[1.]Nebojša RADIĆ, [2.]Dejan JEREMIĆ

ANALYTICAL SOLUTION FOR THERMAL VIBRATIONS OF SINGLE-LAYERED GRAPHENE SHEETS WITH VARIOUS BOUNDARY CONDITIONS

[1-2.] University of East Sarajevo, East Sarajevo, BOSNIA & HERZEGOVINA

Abstract: This paper deals with analysis of thermal vibration behaviour of orthotropic single-layered nanoplate with various boundary conditions. The new first-order shear deformation theory is reformulated using nonlocal differential constitutive relation of Eringen. The governing equations of motion are derived from Hamilton's principle. Using Galerkin method, analytical solution for rectangular nanoplates under various boundary conditions are obtained. Numerical results are presented to show variations of the dimensionless frequency of single-layered nanoplates corresponding to various values of the nonlocal parameter and temperature change.
Keywords: Nanoplates, Thermal vibration, Galerkin method

1. INTRODUCTION

In recent years, nanostructures such as carbon nanotube, nanobeam and nanoplate have received considerable attention because of their mechanical, electrical and other physical and chemical properties. Graphene presents one of the physical forms of carbon and it is the best known nanoplate. Besides graphene, carbon atoms also occur in the form of carbon nanotubes which can be observed as a deformed form of graphene. Graphene sheets and carbon nanotubes present the nanoscale objects and they are very significant parts of nano/micro structures. Nanoplates are expected to have a wide range of applications, including gas sensors [1], medicines [2], electronics [3] and nanoelectro-mechanical systems [4]. The application of Molecular Dynamics (MD) simulation [5] has clearly demonstrated that the small-scale effects in the analysis of the mechanical behaviour of nanostructures cannot be neglected and that the classical continuum theory can be applied, according to the fact that it is a size independent theory. Classical elasticity theory is a scale free theory and cannot handle small effects. As a result, the mechanical behaviour of the nanostructures cannot describes by classical elasticity theory. With the aim to overcome this problem, some size dependent non-classical continuum theories have been developed, such as strain gradient theory [6], modified couple stress theory [7], and nonlocal elasticity theory [8].

These three theories have been developed to include a size effect to the mechanical behaviour of nanostructures by introducing the internal length scale. Among these theories, the nonlocal elasticity theory initiated by Eringen [9] is the most used one. In Eringen's nonlocal elasticity theory the small scale effects have been taken into account assuming that the stress in the observed point depends on the deformations in all other points of the entire domain occupied by the material. To date, buckling of nanoplates [10], their vibrations [11], thermal buckling [12,13], and thermal vibration [14,15] have been studied via the nonlocal elasticity theory of Eringen. In the present study, using Eringen's nonlocal differential constitutive relation local new first-order shear deformation theory (NFSDT) has been reformulated and used for the analysis of the vibration of orthotropic single-layered graphene sheets (SLGSs) embedded in an elastic medium. To the best of the research's knowledge vibration analysis of SLGSs in thermal environment under various boundary conditions has not been investigated in the available literature. Also, the in-plane effects of boundary conditions on thermal vibration of orthotropic SLGSs has not been discussed yet.

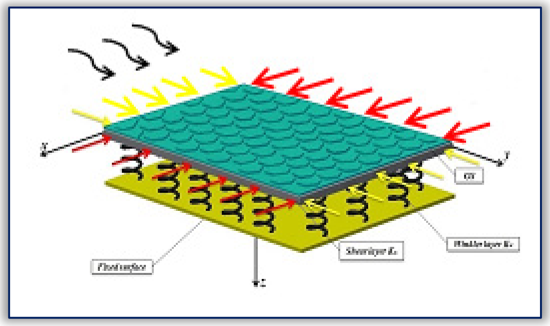


Figure 1. Single-layered orthotropic graphene sheet

2. BASIC FORMULATIONS AND ASSUMPTIONS

A schematic configuration of the orthotropic SLGSs of length of each sheet a, width b, and uniform thickness h resting on a Pasternak foundation has been illustrated on

Figure 1. As can be seen nanoplates are surrounded by an external Pasternak elastic medium, which includes Winkler modulus parameter K_W and shear modulus parameter K_G.

The Cartesian coordinate system is considered such that the x, y and z axes are along the length, width, and thickness of nanoplate.

