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MODELLING AND SIMULATION OF A TYPICAL CROSS FLOW COOLING TOWER

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Abstract: A simplified mathematical model is developed to simulate the thermal performance of a typical 25 MW mechanical draft cross flow cooling tower. The model takes into account the reduction of water flow rate due to evaporation and the variation of air relative humidity along the tower. The model is used to predict the outlet parameters of water and air through the tower at off design conditions. This tower is used to dissipate heat resulting from fission in the core of a nuclear research reactor. The model is used to obtain the demand and characteristic curves of this tower based on the manufacturer data. To evaluate the thermal performance prediction of the present model a family of performance curves at different water flow rates and cooling ranges are plotted and compared with Merkel model curves, where Merkel model curves are the typical manufacturer curves. Typical experimental data is used to validate the present model. Also the tower capability is determined by the present and Merkel models based on the characteristic curve and performance curves methods.
Keywords: cooling tower, modelling, thermal performance prediction

1. INTRODUCTION

There are several types of cooling towers, probably the most common are the mechanical draft towers in which water enters at the top of the tower, sprays and flows downward through the tower. Ambient air is drawn into the tower with the help of fans, and flows in a counter or cross-current manner to the water stream. [1] Different mathematical models have been developed to predict the thermal behavior of these towers. The first practical model to describe the heat and mass transfer mechanisms in wet cooling towers was proposed by Merkel. [2] In order to test the cooling tower performance, it is quite common to use the Merkel theory such as that of Cooling Tower Institute CTI [3] or American Society Of Mechanical Engineers ASME [4] for the computation of tower characteristic (KaV/L) or Number of transfer units (NTU), where the thermal capability of a cooling tower is obtained by performing the thermal acceptance test. In this test the measured data should be evaluated by comparing them correctly with the design conditions that were instructed according to the CTI cooling tower acceptance test code [3]. Incidentally, these data are not only useful in the determination of thermal capability of the tower according to design conditions during the test run period but can also be used to determine the operating characteristics through change in the atmospheric conditions, especially temperatures.[5] Several models were developed to describe the thermal performance of these towers beside Merkel's model. Snyder [6] applied the theory of heat exchanger design to calculate the driving force of a cross flow tower in the same way as was used to calculate the mean temperature difference in a cross flow heat exchanger and obtained the overall enthalpy transfer coefficient. He assumed a linear relationship between the water temperature and enthalpy of saturated air. Zivi and Brand [7] solved these differential equations numerically using a non-linear relationship between the water temperature and enthalpy of saturated air. Schechter and Kang [8] applied the Zivi and Brand's method to more general operating conditions by representing an exponential function to express the equilibrium relation between the water temperature and enthalpy of saturated air at a limited range. Poppe and Rogener [9] developed a new model for cooling towers which did not use the simplifying assumptions made by Merkel. The critical differences between Merkel, e-NTU, and Poppe models were investigated by Kloppers and Kroger [10]. They concluded that when the water outlet temperature is the only important parameter to the tower designer, the less accurate Merkel and e-NTU approaches can be used but when the heat transfer rates are concerned; they give lower values than that predicted by Poppe approach. Hayashi and Hirai [11] approximated the enthalpy of saturated air by a first-order equation with respect to the water temperature, and applied the cross flow heat exchanger calculations to obtain the overall enthalpy transfer coefficient by using a chart. Inazumi and Kageyama [12] proposed a graphical method for calculation of the enthalpy driving force in a cross flow cooling tower. Khan and Zubair [13, 14] considered the effect of Lewis number and heat transfer resistance in the air–water interface and developed a detailed model for counter flow wet cooling towers. Halasz [15, 16] developed a general mathematical model to describe the thermal characteristics of all types of evaporative cooling devices. The main feature of this model is its non-dimensionality which efficiently reduces the required parameters to analyze an evaporative device. He then applied his model to predict the thermal behavior of wet cooling towers and compared the model results with an accurate model. Kairouni et al. [17] applied the Halasz's model to predict the thermal performance of cooling towers in south of Tunisia. Amir and Johann [18] applied a rigorous model to the thermal design of a counter flow cooling tower by obviating the six simplified assumptions in Merkel method; they found that neglecting evaporation losses is the main cause of inaccuracy in Merkel results. In nuclear reactors, the cooling tower is used as the ultimate heat sink, and so the prediction of cooling tower performance during different operating conditions is an essential factor for predicting the thermal-hydraulic performance of the reactor core. Therefore; the objective of this study is to develop a mathematical model to simulate the performance of a typical 25 MW mechanical draft cross flow cooling tower that is used to dissipate heat resulting from nuclear fission in the core of a nuclear research reactor to the environment during both the design and off design operating conditions.

2. METHODOLOGY

Model formulation

By taking a one dimensional control element of height ΔY, width X, and length Z at the bottom of a one half cell of a cross flow cooling tower of height Y, width X, and length Z containing both air and water streams as shown in Figure 1 where water enters at the top and air enters at the left side.

Mass and heat balances in combination with the other thermal and physical relations are applied on the control element. The variations of water and air conditions along width and height of the tower half cell are accounted for. Therefore, the mass and heat balances are performed using the mean air conditions between the inlet and exit air streams through the moved upward control element, and hence the mean water conditions are estimated.

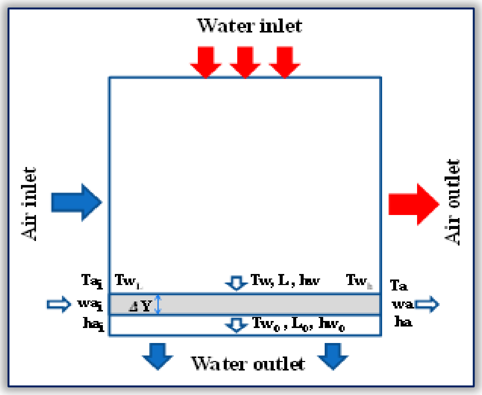

Figure 1. One dimensional control element through a half cell

The following assumptions are considered:

- The cooling tower is operating under steady state conditions.
- There is no recirculation of air between the tower exhaust and intake.
- The thermodynamic properties of the water flow are varied through the tower height, (Y) while the properties of the sideward air flow are estimated at the mean air conditions.
- The Lewis number, defined as the ratio of water air heat transfer to the product of the water air mass transfer and humid heat, is approximated to be unity.
- The water air interface temperature will be considered to be at the bulk water temperature.
- The atmospheric pressure through the tower and the inlet air conditions are constants.

