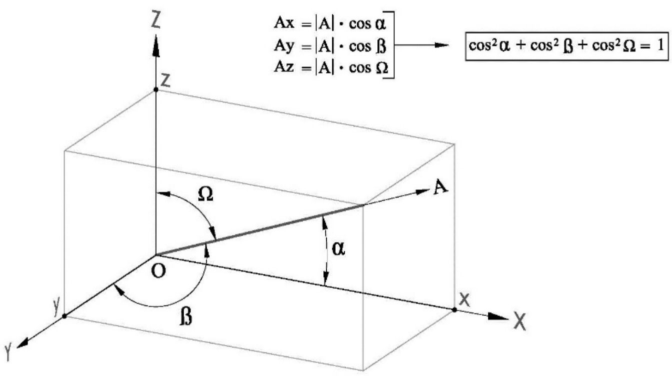

Figure 2. Property of the square-directional cosines of a vector A

In descriptive geometry, Cartesian isoplanes are analogous to a dihedral projection system, and therefore the straight line can be classified into 7 special positions called "Special lines", this is due to the geometrical properties of parallelism and perpendicularity that can be presented in relation to the main planes of dihedral projection [1].

In all these special lines, there are differential rectangular displacements in Cartesian axes called "Delta" (ΔX, ΔY, and ΔZ), which vary from zero "0" to infinite magnitudes; in this way, to each special line that corresponds its characteristic delta.

The oblique line, being neither parallel nor perpendicular to the principal planes of dihedral projection (H, L, and P), will always present delta in its Cartesian axes, and consequently it is not possible to visualize its true length and slope (β) in relation to the isoplane X, Y or horizontal plane (H) (Table 1 and figure 3).

Table 1. Differential rectangular displacements (Δ) of the special lines

Special lines		Differential rectangular displacements			True length of the straight line ISOPLANE	Real angle of inclination (β)
1	horizontal line	Δ X	Δ Y	0	XY(H)	0º
2	front line	Δ X	0	Δ Z	XZ(V)	0º < β < 90º
3	side view line	0	Δ Y	Δ Z	YZ(L)	0º < β < 90º
4	vertical line	0	0	Δ Z	XZ(V) &YZ(L)	90º
5	tip line	0	Δ Y	0	XY(H) &YZ(L)	0º
6	lateral line	Δ X	0	0	XY(H) &YZ(V)	0º
7	oblique line	Δ X	Δ Y	Δ Z	Ø	Ø

In order to determine the true lenght and the real slope of the oblique line (β), it is solved with an auxiliary orthogonal elevation view to the straight line, so that its real magnitudes are projected. It is necessary to construct an auxiliary projection plane which is parallel to the straight line, preferably adjacent to the horizontal view, to obtain both magnitudes requested at the same time; in this way, the auxiliary view is called the "elevation view", which is characterized by the presence of the ground line (G.L.), and the projection of heights or dimensions in true length (figure 4).

With the delta established in the Cartesian axes, it is always evidenced the projection of rectangular triangles in their corresponding isoplanes, and consequently, the Theorem Valencia & Ţălu is fulfilled. "The projected cathetus in the Cartesian isoplanes of an oblique line will always be parallel to the differential rectangular displacements."