According to the NFSDT, the displacement field, taking into account the shear deformation effect, can be expressed as

$$\begin{aligned} u_x(x,y,z,t) &= -z\frac{\partial \phi}{\partial x}, \\ u_y(x,y,z,t) &= -z\frac{\partial \phi}{\partial y}, \\ u_z(x,y,t) &= w(x,y,t) \end{aligned} \tag{1}$$

where w is the transverse displacement of the material point *(x, y)* in the mid-plane and ϕ is function of coordinates *x, y* and time *t.*

Based on the nonlocal continuum theory of Eringen the nonlocal stresses are generally related to the local ones through the following relation :

$$\left[1-(e_0\ell)^2\nabla^2\right]\sigma_{ij}(x) = C_{ijkl}(\varepsilon_{kl} - \alpha_{ij}T) \qquad i,j,k,l = x,y,z \tag{2}$$

where $\sigma_{ij}(x)$ is the nonlocal stress tensor, ∇^2 is the Laplacian operator which is defined by $\nabla^2 = (\partial^2/\partial x^2 + \partial^2/\partial y^2)$, $e_0\ell$ is the nonlocal parameter that takes into account the small scale effects into the constitutive equations, C_{ijkl} is the fourth-order elasticity tensor, ε_{kl} is the strain tensor, α_{ij} is the thermal expansion coefficient, and T is the value of the temperature change. The nonlocal parameter is commonly adjusted based on the predicted frequencies by anappropriate atomic model. Thereby, the nonlocal moments and forces of the nanoplate are related to their local counterparts as,

$$\begin{aligned} M_{xx} - (e_0\ell)^2\nabla^2 M_{xx} &= -\left[D_{11}\frac{\partial^2\phi}{\partial x^2} + D_{12}\frac{\partial^2\phi}{\partial y^2}\right] \\ M_{yy} - (e_0\ell)^2\nabla^2 M_{yy} &= -\left[D_{22}\frac{\partial^2\phi}{\partial y^2} + D_{12}\frac{\partial^2\phi}{\partial x^2}\right] \\ M_{xy} - (e_0\ell)^2\nabla^2 M_{xy} &= -2D_{66}\frac{\partial^2\phi}{\partial x\partial y} \end{aligned} \tag{3}$$

$$\begin{aligned} Q_{xz} - (e_0\ell)^2\nabla^2 Q_{xz} &= H_{55}\left(\frac{\partial w}{\partial x} - \frac{\partial \phi}{\partial x}\right) \\ Q_{yz} - (e_0\ell)^2\nabla^2 Q_{yz} &= H_{44}\left(\frac{\partial w}{\partial y} - \frac{\partial \phi}{\partial y}\right) \end{aligned} \tag{4}$$

where

$$\begin{aligned} & D_{11} = \frac{Q_{11}h^3}{12}, \quad D_{22} = \frac{Q_{22}h^3}{12}, \quad D_{12} = \frac{Q_{12}h^3}{12}, \quad D_{66} = \frac{Q_{66}h^3}{12}, \\ & H_{44} = K_S Q_{44} h, \quad H_{55} = K_S Q_{55} h, \\ & Q_{11} = \frac{E_1}{1-\nu_{12}\nu_{21}}, \; Q_{12} = \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}} = Q_{21} = \frac{\nu_{21}E_1}{1-\nu_{12}\nu_{21}}, \; Q_{22} = \frac{E_2}{1-\nu_{12}\nu_{21}}, \\ & Q_{44} = G_{23}, \; Q_{55} = G_{31}, \; Q_{66} = G_{12} \end{aligned} \tag{5}$$

where K_S is the shear correction factor. As widely accepted, the approximate value for shear correction factor is $K_S = 5/6$.

By adopting Hamilton's principle, the equations of motion can be expressed as follows:

$$\begin{aligned} \delta w &: \frac{\partial Q_{yz}}{\partial x} + \frac{\partial Q_{zx}}{\partial y} - K_W w + K_G\nabla^2 w + N_{xxT}\frac{\partial^2 w_1}{\partial x^2} + N_{yyT}\frac{\partial^2 w_1}{\partial y^2} = \rho h\ddot{w} \\ \delta\phi &: \frac{\partial^2 M_{xx}}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x\partial y} + \frac{\partial^2 M_{yy}}{\partial y^2} - \frac{\partial Q_{yz}}{\partial x} - \frac{\partial Q_{xz}}{\partial y} = -\frac{\rho h^3}{12}\nabla^2\ddot{\phi} \end{aligned} \tag{6}$$