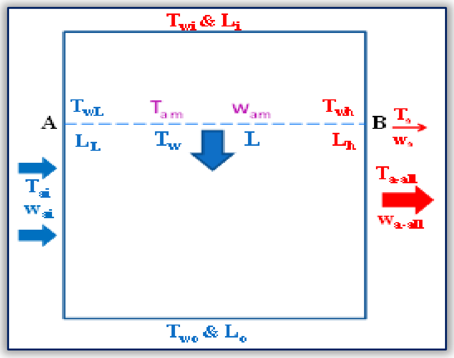

Figure 2. Water and air conditions at section A-B

The schematic diagram shown in Figure 2 illustrates the mean conditions of water and air streams at section A-B where the cold incoming air at point A maximizes the heat and mass transfer. On the other hand the hot exit air at point B inhibits the efficiency of heat and mass transfer. Generally the left hand side contains lower water flow rates and temperatures relatively rather than the right hand side along the height of the tower below water inlet. In Figure 2 we noted: L_L is the low water flow rate at point A, L_h is the high water flow rate at point B, T_{wL} is the low water temperature at point A, T_{wh} is the high water temperature at point B, L is the mean water flow rate at section A-B, T_w is the mean water temperature at section A-B, T_{ai} is inlet air dry bulb temperature, w_{ai} is the inlet air humidity ratio, T_a is the exit air temperature at point B, w_a is the exit air humidity ratio at point B, T_{am} is the mean air temperature at section A-B, w_{am} is the mean air humidity ratio at section A-B, $T_{a\text{-}all}$ is the overall exit air dry bulb temperature, and $w_{a\text{-}all}$ is the overall exit air humidity ratio.

» Mass and heat balances

Applying mass and heat balances for air stream through the control element where both of the mean air humidity ratio and enthalpy will be changed:

$$G\,dw_{am} = dL \quad (1)$$

$$G\,dh_{am} = d(L\,h_w) \quad (2)$$

where G is the air mass flow rate, kg/s, L is the mean water mass flow rate, kg/s, w_m is the mean moist air humidity ratio, kg water/kg air, h_{am} is the mean air enthalpy, KJ/kg, and h_w is the mean water enthalpy, KJ/kg.

Equation (1) represents the evaporation losses as humidity ratio added to ambient air, and equation (2) represents the heat removed by ambient air including evaporation losses.

Since:

$$d(L\, h_w) = L dh_w + h_w dL \tag{3}$$

Substituting equation (3) in equation (2);

$$G\, dh_{am} = L dh_w + h_w dL \tag{4}$$

Applying mass and heat balances for water stream through the control element where the mean water flow rate (L) is changed at the two boundaries of moist air humidity ratios (w_{ai} and w_a);

$$dL = Ka\ \ S\, dY\, (w_i - w_{am}) \tag{5}$$

where K a is the volumetric mass transfer coefficient, kg/m^3 s, Y is the cell height, m, and w_i is the saturated air humidity ratio at water air interface, kg water / kg air.

The mean water enthalpy (h_w) is changed at the two boundaries of moist air dry bulb temperatures (T_{ai} and T_a);

$$L\, dh_w = h_{fg} dL + K_g\, a\, S\, dY\, (T_w - T_{am}) \tag{6}$$

where T_{am} is the mean moist air dry bulb temperature, ˚C, h_{fg} is the latent heat of vaporization, KJ/kg, and K_g a is the volumetric heat transfer coefficient, KW/m^3 ˚C.

Equation (5) gives the mass transfer between water and air. The difference between the mean air humidity ratios at the water air interface and at the bulk air stream is used as the driving force (Merkel equation). Equation (6) represents the heat transfer between water and air in terms of sensible heat and latent heat, where the sensible heat is estimated at the mean of water temperature (T_w) and moist air dry bulb temperature (T_{am}) while the latent heat is estimated at the mean water temperature (T_w).

By substituting for dL and Ldh_w from equations (5) and (6) in equations (1) and (4);

$$G\, dw_{am} = K\ \ a\, S\, dY\, (w_i - w_{am}) \tag{7}$$

$$G\, dh_{am} = K\ \ a\, S\, dY\, (w_i - w_{am})\big(h_w + h_{fg}\big) + K_g\, a\, S\, dY\, (T_w - T_{am}) \tag{8}$$

The mean moist air dry bulb temperature, ˚C is;

$$T_{am} = (T_a + T_{ai})/2 \tag{9}$$

where T_a is the exit moist air dry bulb temperature from an element.

The overall exit air temperature, ˚C of all elements ($T_{a\text{-}all}$) is;

$$T_{a-all} = \left(\textstyle\sum_{r=1}^{r} T_{ar}\right)/r \tag{10}$$

where r is the number of elements along the cell height.

The mean moist air humidity ratio, kg water/kg air is;

$$w_{am} = (w_a + w_{ai})/2 \tag{11}$$

where w_a is the exit moist air humidity ratio from an element.

The overall exit moist air humidity ratio, kg water/kg air of all elements ($w_{a\text{-}all}$) is;

$$w_{a-all} = \left(\textstyle\sum_{r=1}^{r} w_{ar}\right)/r \tag{12}$$

The mean moist air enthalpy, KJ/kg is;

$$h_{am} = (h_a + h_{ai})/2 \tag{13}$$

where, h_a is the exit moist air enthalpy from an element.

The overall exit moist air enthalpy, KJ/kg of all elements ($h_{a\text{-}all}$) is;

$$H_{a-all} = \left(\textstyle\sum_{r=1}^{r} h_{ar}\right)/r \tag{14}$$

The mean water temperature, ˚C is;

$$T_w = (T_{wL} + T_{wh})/2 \tag{15}$$

where T_{wL} is the water temperature, ˚C at the inlet air boundary and T_{wh} is the water temperature, ˚C at the exit air boundary.