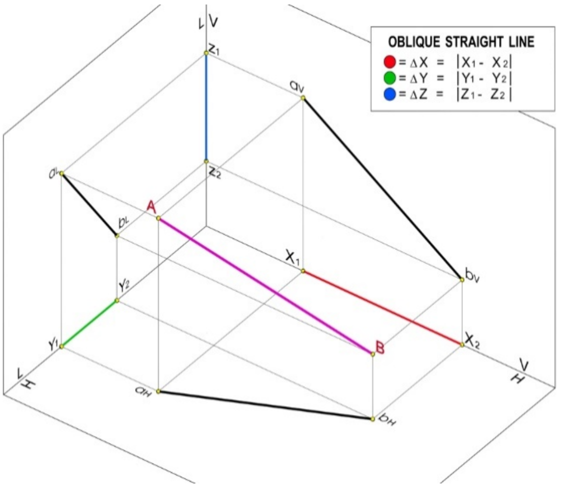

Figure 3. Differential rectangular displacements of an oblique line AB

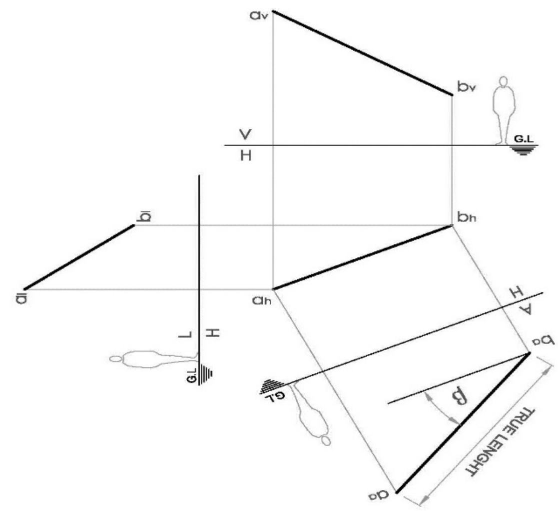

Figure 4. True lenght, and real slope angle of the line AB

The most significant right triangle is the one projected in the horizontal plane, since its hypotenuse always coincides with the strike of the line in relation to the Y axis (North-South Line); in order for this to be true, the projected opposite side CO should always be assumed to be parallel to ΔX, and the adjacent side CA should be parallel to ΔY, in order to obtain the strike of the straight line located in space, by applying the tangent function of the angle θ of the resulting quotient of its sides, figure 5.

According to the paragraph described above, the next step is to apply the Pythagorean theorem in the right triangle projected in the horizontal plane H, to find the value of real hypotenuse of the straight line AB (Topographic distance AB).

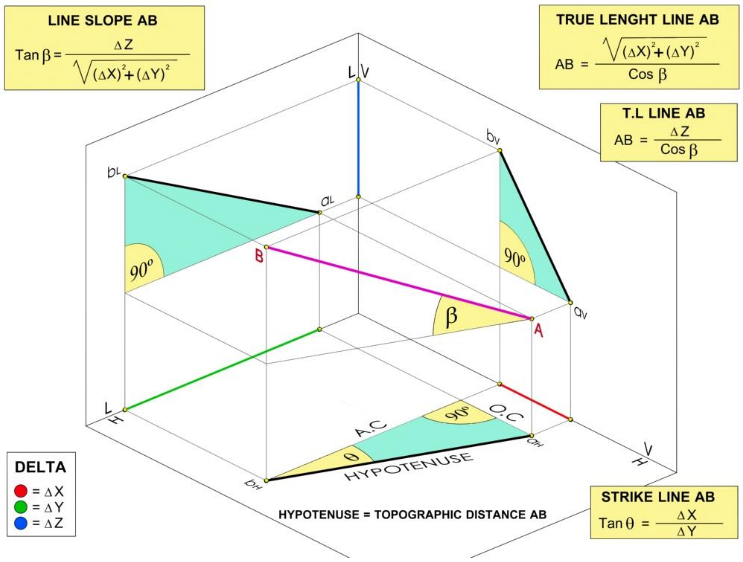

Figure 5. Utility of the right triangles of a straight line projected in the Cartesian isoplanes

$$\text{Hypotenuse} = \sqrt{(\Delta X)^2 + (\Delta Y)^2} \tag{1}$$

Knowing ΔZ, we can proceed to determine the angle (β) of real slope in relation to the isoplane XY with the tangent function.

$$\text{LINE SLOPE AB} = \text{Tan } \beta = \frac{\Delta Z}{\sqrt{(\Delta X)^2 + (\Delta Y)^2}} \tag{2}$$

With the found value of the angle β, we proceed to determine the true magnitude of the straight line AB, and the strike with the following formulae:

$$\text{TRUE LENGTH LINE AB} = \frac{\sqrt{(\Delta X)^2 + (\Delta Y)^2}}{\cos\beta} \tag{3}$$

$$\text{TRUE LENGTH LINE AB} = \frac{\Delta Z}{\cos\beta} \tag{4}$$

$$\text{STRIKE LINE AB, Tan}\theta = \frac{\Delta X}{\Delta Y} \tag{5}$$

With the above formulae, it is intended to leave a legacy with good bases for future applications in Mathematics, Analytical Geometry, Trigonometry, Calculus, Physics etc.

3. CONCLUSIONS

According to the methodology described in this document, the proposed mathematical formulae are correct and can be applied in several areas of specialized study, where the vectors located in the 3-D Cartesian coordinate system are analyzed.

The use of differential rectangular displacements allows to decompose a vector located in space in right triangles projected in the Cartesian isoplanes, to which, trigonometric functions based on the Pythagorean Theorem are applied.

The formulae proposed in the previous chapter facilitate the current processes of complex calculus to determine the true length and real slope of a straight line located in a 3-D Cartesian coordinate system.

The methodology and simplicity of the formulas described in this document could be implemented as an educational resource in the curricula of secondary, university and specialized studies for spatial comprehension and analysis of vectors located in space.

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