[1.]Atul BHASKAR, [1.]Enrique CUAN-URQUIZO, [1.]Alessandra BONFANTI, [1.]Hayk VASILYAN, [1.]Tigran SAGHATELYAN, [1.]Loris DOMENICALE, [2.]S.J.A. RIZVI, [1.]Naresh BHATNAGAR

POROUS AND LATTICE MATERIALS: MECHANICS & MANUFACTURE

[1.] University of Southampton, UNITED KINGDOM
[2.] Indian Institute of Technology, New Delhi, INDIA

Abstract: We consider a host of regular lattice architectures and present analytical and computational approaches to derive the structure-property relationship for such structured material by exploiting the translational periodicity of infinite lattices. Two specific geometries – the so-called hexagonal honeycomb and the woodpile lattice – are studied analytically and computationally. The elasto-plastic response in the case of the first, and the bending response of lattice beams for the second, is considered. These specific problems have been motivated by biostructures relevant to medical implants and scaffolds. We also present novel methods to additively manufacture such lattices. When possible, the response is obtained as an analytical function of the microstructural parameters described by the geometry of the repetitive elements of the lattice, such as characteristic diameter, length, or thickness. Alternative methods of manufacturing materials with random internal architectures are also presented. The relative strengths and weaknesses of the two classes of materials with respect to analysis and manufacture are discussed.
Keywords: Lattice materials & structures, scaffolds & implants, additive manufacturing

1. INTRODUCTION

Material with regular or irregular internal architecture is frequently found in nature and in engineering. Multiple scales of microstructure enable the material to possess properties that are not otherwise observed naturally. This ability to engineer properties, by cleverly designing the material arrangement, has led to exciting developments in the area of mechanics of structured materials and also manufacturing of this class of materials. The possibility of being able to synthesize a material with unusual properties, by placing material in interesting ways, is further strengthened by recent developments in additive manufacturing. A host of applications spanning lightweight material for aerospace engineering, biomedical implants & scaffolds, elastic & acoustic meta-materials are currently driving theoretical and experimental research to realise a new class of materials and structures, never exploited to their full potential before. The present work is brings together developments in this area of mechanics, materials, design and manufacturing.

Practical structured materials often have complex architecture such as those found in the repetitive design of cardiovascular stents, tissue engineering scaffolds and craniofacial implants. While theoretically assessing their response for a given geometry, we may face difficulty posed by the complexity of geometry becoming prohibitive for analytical treatment. One resorts to computational approaches such as the finite element analysis in such situations. A computational approach is satisfactory, and often desired, when one has already committed to a lattice design; however, while searching for interesting design alternatives, analytical information proves to be invaluable because it possesses sensitivity information with respect to certain critical design parameters such as cell wall thickness, filament diameter, etc., or material properties such as the Young's modulus, yield stress, etc. With this as the main motivation, here we present analyses for two idealised geometries that possess generic features common to some of the architectures frequently found in the applications mentioned above.

The first lattice geometry considered here is the well-known two dimensional hexagonal honeycomb. Elastic response of such lattices has been studied for a while now. Here we present recent development covering the apparent elasto-plastic response of such lattice material. The second class of lattice material is the so-called woodpile arrangement that possesses a 3-dimensional geometry. Such woodpiles are frequently used in tissue engineering scaffolds. Despite the absence of a clear two-dimensionality, we present a class of important problems where simple mechanics enable us to establish the structure-property relationships analytically.

The present paper is organized as follows. The apparent elasto-plastic response of honeycombs and linear elastic response of woodpile structures are presented in section 2 where controlled additive manufacture of woodpile lattices is also discussed. Further sub-sections within Section 2 include alternative approaches to manufacture porous materials such as via injection moulding that makes use of cycles of expansion and compression to facilitate cellular morphology. Conclusions are presented in section 3.