If we apply the operator $\Re = 1-(e_0\ell)^2\nabla^2$ to Eq.(6) and substitute Eqs. (3) and (4) into the resulting equation then equations of motion can be expressed via the displacement (w,ϕ).

$$\begin{aligned} &H_{55}\left(\frac{\partial^2 w}{\partial x^2}-\frac{\partial^2\phi}{\partial x^2}\right)+H_{44}\left(\frac{\partial^2 w}{\partial y^2}-\frac{\partial^2\phi}{\partial y^2}\right)-K_W w+K_G\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)+\\ &+N_{xxT}\frac{\partial^2 w}{\partial x^2}+N_{yyT}\frac{\partial^2 w}{\partial y^2}-(e_0\ell)^2\left[-K_W\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)+K_G\left(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}\right)\right] \\ &+N_{xxT}\left(\frac{\partial^4 w}{\partial x^4}+\frac{\partial^4 w}{\partial x^2\partial y^2}\right)+N_{yyT}\left(\frac{\partial^4 w}{\partial y^4}+\frac{\partial^4 w}{\partial x^2\partial y^2}\right)=\rho h\ddot{w}-(e_0\ell)^2\rho h\left(\frac{\partial^2\ddot{w}}{\partial x^2}+\frac{\partial^2\ddot{w}}{\partial y^2}\right)\end{aligned} \tag{7}$$

$$\begin{aligned} &D_{11}\frac{\partial^4\phi}{\partial x^4}+2(D_{12}+2D_{66})\frac{\partial^4\phi}{\partial x^2\partial y^2}+D_{22}\frac{\partial^4\phi}{\partial y^4}+H_{55}\left(\frac{\partial^2 w}{\partial x^2}-\frac{\partial^2\phi}{\partial x^2}\right)+H_{44}\left(\frac{\partial^2 w}{\partial y^2}-\frac{\partial^2\phi}{\partial y^2}\right)\\ &=\frac{\rho h^3}{12}\left(\frac{\partial^2\ddot{\phi}}{\partial x^2}+\frac{\partial^2\ddot{\phi}}{\partial y^2}\right)-(e_0\ell)^2\frac{\rho h^3}{12}\left(\frac{\partial^4\ddot{\phi}}{\partial x^4}+2\frac{\partial^4\ddot{\phi}}{\partial x^2\partial y^2}+\frac{\partial^4\ddot{\phi}}{\partial y^4}\right)\end{aligned} \tag{8}$$

3. ANALYTICAL SOLUTION

In this study, we will analyse four types of boundary conditions (SSSS, SCSC, CCCC, C1CC1C). The mark SCSC means that the edges of the nanoplate $x=0,a$ are simply supported-immovable (S), while the edges $y=0,b$ are clamped-immovable (C). In C1 supports, the free displacement in the plane of the nanoplate is allowed due to the increase of temperature, so the nanoplate in these supports cannot be loaded.

The analytical solution of Eqs. (7)-(8) that satisfy the above boundary conditions can be obtained for rectangular nanoplates under various boundary conditions by using the following expressions of generalized displacements:

$$\begin{aligned} w(x,y) &= W_{mn}X(x)Y(y)e^{i\omega t}\\ \phi(x,y) &= \phi_{mn}X(x)Y(y)e^{i\omega t}\end{aligned} \tag{9}$$

In this paper, the Galerkin's method is employed for solving the equations of motion for SLGSs with various boundary conditions. By using Eq. (9) and applying the Galerkin's method to Eqs. (7)-(8), the analytical solution can be obtained from

$$\left(\begin{bmatrix}K_{11} & K_{12}\\ K_{21} & K_{22}\end{bmatrix}-\omega^2\begin{bmatrix}m_{11} & 0\\ 0 & m_{22}\end{bmatrix}\right)\begin{Bmatrix}W_{mn}\\ \theta_{mn}\end{Bmatrix}=\begin{Bmatrix}0\\ 0\end{Bmatrix} \tag{10}$$