The cross sectional area of the control element, m^2 is;

$$S = X\, Z \tag{16}$$

For air water system, the Lewis number is considered to be one and the volumetric heat transfer coefficient (Kg a) can be determined using Lewis number relation as; [2]

$$\frac{K_g\, a}{K a\ \ Cp_m} = Lew = 1.0 \tag{17}$$

Packing fill volume (V) of the N half cells, m^3 is;

$$V = N\, (X\ \ Y\ \ Z) \tag{18}$$

And the tower characteristic number or the number of transfer units (NTU) can be obtained as; [2]

$$NTU = K\, a\, V/L \tag{19}$$

» Thermal relations

The thermal relations used in the model are defined as:
The latent heat of vaporization (h_{fg}), KJ/kg is estimated using the following formula; [19]

$$h_{fg} = 2501 - 2.42\ T_w \tag{20}$$

The heat load (HL), KW is calculated according to equation (2);

$$HL\ = L_i h_{wi} - L_o h_{wo} \tag{21}$$

where, L_i is the inlet water flow rate, kg/s, h_{wi} is the inlet water enthalpy, KJ/kg, L_o is the outlet water flow rate, kg/s and h_{wo} is the outlet water enthalpy, KJ/kg.
The change of water flow rate (dL), kg/s is;

$$dL = (L - L_o\ ) \tag{22}$$

The exit moist air enthalpy (h_a), KJ/kg of individual element is estimated as a function of the moist air humidity ratio (w_a) and dry bulb temperature (T_a); [19]

$$h_a = \left(C_{pa} + C_{pv} w_a\right) T_a + h_{fg0} w_a \tag{23}$$

The mean moist air enthalpy (h_{am}), KJ/kg at bulk air is estimated as a function of the mean moist air humidity ratio (w_{am}) and dry bulb temperature, (T_{am});

$$h_{am} = \left(C_{pa} + C_{pv} W_{am}\right) T_{am} + h_{fg0} W_{am} \tag{24}$$

The mean water enthalpy (h_w), KJ/kg is; [19]

$$h_w = C_{pw}\ T_w \tag{25}$$

The latent heat of vaporization (h_{fg0}) for air enthalpy calculations, KJ/kg is taken as; [19]

$$h_{fg0} = 2501 \tag{26}$$

The saturated vapour pressure, vapour pressure, relative humidity, humidity ratio, and Wet-bulb temperature is estimated with a very good approximation by combination of the following equations: [20]

$$P_s = 0.6112\ \exp\ (17.67\ T_a/(T_a + 243.5)) \tag{27}$$

$$P_v = 0.6112\ \ \exp\ \{\left(\frac{17.67\ T_{wb}}{T_{wb}+243.5}\right) - (P(T_a - T_{wb})\ 0.00066 \times (1 + (0.00115\ T_{wb})))\} \tag{28}$$

$$R = P_v/P_s \tag{29}$$

$$R = w_a/w_s \tag{30}$$

$$P_s = w_s\ P/(w_s + R_G/R_V) \tag{31}$$

$$P_v = w_a\ P/(w_a + R_G/R_V) \tag{32}$$

where P_s is the saturated air vapour pressure, kpa, P_v is the moist air vapour pressure, kpa, T_{wb} is the air wet bulb temperature, ˚C, R is the air relative humidity, w_s is the saturated air humidity ratio, kg_{water}/kg_{air}, R_G is the specific air gas constant, KJ/kg ˚C, and R_V is the water vapour specific gas constant, KJ/kg ˚C.
The evaporation losses (EV_{loss}), m^3/hr is calculated as;

$$Ev_{loss} = (L_i - L_o) \times 3600/\rho_w \tag{33}$$

Where, ρ_w is the water density, kg/m^3 at inlet water temperature (T_{wi}), ˚C.

The cooling range (Range), ˚C is; $\text{Range} = (T_{wi} - T_{wo})$ (34)

The approach temperature (Appr), ˚C is;

$$Appr = (T_{wo} - T_{wbi}) \tag{35}$$

Model solution

The model is used to predict the tower demand curve, characteristic curve, and performance curves. The model also predicts the air and water parameters along the tower at off design conditions. The prediction is based on the corresponding volumetric mass transfer coefficient (K a), which is determined using the tower characteristic curve.

» Demand curve prediction

Engineering Equation Solver (EES) software is used to solve the mathematical model. EES uses the equation based integral function to solve the set of algebraic and nonlinear differential equations simultaneously, where the integration variable of the differential equations is taken as the height of the tower (Y). The input data are the incoming conditions of moist air and water such as: air mass flow rate (G), inlet air dry-bulb temperature (T_{ai}), inlet air relative humidity (R_i), inlet and outlet water temperatures (T_{wi}) and (T_{wo}) respectively, inlet water mass flow rate (L_i), and the total pressure (P) in addition to the tower characteristics; the cross sectional area (S) and number of the half cells (N). The calculation is performed by using iterative values of the volumetric mass transfer coefficient (K a) and the outlet water flow rate (L_o).
The calculations are performed according to the following algorithm:

A. Input the design variables; G , T_{ai} , R_i ,T_{wi} , T_{wo} , P , S , N , and Y
   Input L/G = 0.9

B. Calculate $L_i = G * L/G$
Calculate inlet air wet-bulb temperature (T_{wbi}) using equations (27), (28), and (30).
Calculate inlet air humidity ratio (w_{ai}) using equation (27), and inlet air enthalpy (h_{ai}) using equation (23).
C. Suppose values of the volumetric mass transfer coefficient (K a).
D. Suppose values of the outlet water mass flow rate (L_o).
E. Solving the set of non-linear differential equations (5, 6, 7, and 8) and the algebraic equations (24 & 25) simultaneously to estimate: the mean water temperature (T_w) and mean water flow rate (L).
F. Calculate tolerances $\zeta\, T_w$
G. If $\zeta\, T_w$ is within accepted values, then go to (H);
Otherwise proceed to new trial of K a and return to (C).
Stopping criteria are defined as follows:

$$\zeta\, \mathrm{Tw} = \frac{|T_w - T_{wi}|}{T_{wi}} \leq 0.001$$

H. Calculate tolerances ζ L;
I. If ζ L is within accepted values, then go to (K);
Otherwise proceed to new trial of L_o and return to (D).
Stopping criteria are defined as follows:

$$\zeta\, L = \frac{|L - L_i|}{L_i} \leq 0.001$$

J. Calculate KaV/L
K. Record each of L/G , and KaV/L
L. Repeat for L/G = (1.0, 1.1, 1.2, ... to 2.1) and return to (A).
M. Plot the relation between L/G vs. KaV/L

» Characteristic curve prediction

The tower characteristic KaV/L is referred to as an accepted concept of cooling tower performance. The cooling tower KaV/L depends on the L/G ratio. KaV/L value of a tower operating at off design conditions will not be the same as its value at design conditions. An empirical equation is used to predict KaV/L at off design conditions;

$$\mathrm{NTU} = (\mathrm{K\,a\,V/L}) = \mathrm{C}\,(\mathrm{L/G})^{-n} \tag{36}$$