2. MATERIAL WITH TRANSLATIONAL PERIODICITY: APPARENT PROPERTIES & MANUFACTURE

The apparent response of structured materials is of great practical interest because it enables us to homogenize material with complex internal architecture by replacing such heterogeneous matter by equivalent continuum in theoretical and computational analyses. There is a significant body of work relating linear elastic response of honeycombs [1], thus providing the apparent elastic properties of such material. However, attempts to assess non-linear apparent response are far and few between. This is especially true of non-linearity due to elasto-plastic constitutive relationships where such studies have been limited to determining the remote collapse stress. This is considered in section 2.1. Additive manufacture of woodpile lattices is considered in Section 2.2 using fused deposition modelling. Experimentally realized structures with controlled porosity are presented and their internal architecture investigated using scanning electron microscopy.

Porosity is a desired morphological feature for many biomedical scaffold applications where regularity of the geometry is unimportant. Alternative approaches using foaming by means of injection molding is presented in Section 2.3. Following non-linear response of planar lattice in Section 2.1, we consider lattices in woodpile arrangement and their linear elastic response. The geometry has translational symmetry, but the periodicity is not within a plane. We present bending and shear deformation of slender beams made of such lattice material in section 2.4.

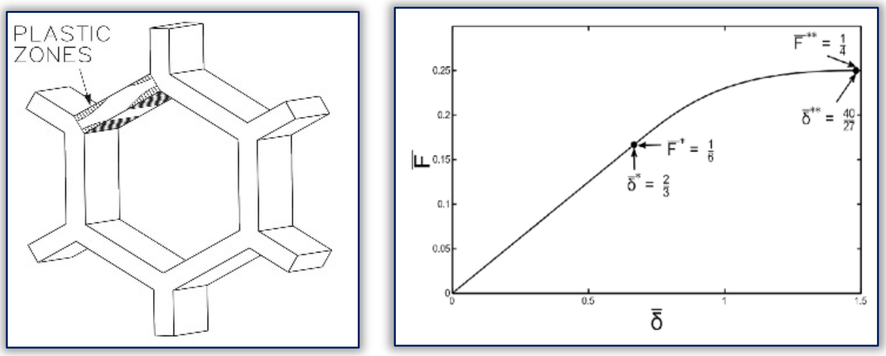


Figure 1. A hexagonal cell from an infinite honecomb showing details of the plastic zones formed during elasto-plastic deformation of a remotely loaded honeycomb sheet (left), a "master-curve" for response of such lattices (right [4]).

Elastoplastic response of honeycombs

Consider an infinite planar hexagonal honeycomb lattice whose typical hexagonal cell is shown in figure 1 (left). Note that such a "cell" is not a unit cell in the crystallographic sense—a unit cell must not overlap on to itself when translated within the plane, which the hexagonal "cell" does.

The essential simplification in the elasto-plastic analysis of such cellular structures arises from the translational symmetry. Additionally, when remote stress is applied horizontally, only cell walls that are inclined respond to this stress (primarily in flexure) and all the cell walls running vertically remain undeformed. This enables us to analytically calculate response during different phases of deformation – elastic through to non-linear elasto-plastic and finally collapse. Details of this approach can be found in [2], [3], and [4] for a range of problems in this class involving different constitutive relationships (e.g. elastic-perfectly-plastic, one with non-linear strain hardening, etc.). A response curve normalised on the basis of scaling arguments incorporating dimensional homogeneity as well as physical arguments (see [2], [3] for details) is presented in Figure 1 (right). The curve could be treated as a master-curve for all response evaluations concerning the elasto-plastic response of hexagonal honeycomb lattices. While axial as well as bending deformation within cell walls were included in the analysis, computational experiments confirmed that ignoring axial deformation within cell walls is adequate for honeycombs with thin cell walls under elasto-plastic deformation.