The solution Eq. (10) has the following form:

$$\omega=\sqrt{\frac{-S_1+\sqrt{S_1^2-4S_2S_3}}{2S_2}} \tag{11}$$

where

$$\begin{aligned} S_1 &= \frac{\rho h^3}{12}(V_1-g^2V_2)(B_1+B_2)+\rho h(V_3-g^2V_1)(B_3-B_2)\\ S_3 &= B_1B_3+B_2(B_3-B_1)\\ S_2 &= \frac{\rho^2h^4}{12}(V_1-g^2V_2)(V_3-g^2V_1)\\ B_1 &= C_1+C_2\\ C_1 &= K_G(V_1-g^2V_2)+K_W(g^2V_1-V_3)\\ C_2 &= N_{xxT}\left[T_1-g^2(T_3+T_6)\right]+N_{yyT}\left[T_2-g^2(T_5+T_6)\right]\\ B_2 &= K_S h(G_{13}T_1+G_{23}T_2)\\ B_3 &= D_{11}T_3+(D_{12}+2D_{66})T_4+D_{22}T_5\end{aligned} \tag{12}$$

n Eq.(12) the mark for nonlocal parameter $g = e_0\ell$ has been used.

The values $T_1, T_2, T_3, T_4, T_5, T_6, V_1, V_2$ and V_3 depend on boundary conditions:

a) SSSS nanoplates

$$T_1 = -\frac{ab}{4}\alpha^2,\quad T_2 = -\frac{ab}{4}\beta^2,\quad T_3 = \frac{ab}{4}\alpha^4,\quad T_4 = \frac{ab}{2}\alpha^2\beta^2,\quad T_5 = \frac{ab}{4}\beta^4,\quad T_6 = \frac{ab}{4}\alpha^2\beta^2,$$

$$V_1 = -\frac{ab}{4}\left(\alpha^2+\beta^2\right),\quad V_2 = \frac{ab}{4}\left(\alpha^2+\beta^2\right)^2,\quad V_3 = \frac{ab}{4}$$

b) SCSC nanoplates

$$T_1 = -\frac{3ab}{16}\alpha^2,\quad T_2 = -\frac{ab}{4}\beta^2,\quad T_3 = \frac{3ab}{16}\alpha^4,\quad T_4 = \frac{ab}{2}\alpha^2\beta^2,\quad T_5 = ab\beta^4,\quad T_6 = \frac{ab}{4}\alpha^2\beta^2,$$

$$V_1 = -\frac{ab}{4}\left(\frac{3}{4}\alpha^2+\beta^2\right),\quad V_2 = ab\left(\frac{3}{16}\alpha^4+\frac{1}{2}\alpha^2\beta^2+\beta^4\right),\quad V_3 = \frac{3ab}{16}$$

c) CCCC, C1CC1C nanoplates

$$T_1 = -\frac{3ab}{16}\alpha^2,\quad T_2 = -\frac{3ab}{16}\beta^2,\quad T_3 = \frac{3ab}{4}\alpha^4,\quad T_4 = \frac{ab}{2}\alpha^2\beta^2,\quad T_5 = \frac{3ab}{4}\beta^4,\quad T_6 = \frac{ab}{4}\alpha^2\beta^2,$$

$$V_1 = -\frac{3ab}{16}\left(\alpha^2+\beta^2\right),\quad V_2 = \frac{ab}{2}\left(\frac{3}{2}\alpha^4+\alpha^2\beta^2+\frac{3}{2}\beta^4\right),\quad V_3 = \frac{9ab}{64}$$

where $\alpha = m\pi/a$, $\beta = n\pi/b$, *m* and *n* are the half wave number.

4. NUMERICAL RESULTS AND DISCUSSION

In this section, based on nonlocal NFSDT, numerical results for vibration are presented for the orthotropic SLGSs embedded in an elastic medium with various cases of boundary conditions. Geometrical and material properties of the orthotropic graphene sheet are presented as

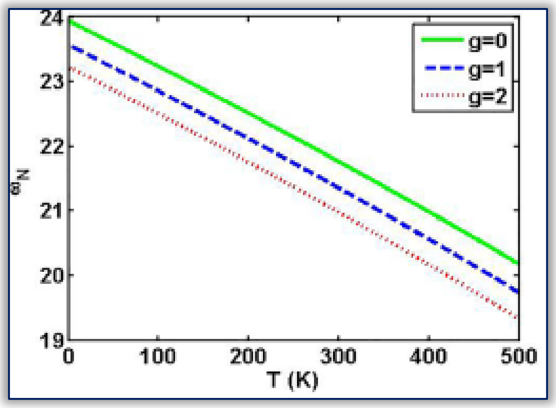
$$E_1 = 1765\,GPa,\quad E_2 = 1588\,GPa,\quad G_{12} = \frac{E_1}{2\left(1+\nu_{12}\right)} = 678.85\,GPa,$$