According to the fact that: the value of C for a designed cooling condition is the same regardless the change of water flow rate. From this rule, the value of the constant (n) can be derived as;

$$n = \log\left(\frac{(\mathrm{KaV/L})_1}{(\mathrm{KaV/L})_2}\right) / \log\left(\frac{(\mathrm{L/G})_2}{(\mathrm{L/G})_1}\right) \tag{37}$$

where $(KaV/L)_1$ is the characteristic number at L/G ratio of 100 % water flow rate $(L/G)_1$, and $(KaV/L)_2$ is the characteristic number at L/G ratio of 110 % water flow rate $(L/G)_2$.
Both $(KaV/L)_1$ and $(KaV/L)_2$ is obtained from the manufacturer characteristic curve where Merkel model is used to determine the corresponding $(T_{wo})_1$ and $(T_{wo})_2$ respectively.
$(KaV/L)_1$ and $(KaV/L)_2$ of the present model are determined by using iterative values of the volumetric mass transfer coefficient (K a) and the outlet water flow rate (L_o) while $(T_{wo})_1$ and $(T_{wo})_2$ are specified. NTU_1 and NTU_2 of the present model are determined as the following algorithm:

A. Input the design variables; G, T_{ai} , R_i ,$(L_i)_1$, Range , $(T_{wo})_1$, P , S , N , and Y
B. Calculate $(L/G)_1 = (Li)_1 /G$
Calculate $(T_{wi})_1 = (T_{wo})_1 +$ Range
Calculate inlet air wet-bulb temperature (T_{wbi}) using equations (27), (28), and (29).
Calculate inlet air humidity ratio (w_{ai}) using equation (30), and inlet air enthalpy (h_{ai}) using equation (23).
C. Suppose value of the volumetric mass transfer coefficient (K a).
D. Suppose value of the outlet water mass flow rate (L_o).
E. Solving the set of non-linear differential equations (5, 6, 7, and 8) and the algebraic equations (24 & 25) simultaneously to estimate; the mean water temperature (T_w) and mean water flow rate (L).
F. Calculate tolerance $\zeta\, T_{w;}$
G. If $\zeta\, T_w$ is within accepted values, then go to (H).
Otherwise proceed to new trial of (K a) and return to (C).
Stopping criteria is defined as follows:

$$\zeta\, \mathrm{Tw} = \frac{|T_w - T_{wi}|}{T_{wi}} \leq 0.001$$

H. Calculate tolerance ζ L;

I. If ζL is within accepted values, then record results;
Otherwise proceed to new trial of (L_o) and return to (D).
Stopping criteria is defined as follows:

$$\zeta L = \frac{|L - L_i|}{L_i} \leq 0.001$$

J. Calculate: $NTU_1 = KaV/L$
K. Replace $(L_i)_1$, $(T_{wo})_1$, and $(L/G)_1$ by $(L_i)_2$, $(T_{wo})_2$, and $(L/G)_2$ respectively and repeat steps, from A to J.
L. The constants C and n are determined by substituting $\{(L/G)_1, NTU_1, (L/G)_2$, and $NTU_2\}$ in equations (36) and (37).

» Model prediction at off design conditions

The model can be used to predict the performance curves and outlet parameters of water and air at different operating conditions. The prediction is based on iterative values of the outlet water temperature (T_{wo}) and the outlet water flow rate (L_o) while the volumetric mass transfer coefficient (K a) is specified at the operating L/G ratio. The calculations are performed according to the following algorithm:

A. Input the variables; G , L_i , T_{ai} , R_i , T_{wi} , P , S ,C , n , and Y
B. Determine the characteristic number (KaV/L) according to the inlet L_i/G ratio.
C. Calculate the volumetric mass transfer coefficient (K a) according to the characteristic number (KaV/L), inlet water mass flow rate (L_i), and packing fill volume (S Y).
D. Calculate inlet air wet-bulb temperature (T_{wbi}) using equations (27), (28), and (29).
Calculate inlet air humidity ratio (w_{ai}) using equation (30), and inlet air enthalpy (h_{ai}) using equation (23).
E. Suppose value of outlet water temperature (T_{wo}) and estimate (h_{wo}) using equation (25).
F. Suppose value of outlet water mass flow rate (L_o).
G. Solving the set of non linear differential equations (5, 6, 7, and 8) and the algebraic equations (24 & 25) simultaneously to estimate; the mean water temperature (T_w), mean water flow rate (L),
H. Calculate tolerance ζ_{Tw};
I. If ζT_w is within accepted values, then go to (J).
Otherwise proceed to new trial of T_{wo} and return to (E).
Stopping criteria is defined as follows:

$$\zeta Tw = \frac{|T_w - T_{wi}|}{T_{wi}} \leq 0.0001$$

J. Calculate tolerance ζL;
K. If ζL is within accepted values, then record results;
Otherwise proceed to new trial of L_o and return to (F).
Stopping criteria is defined as follows:

$$\zeta L = \frac{|L - L_i|}{L_i} \leq 0.0001$$

L. Calculate the overall air dry-bulb temperature ($T_{a\text{-}all}$), overall air humidity ratio ($w_{a\text{-}all}$), and overall moist air enthalpy ($h_{a\text{-}all}$), using equations (10), (12), (14) respectively.