Additive manufacture of woodpile lattices and scaffolds

Manufacture of biomedical scaffolds is of great current interest. One of the requirements for such structures is the porosity to allow cells to attach, proliferate and grow. Lattice materials, such as those that possess woodpile arrangement, naturally provide such voids. Such architectures are most readily attained by the use of additive manufacturing. In the present work, we report controlled manufacture of lattice structures using fused deposition modelling. The spacing of the filaments is controlled by generating the machine instructions in the form of G-codes, which, in turn, were generated by our own computer codes. The process of controlled layer-wise manufacture is shown schematically in Figure 2. The spacing between scan lines in the *x-y* plane can be controlled (Figure 2, center) and an SEM image of the resulting lattice structure is shown in the figure on the right of Figure 2. The adhesion between layers is during the cooling period of the filaments in the fused state and their extent depends on the process parameters such as the temperature of the nozzle and also the cooling rate. The out-of-plane strength and the stiffness (especially in shear) are expected to strongly depend on the overlap between the filaments.

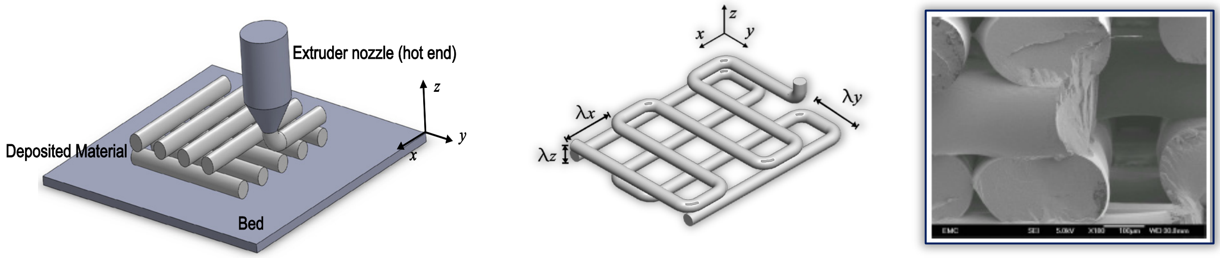


Figure 2. Additive manufacture of porous lattices via a controlled fused deposition modelling process. Schematic diagram of the process (left), control of porosity via lattice spacing (center), SEM micrograph of the manufactured lattice (right).

Controlled manufacture of lattices enables us to realize material with unusual properties. One such sheet of material is shown in Figure 3. The first (Figure 3, left) is a hexagonal honeycomb manufactured using 3D printing. The apparent Poisson's ratio of regular hexagonal lattice in the plane is known to be +1 [1]. This is confirmed by the out-of-plane bending upon the application of moments at the opposite edges, which exhibits an *anticlastic curvature*. As opposed to this, the central sub-figure (Figure 3) shows a *synclastic curvature* upon the application of similar edge moment, which indicated negative apparent Poisson's ratio. A synclastic curvature is desirable in many applications such as skin tissue engineering or cardiac patches.

Regular structures are often not necessary for applications such as tissue engineering scaffolds as the morphology of cells with tissues is hierarchical but not perfectly repetitive. Motivated by this, research is being pursued to create random architecture in a controlled way. This is also achievable by the use of additive manufacturing such as fused deposition modelling by using an appropriate algorithm to control the architecture of the porous structures. Such a random material, 3D-printed by us, is shown in Figure 3 (right).

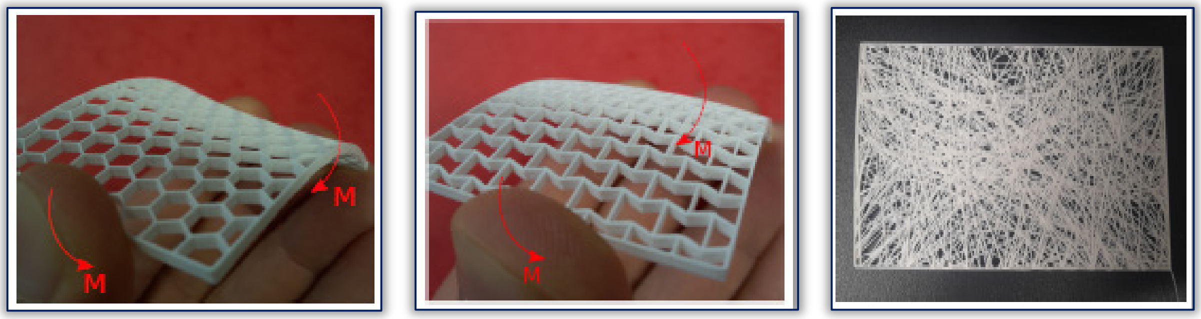


Figure 3. Additively manufactured sheet material. Hexagonal honeycomb (left), auxetic sheet (center), random porous structure (right).