Merkel model

Determination of the demand curve using Merkel's model of counter flow tower can be performed by solving equation (38). This model is based on the assumption of saturated bulk air at the tower exit and neglecting evaporation losses. The design conditions are used to determine the characteristic number (KaV/L) versus different L/G ratios, by applying Chebyshev numerical integration method as follows: [21]

$$\frac{Ka\,V}{L} = C_{pw} \times \int_{T_{wo}}^{T_w} \frac{dT}{(h_i - h_b)} \quad (38)$$

$$= C_{pw}(T_w - T_{wo}) * \left[\left(\frac{1}{Dh1}\right) + \left(\frac{1}{Dh2}\right) + \left(\frac{1}{Dh3}\right) + \left(\frac{1}{Dh4}\right)\right]/4 \quad (39)$$

where;

$$Dh1 = (h_{i1} - h_{b1}) \quad (40)$$

h_{i1} is obtained at $T_{i1} = (T_{wo} + 0.1\text{ Range})$ & $h_{b1} = h_{bi} + 0.1 * C_{pw} * (L/G) * \text{Range}$

$$Dh2 = (h_{i2} - h_{b2}) \quad (41)$$

h_{i2} is obtained at $T_{i2} = (T_{wo} + 0.4\text{Range})$ & $h_{b2} = h_{bi} + 0.4 * C_{pw} * (L/G) * \text{Range}$

$$Dh3 = (h_{i3} - h_{b3}) \quad (42)$$

h_{i3} is obtained at $T_{i3} = (T_{wo} + 0.6\text{Range})$ & $h_{b3} = h_{bi} + 0.6 * C_{pw} * (L/G) * \text{Range}$

$$Dh4 = (h_{i4} - h_{b4}) \quad (43)$$

h_{i4} is obtained at $T_{i4} = (T_{wo} + 0.9\text{Range})$ & $h_{b4} = h_{bi} + 0.9 * C_{pw} * (L/G) * \text{Range}$

The saturated air enthalpy at interface (h_{in}), KJ/kg is estimated at bulk water temperature;

$$h_{in} = (C_{pa} + C_{pv}w_{in})T_{in} + h_{fg0}w_{in} \quad (44)$$

where; h_{bi} is the initial saturated bulk air enthalpy, KJ/kg, in refers to the air water interface at the four steps, (in = i1, i2, i3, i4), bn refers to the saturated bulk air at the four steps (bn = b1, b2, b3, b4) and w_{in} is determined at T_{in} and saturation relative humidity (R = 1.0).

The exit saturated bulk air enthalpy (h_b), KJ/kg is estimated as;

$$Gh_b = LC_{pw}\,(T_{wi} - T_{wo}) \quad (45)$$

$$h_b = h_{bi} + C_{pw}(L/G)\,(T_{wi} - T_{wo}) \quad (46)$$

Determination of cooling tower capability

The cooling tower capability is determined according to cooling tower institute (CTI) by two methods; the Characteristic curve method and the Performance curves method.

The characteristic curve method is applied by plotting the test point on the tower demand curve and drawing a parallel line at the test point to the characteristic curve that intersects the demand curve at (L/G) intersection. The tower capability is determined by using the characteristic curve method as: [3]

$$Q = \frac{(L/G)\text{intersection}}{(L/G)\text{design}} \times 100 \quad (47)$$

To calculate the tower capability by the method of tower performance curves, it is required to convert the test water flow rate to the water flow rate at the design conditions. Equation (49) is necessary to predict the amount of water that the tower can cool, at test temperatures, if the fan drives were loaded to design power. The performance curves could be prepared by the simple method and detail method. Performance prediction of the cooling tower using the simple method is made by a few design parameters as well as; water flow rate (L_i), (L/G) ratio, (KaV/L), cooling range, outlet water temperature (T_{wo}), and wet bulb temperature (T_{wbi}) while the performance prediction by the detail method is requiring all the actual cooling tower dimensions, thermal rating conditions, and all the mechanical rating conditions. The difference between the results of the two methods is very minor. So, the simple method is strongly recommended to use in practice. The tower capability can be determined using the performance curves method as: [21]

$$Q = \frac{L_{adjusted}}{L_{predicted}} \times 100 \quad (48)$$

$$\left(\frac{L}{G}\right)_{test} = \left(\frac{L}{G}\right)_{design}\left(\frac{L_{test}}{L_{design}}\right)\left(\frac{BHP_{design}}{BHP_{test}}\right)^{\frac{1}{3}}\left(\frac{\rho_{test}}{\rho_{design}}\right)^{\frac{1}{3}}\left(\frac{\upsilon_{test}}{\upsilon_{design}}\right) \quad (49)$$

where: ρ = moist air density, kg/m^3, υ = dry air specific volume m^3/kg, and BHP = fans brake horse power, Hp

$$L_{adjusted} = L_{test} * \left(\frac{BHP_{design}}{BHP_{test}}\right)^{\left(\frac{1}{3}\right)}\left(\frac{\rho_{test}}{\rho_{design}}\right)^{\frac{1}{3}}\left(\frac{\upsilon_{test}}{\upsilon_{design}}\right) \quad (50)$$

where: ($L_{adjusted}$) is the water flow rate corresponding to the design conditions at the assumption of constant gas. [21]

3. RESULTS AND DISCUSSION

The present model is applied on a typical 25.0 MW mechanical draft cross flow cooling tower, this tower is used to dissipate the heat resulting from fission in the core of a nuclear research reactor. The maximum total heat resulting from this reactor and its supplementary units is 25.0 MW. This heat is dissipated primarily across many heat exchangers before forced to the cooling tower, which give it off to the atmosphere. The tower consists of six cells where each cell has its own water inlet and fan. Typical manufacturer design data of the cooling tower is shown in Table 1.

Table 1. Typical manufacturer data of the cross flow tower

Parameter	Design value
Inlet water temp, °C	37
Inlet air dry-bulb temp, °C	32.74
Inlet air wet-bulb temp, °C	24
Inlet relative humidity	48
Inlet water flow rate, m^3/hr	3120
L/G	1.366
KaV/L	1.05
Range, °C	7.0
Approach, °C	6.0
Height of the tower cells (Y), m	3.0
Cross sectional area of a one half cell (S), m^2	7.35
Volume of a one half cell (V), m^3	22.05
Moist air density (ρ), kg/m^3	1.17426
Dry air specific volume (υ), m^3/kg	0.86768
Brake horse power (BHP), hp	48.1

Demand and characteristic curves

Based on the iteration procedure described in section 2.2.1 the present model predicts a values of 3.59 kg/m^{3}s and 1.103 for the volumetric mass transfer coefficient (k a) and the characteristic number (KaV/L) at the manufacturer design conditions while the corresponding values obtained by Merkel model are 3.417 kg/m^{3}s and 1.05. The Constants (n and C) of the characteristic equation are determined according to equations (36) and (37) as described in section 2.2.2 using the manufacturer characteristic curve. Table 2 shows the parameters used to determine the slope value (n) at the present and Merkel models.

Figure 3 shows the demand and characteristic curves obtained by both the present and Merkel models. The manufacturer demand curve is also plotted. It is found that, the manufacturer curve is typical as Merkel curve and the present demand curve is shifted slightly above them. This is attributed to the implementation of evaporation losses in the present model as well as using the variation of the moist air conditions along the tower.

Table 2. Slope determination at two different L/G ratios

Item	L/G	Present model	Merkel model
$(KaV/L)_1$	1.366	1.103	1.05
$(KaV/L)_2$	1.5026	1.029	0.9757
Slope (n)		- 0.73	- 0.77

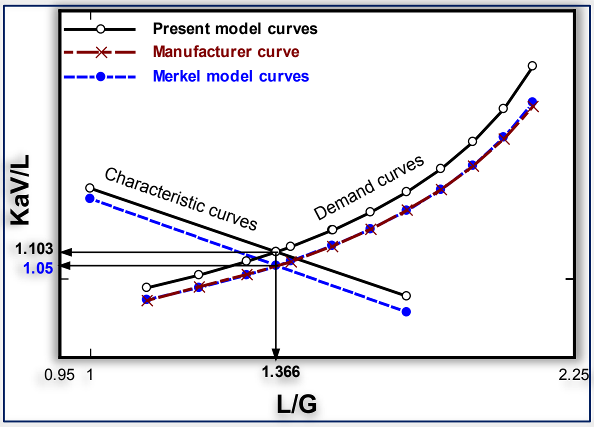

Figure 3. Demand and characteristic of the cross flow cooling tower

Performance curves

Cooling towers operate most of the time at different conditions than their design conditions. The characteristic number (KaV/L) will remain unchanged as long as the L/G ratio is constant; however the tower performance is changed according to the ambient air conditions.

Therefore a thermal performance curve is plotted at the design water flow rate (L_i) and cooling range (Range) by using the present model and compared by the manufacturer curve at various ambient air conditions as shown in Figure 4.

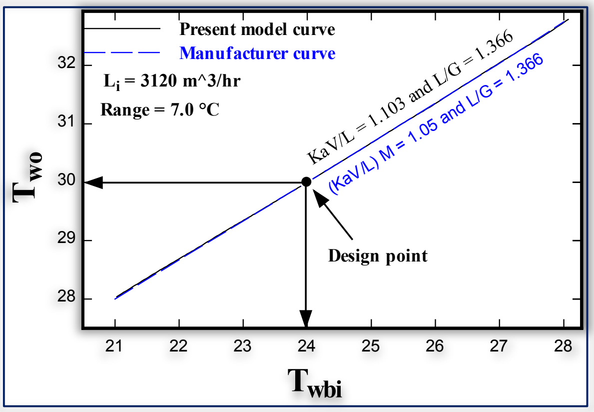

Figure 4. The design thermal performance curves

For different water flow rates and cooling ranges, a family of performance curves should be produced at different ambient air conditions. In the present model, the ambient air conditions are determined by two air parameters rather than one parameter in Merkel model; the wet bulb temperature. So the relative humidity is specified at the design value.

Three sets of performance curves are produced using the simple method described before. Both the present and Merkel model curves are plotted together at different inlet water flow rates, cooling ranges and air wet bulb temperatures. Three inlet water flow rates are considered; 90 % of the design flow rate, 100 % of the design flow rate, and 110 % of design flow rate. Cooling ranges are taken as (+/- 20 %) of the design cooling range; 7.0˚C. Wet bulb temperatures are taken through a range of 12 to 28 ºC.

It is found that, the differences between the performance curves obtained by both the present and Merkel models are negligible as shown in Figures 5, 6 and 7.

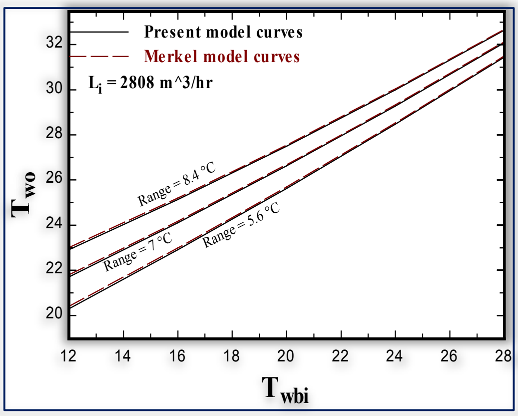

Figure 5. Performance curves at 90 % water flow rate

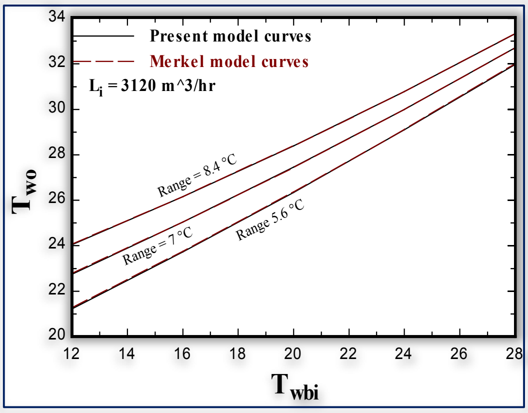

Figure 6. Performance curves at 100 % water flow rate

Model verification

El-Morshedy [22] collected an experimental data of the cooling tower under this study in order to perform a thermal acceptance test according to code (ATC-105) of Cooling Tower Institute (CTI) to determine its capability. The model perdition for the parameters of the test is performed according to the procedure described in section 2.3.

The present and Merkel models predict a cold water temperature (T_{wo}) of 26.78˚C and 26.81˚C respectively while the test value was 26.53 ˚C. The difference between the two models T_{wo} is only 0.03˚C.

The test data and the present model results are tabulated as shown in Tables 3 and 4 respectively where $(L/G)_{test}$ is calculated by using equation (49) to be:

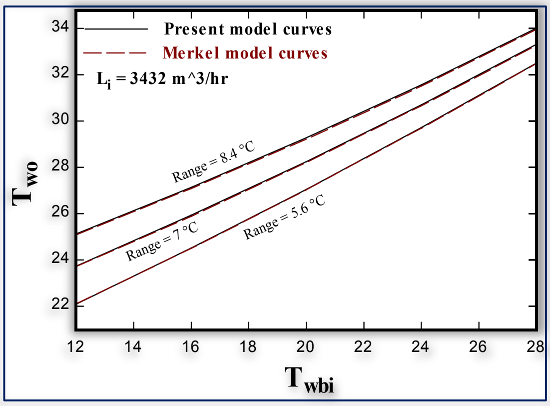

Figure 7. Performance curves at 110 % water flow rate

$$\left(\frac{L}{G}\right)_{test} = 1.366\left(\frac{2804}{3120}\right)\left(\frac{48.1}{45.94}\right)^{1/3}\left(\frac{1.18997}{1.17426}\right)^{1/3}\left(\frac{0.85325}{0.86768}\right) = 1.2313$$