Injection moulding with expandable cavity mould

An alternative approach of manufacturing porous scaffolds is to encourage foaming during the manufacturing process. One such approach is that of injection moulding with the use of expandable cavity mould. In this process, a bespoke arrangement for expandable cavity mould was set up. The basic idea is to charge a polymer nitrogen gas mixture in the molten state and expand the volume of the cavity so that a pressure drop in the polymer melt leads to the formation of bubbles. The size, morphology and the distribution of the bubbles can be controlled by process parameters that characterize injection moulding.

A schematic diagram of the stages of foaming is shown in Figure 4 (top left). The four stages of void formation show cell nucleation, primary expansion followed by melt compression and secondary rapid expansion. The effect of expansion rate on the morphology of expandable cavity moulded propylene is shown in the group of four micrographs in Figure 4 (top right). An advantage of porous material manufactured using such a process is the scalability and speed of processing as opposed to additive manufacturing, which is relatively slow but offers the advantage of changing the architecture with relative ease.

The effect of stroke of the expansion was studied (the group of eight micrographs, Figure 4, bottom left). It was found that cell density tends to increase with stroke first then it falls. A high-resolution micrograph (Figure 4, bottom right) shows the effect of expansion-compression-expansion cycle on the cell morphology.

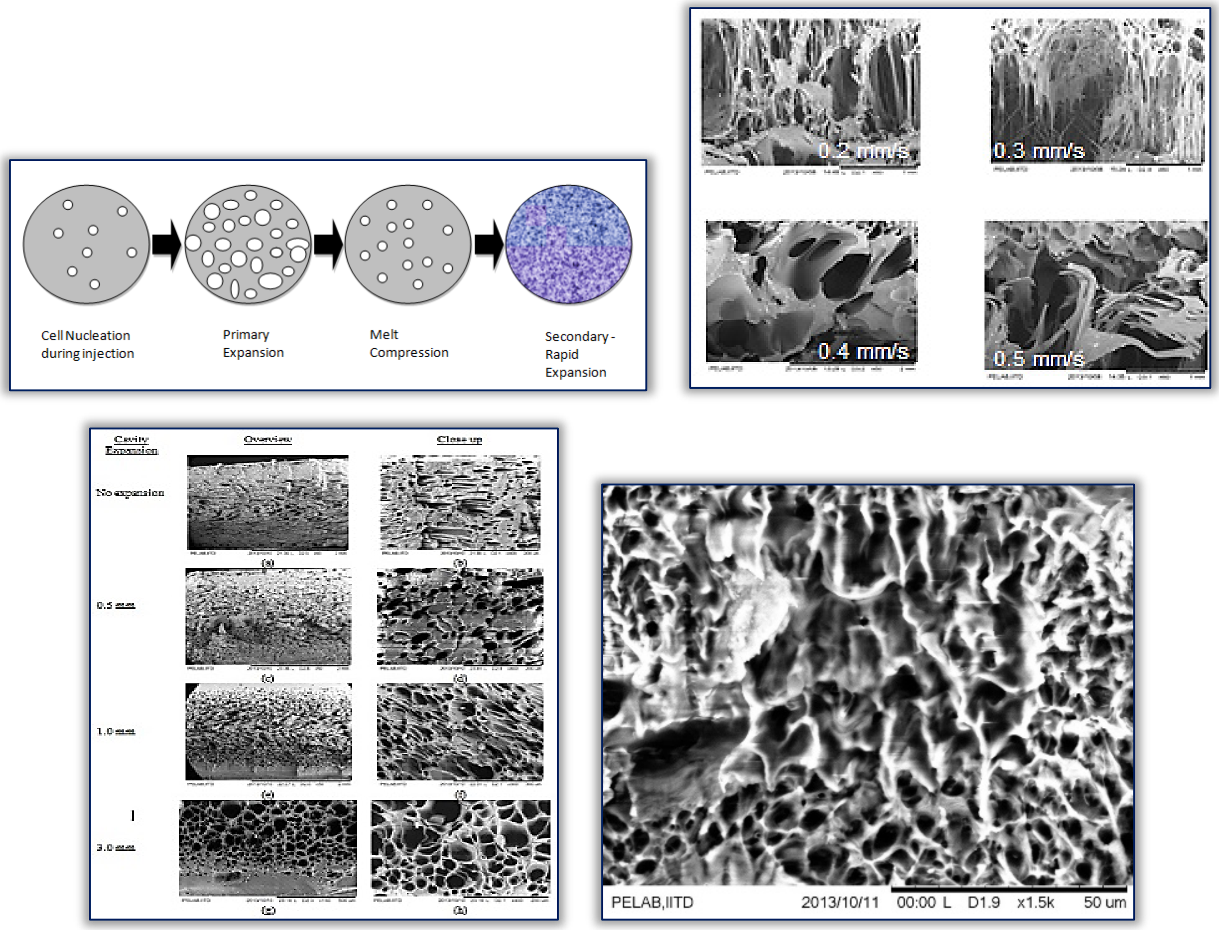


Figure 4. Porous material manufactured via expandable cavity moulding. Schematic of the process (top left), the effect of expansion rate on cell morphology of propylene foam (group of four micrographs, top right), the effect of expansion stroke on cell morphology of PLA foam (group of eight micrographs, bottom right), high-resolution detail (bottom right).

Mechanics of woodpile lattice beams