Table 3. Typical experimental data of the cross flow tower

Parameter	Test value
Water flow rate, m^3/hr	2804
Inlet water temperature, ˚C	33.32
Outlet water temperature, ˚C	26.53
Inlet air dry bulb temperature, ˚C	27.63
Inlet wet bulb temperature, ˚C	20.68
Inlet relative humidity	0.53
Cooling range, ˚C	6.79
Approach, ˚C	5.85
Brake horse power (BHP), hp	45.94
Moist air density (ρ), kg/m^3	1.18997
Dry air specific volume (υ), m^3/kg	0.85325
Height of the tower cells (Y), m	3.0
Cross sectional area of a one half cell (S), m^2	7.35
Packing fill volume of a one half cell (V), m^3	22.05

In spite of this negligible difference in predicting of T_{wo} and in turn the tower capability (Q), the differences in predicting the other parameters is considerable where:

The predicted heat load or the exit air enthalpy by the present model is 26.47 MW while Merkel model value is 25.2 MW with a difference of 5.04 %. This result is compatible with Kloppers and Kroger [10] finding {They concluded that when the water outlet temperature is the only important parameter to the tower designer, the less accurate Merkel and e-NTU approaches can be used but when the heat transfer rates are concerned; they give lower values than that predicted by Poppe approach}.

Table 4. The present model parameters

Step	height	L	h_w	T_w	h_a	T_a	w_a	dL	KaV/L	L/G
1	0	774.7	139.4	33.32	129.3	32.64	0.03768	0	1.201	1.231
2	0.1034	774.4	138.4	33.09	126.8	32.31	0.03685	0.3209	1.201	1.231
3	0.2069	774.2	137.5	32.86	124.4	31.99	0.03602	0.636	1.201	1.23
4	0.3103	773.9	136.5	32.64	122	31.68	0.0352	0.9457	1.202	1.23
26	2.586	767.9	116	27.62	69.15	27.48	0.01625	6.957	1.211	1.221
27	2.69	767.6	115	27.41	66.67	27.48	0.01529	7.216	1.212	1.220
28	2.793	767.3	114	27.2	64.18	27.5	0.0143	7.475	1.212	1.220
29	2.897	767	113	26.99	61.68	27.55	0.0133	7.735	1.213	1.219
30	3	766.7	112	26.78	59.16	27.63	0.01228	7.995	1.213	1.219
Overall exit air parameters					94.38	29.19	0.02544			

Tower capability

The tower capability is determined by two methods: the characteristic curve and performance curves methods using the present and Merkel models as follow:

» Characteristic curve method

According to the characteristic curve method described in section 2.4, the KaV/L of the experimental test is required to be plotted on the demand curve. The $(KaV/L)_{test}$ is obtained according to the volumetric mass transfer coefficient K a which is determined at the experimental test conditions by iteration technique to satisfy the test value of the cold water temperature; T_{wo} = 26.53 ˚C. It is found that: the exact K a is 3.515 kg/m^{3}s and $(KaV/L)_{test}$ is 1.32. The present model demand and characteristic curves are used to determine the tower capability. As shown in Figure 8 the test point curve intersects the demand curve at L/G intersection of 1.455 where the tower capability is determined using equation (47) to be:

$$Q = 106.52\ \%$$

The model predicted KaV/L which represents 100 % capability is obtained at the test L/G ratio which intersects the characteristic curve at 1.201 where Two = 26.78 ˚C.

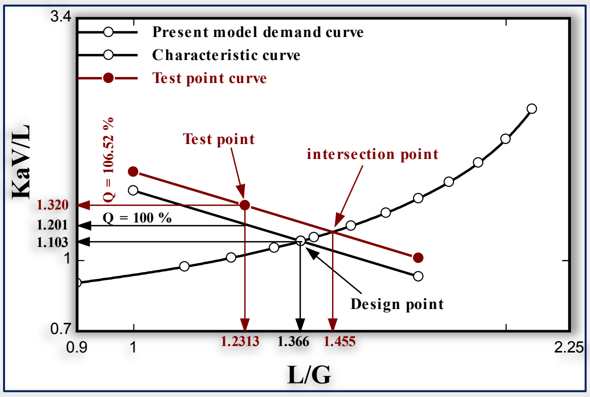

Figure 8. Characteristic curve method using the present model curves

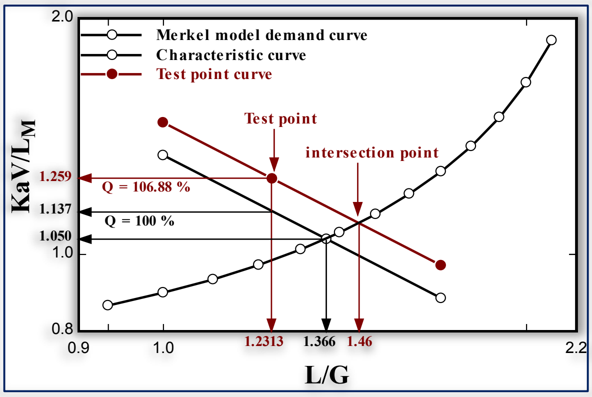

Figure 9. Characteristic curve method using Merkel model curves

The characteristic curve method is repeated using Merkel model curve to determine the (L/G) intersection as shown in Figure 9 where $(KaV/L)_M$ under the conditions of the experimental test is found to be 1.259. The L/G intersection is 1.46 and the tower capability is:

$$Q_M = 106.88\ \%$$

» Performance curves method

The approximately typical performance curves shown in Figures 5,6 and 7 of the present and Merkel models are used to plot the relation between the outlet water temperature (T_{wo}) versus the cooling range (Range) as shown in Figure 10 where the test cooling range 6.79 ˚C is plotted on the obtained curves and intersects the water flow rates (L_i) at different outlet water temperatures (T_{wo}).