The structure-property relationship of material in woodpile arrangement is of great current interest because of its relevance to tissue engineering and biomedical scaffolds [7]. Consider the bending of thin beams made of woodpile lattice material. When such a lattice beam bends laterally in the *x-z* plane, filaments running parallel to the *x*-axis are in tension and compression as well as in bending. Filaments farther from the neutral axis of the equivalent homogeneous beam of the same shape as that of the lattice beam are primarily in tension and compression whereas those nearer the neutral axis dominantly in bending. This state of stress within each filament is set up running linearly over its cross section but with a net axial force. This stress distribution within the filaments can be used to develop an analytical model incorporating the filament micromechanics while treating each filament as an Euler-Bernoulli beam with an axial loading superposed on it. The result is an analytical expression for the apparent bending stiffness involving the apparent second moment of the cross-section of the lattice beam, given by

$$\langle I \rangle = 8\pi r^4 N_y \left(\frac{1}{6} N_z^3 - \frac{13}{96} N_z \right),$$

where r is the radius of the filament, N_y & N_z are the number of layers in the *y* and *z* stacking directions respectively. Further refinement could be made for more detailed modelling for the filament micromechanics, e.g. by including shear deformation within them, especially important when the spacing between the filaments is moderately small and/or filaments may be relatively thick [8].

Unusual properties and architectures could potentially be achieved by programming the tool path as well as controlling the porosity as afforded by additive manufacturing. While the analysis above was restricted to uniform lattices, functionally graded lattices could also be manufactured (see Figure 5, top left) – the property variation along the length is manifested by the asymmetric bending, which is visually apparent. Similarly, flat films (figure 5, bottom left), as well as those with inherent curvatures, could also be additively manufactured.

The "lattice", over a surface that possesses anticlastic curvature, was 3D-printed by programming the tool path in all the three directions of the printing nozzle (Figure 5, bottom right). Such precise control of porosity has opened exciting new opportunities in the field of elastic meta-materials.

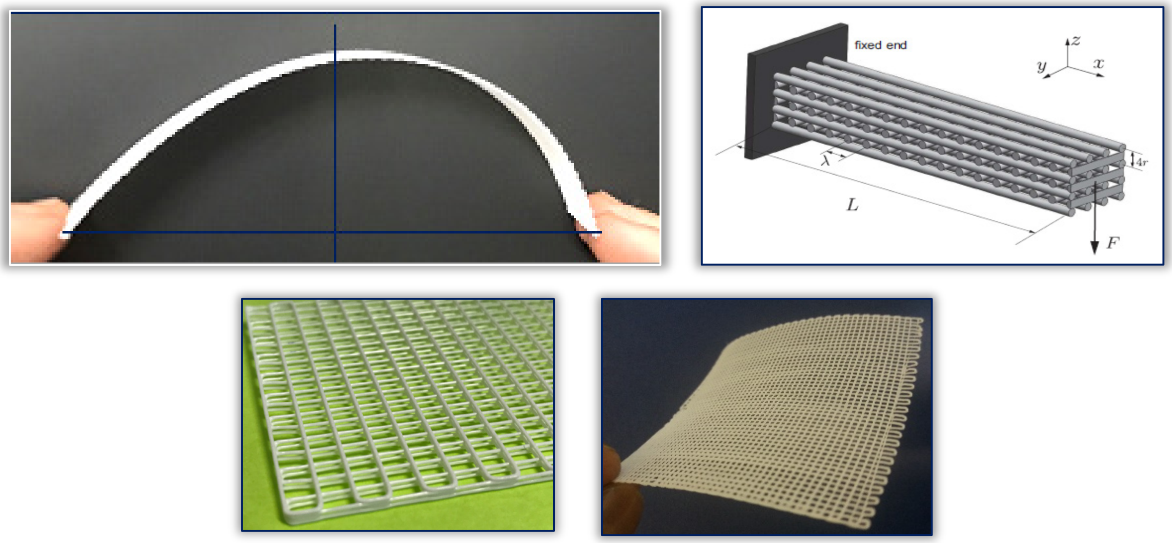


Figure 5. Porous and film-like material. Uniform woodpile cantilever beam (top left), functionally graded film (top right), flat lattice film (bottom right), 3D-printed "shell-like" lattice with anticlastic curvature in its unstressed state.

The model described above proves to be adequate for slender lattice beams in bending where the micromechanics of filament accounts for stretch and flexure. When the overall lattice beam is short, the apparent shear of the lattice becomes increasingly important. The importance of shear in the lattice is of even greater significance as compared to that within homogeneous beams of the same external dimensions. This is apparent from the results of numerical experiments presented in Figure 6. The effect is greater for progressively shorter beams (first two figures in Figure 6 vs the last two). The mechanism of lattice shear is traced in filament bending and if confirmed by the cubic scaling of the apparent shear modulus of such lattices with the volume fraction given by <G>. Detailed analysis of the filament flexure leading to lattice shear is presented in [8], [9].

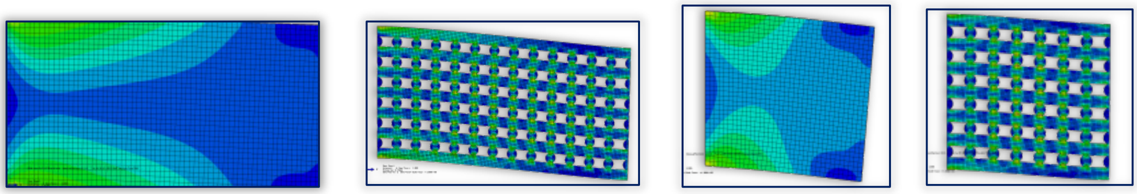
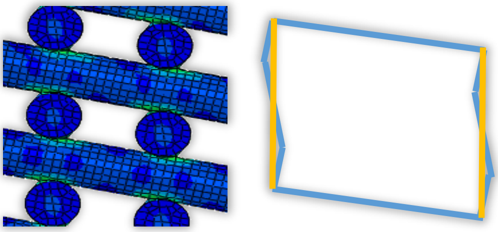