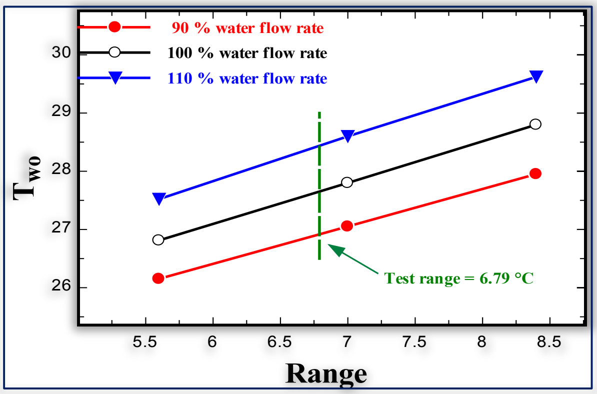

Figure 10. The test range and (T_{wo} versus Range)

The predicted water flow rate is found to be 2660 m3/hr while the adjusted water flow rate is estimated by using equation (50) to be:

$$L_{adjusted} = 2804\left(\frac{48.1}{45.94}\right)^{1/3}\left(\frac{1.18997}{1.17426}\right)^{1/3}\left(\frac{0.85325}{0.86768}\right) = 2847.3\ m^3/hr$$

Then the tower capability is calculated using equation (48) to be:

$$Q = Q_M = \frac{2812.38}{2660} * 100 = 105.73\ \%$$

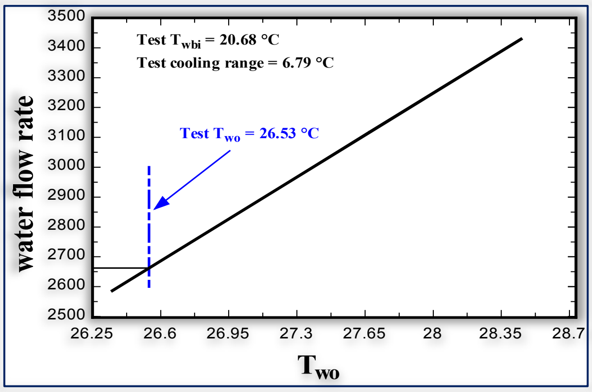

Figure 11. The test T_{wo} and (L_i versus T_{wo})

The plot of inlet water flow rate (L_i) vs. Outlet water temperature (T_{wo}) is used to determine the corresponding water flow rate at the test (T_{wo} = 26.53 ˚C) as shown in Figure 11.

4. CONCLUSIONS

A mathematical model is developed to simulate a typical 25 MW mechanical draft cross flow cooling tower based on mass and heat balances around one dimensional control element through a one half cell of the tower. A realistic approach is followed in the model where the reduction of water flow rate due to evaporation and the variation of air relative humidity along the tower are taken into consideration. The tower demand and characteristic curves are obtained by both the present and Merkel models, where Merkel model curves are the manufacturer curves. The present model curves are shifted above Merkel model curves. This means that, the implementation of evaporation losses and the variation of air relative humidity has remarkable effect on the characteristic number. The tower performance curve obtained by the present model at the design cooling range and water flow rate is plotted against the manufacturer curve where the two curves are very closely. Furthermore, performance curves of the present and Merkel models at different cooling ranges and water flow rates are plotted together where approximately typical curves are obtained. Although the same outlet water temperature obtained by the two models there is a considerable difference in estimating the exit air enthalpy due to include evaporation losses in the present model. This is compatible with Kloppers and Kroger [10] finding where They concluded that: {when the water outlet temperature is the only important parameter to the tower designer, the less accurate Merkel and e-NTU approaches can be used but when the heat transfer rates are concerned; they give lower values than that predicted by Poppe approach}. The model is validated by using exact experimental data, where it predicts outlet water temperature 26.78°C, while the test temperature is 26.53°C with a difference of 0.25°C. The tower capability values of 106.52 % and 106.88 % are obtained by the present and Merkel models respectively using the characteristic curve method. While a value of 105.73 % is obtained by both models using the performance curves method.

Nomenclatures:

a - Surface area per unit volume, m^{-1}
Appr - Temperature approach, °C
BHP - Brake horse power, Hp
C_{pa} - Dry air specific heat, KJ/kg °C
C_{pw} - Water specific heat, KJ/kg °C
C_{pv} - Water vapour specific heat, KJ/kg .°C
C_{pm} - Moist air specific heat, KJ/kg °C
Ev_{loss} - Evaporation losses, m^3/hr
G - Air mass flow rate, kg/s
h_a - Moist air enthalpy, KJ/kg
h_i - Saturated air enthalpy at interface, KJ/kg
h_b - Saturated air enthalpy at bulk air, KJ/kg
h_w - Mean water enthalpy, KJ/kg
h_{wo} - Outlet water enthalpy, KJ/kg
h_{fg0} - Standard latent heat of vaporization, KJ/kg
h_{fg} - Latent heat of vaporization at T_w , KJ/kg
HL - Heat load, KW
K - Over all mass transfer coefficient, kg/m^2 s
K a - Volumetric mass transfer coefficient, kg/m^3 s
K_g - Over all heat transfer coefficient, KW/m^2 °C
K_g a - Volumetric heat transfer Coefficient, KW/m^3 °C
KaV/L - Characteristic number (NTU)
$(KaV/L)_M$ - Merkel's model Characteristic number $(NTU)_M$
L - Water mass flow rate, kg/s
L_o - Outlet water mass flow rate, kg/s
Lew - Lewis number
N - Number of half cells
L/G - Water flow rate by air flow rate, kg / kg
$(L/G)_{eq}$ - Intersection water to air mass flow ratio
n - Slope of the characteristic curve
NTU - Number of transfer units
P - Atmospheric pressure, 101.32 kpa
P_s - Water saturation vapour pressure, kpa
P_v - Water vapour partial pressure, kpa
Q - Cooling tower capability
Range - Cooling range, °C
R_G - Air gas constant, KJ/kg °C
R_v - Water vapor gas constant, KJ/kg °C
R - Relative humidity
S - Cross sectional area of a half cell packing, m^2
T_{wo} - Outlet water temperature, °C
T_w - Mean water temperature, °C
T_i - Water-air interface temperature, °C
T_{ai} - Inlet air dry-bulb temperature, °C
T_a - Air dry-bulb temperature, °C
T_{am} - Mean air dry-bulb temperature, °C
T_{wbi} - Inlet air wet-bulb temperature, °C
T_{wb} - Air wet-bulb temperature, °C
V - Volume of one half cell, m^3
w_a - Moist air humidity ratio, kg/kg
w_s - Saturated air humidity ratio, kg/kg
w_i - Saturated air humidity ratio at interface, kg/kg
ρ_w - Water density, kg/m^3

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