Figure 6. The role of lattice shear in porous solids vis-à-vis that in dense solids

A closer look at the deformed shapes as obtained from the numerical experiments using finite element analysis reveals greater rotation of the right face with respect to the left face for dense solids when compared with the porous lattice. A zoomed in view of this is shown in Figure 7 (left) where the presence of lattice shear is apparent. This could be quantified by considering the filament mechanics. Finally, the effect of adhesion between orthogonally placed filaments could have a significant influence on the shear response, as well as on the extensional response in the stacking direction (which is perpendicular to the two fundamental directions along which filaments run in different layers). This is highlighted by the localized stress field at such joints (Figure 7, right; showing localized response).



$$\gamma = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$

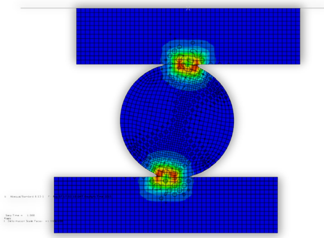


Figure 7. The role of lattice shear in porous solids vis-à-vis that in dense solid (left). Localised stresses at the the interface of two consecutive layers within woodpile structures (right).

3. CONCLUSIONS

Classes of materials that possess porosity as an essential feature of their mesoscopic architecture were considered. When the structure possesses translational symmetry, it is often possible to derive the apparent response in a number of moderately complex situations of loading and response, when closed form analysis involving the mechanics of individual members of the repetitive structure – such as a cell wall –is possible. Two alternative approaches of introducing porosity were considered: when additive manufacturing is employed, the process offers great flexibility on the details of the achievable architecture but it could be a relatively slow process. As opposed to this, approaches of foaming such as that using expandable cavity moulding are a relatively scalable process but offer control on the architecture only statistically (e.g. average pore size). The two approaches could complement each other depending upon the specific applications, especially for biomedical implants and tissue engineering where interconnected pores are required to facilitate cell invasion, attachment, proliferation and growth. It was also noted that unusual mechanical properties of materials and structures could be attained by the controlled use of additive manufacturing for possible applications other than biomedical engineering, e.g. fabrication of elastic meta-materials.

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References

[1] Gibson, L.J., Ashby, M.F. (1997) Cellular Solids: structure and properties, 2nd ed. Cambridge University Press, Cambridge.
[2] Bonfanti, A (2016) Mechanics of Structured Materials and Their Biomedical Applications, PhD Thesis, University of Southampton.
[3] Bonfanti, A, Stavros, S, and Bhaskar, A. (2017) Response and residual curvature of bent-stretched circular rods with applications to metal forming: Closed-form solutions for elastic-perfectly plastic and hyperbolic hardening materials." International Journal of Mechanical Sciences 123: 340-349.
[4] Bonfanti, A., Bhaskar, A. "Elastoplastic response and recoil of honeycomb lattices". European Journal of Mechanics/A. 2017. Under review.
[5] Rizvi, S.J.A., Bhatnagar, N. (2015) Open cell foaming in injection molding with expandable cavity mold. FOAMS 2015, Conference.
[6] Bhaskar, A. (2009) The effective Poisson ratio of random cellular matter having bending dominated architecture *EPL (Europhysics Letters)*, 87, 1-6.
[7] Cuan-Urquizo, E., Yang, S., & Bhaskar, A. (2015). Mechanical characterisation of additively manufactured material having lattice microstructure. In IOP Conference Series: Materials Science and Engineering (Vol. 74, No. 1, p. 012004). IOP Publishing.
[8] Cuan-Urquizo, E. (2016) Mechanical properties of woodpile lattice materials fabricated using additive manufacturing, PhD Thesis, University of Southampton.
[9] Cuan-Urquizo, E., Bhaskar, A. Flexural elasticity of woodpile lattice beams, European Journal of Mechanics - A/Solids: under review.

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