$$G_{13} = G_{23} = \frac{5}{6}G_{12} = 565.71\,GPa,$$

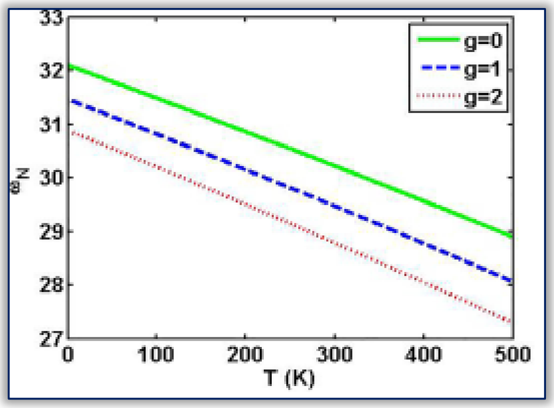
$$\rho = 2300\,kg/m^3,\quad h = 0.34\,nm,\quad \alpha_{11} = 1.6\times10^{-6}\,K^{-1},\quad \alpha_{22} = 1.4\times10^{-6}\,K^{-1},$$

The following dimensionless parameters are used:

$$\omega_N = \omega a^2\sqrt{\frac{\rho h}{D_{11}}},\quad K_{WN} = \frac{K_W a^4}{D_{11}},\quad K_{GN} = \frac{K_G a^2}{D_{11}}$$



a)



b)

Figure 2 Variation of dimensionless frequency versus temperature rise for various nonlocal parameter corresponding to a) SSSS nanoplates and b) SCSC nanoplates

Figures 2a-b depict the effect of the temperature rise on the dimensionless frequency for different boundary conditions (SSSS, SCSC). For both boundary conditions, with the increase value of temperature and nonlocal parameter the dimensionless frequency value decreases.

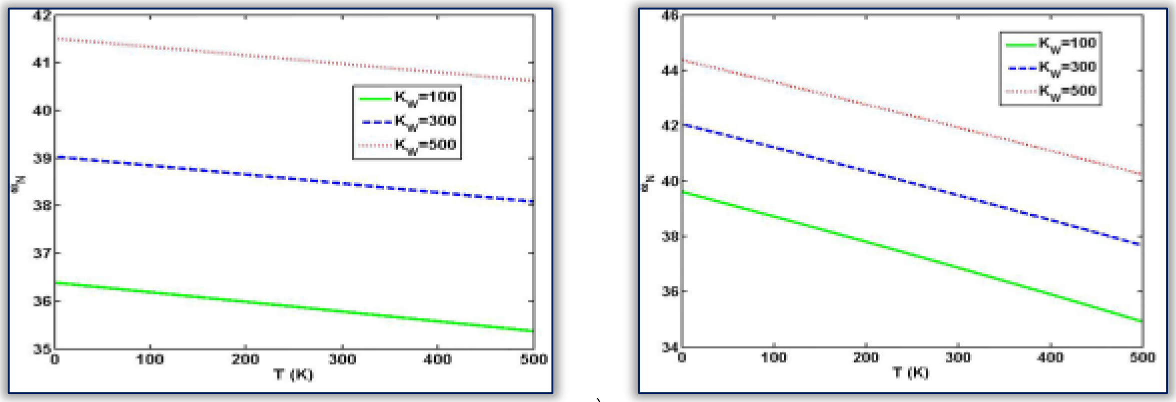


Figure 3. Variation of dimensionless frequency versus temperature rise for various Winkler parameter corresponding to a) CCCC nanoplates and b) C1CC1C nanoplates

The influence of the temperature rise and elastic foundation stiffness (Winkler parameter) on the dimensionless frequency for two different boundary conditions (CCCC, C1CC1C) is shown in Figs.3a-b. For both boundary conditions, with the increase value of temperature the dimensionless frequency value decreases. For both boundary conditions, with the increase value of Winkler parameter the dimensionless frequency value increases.

6. CONCLUSION

Based on the Eringen's nonlocal elasticity theory and the new first order shear deformation theory, the general equations for transverse vibrations of orthotropic single-layered graphene sheets embedded in an elastic medium and subjected in-plane edge thermal loads with different boundary conditions are formulated.

Using the Galerkin's method, the analytical solutions for vibrational frequency of the system with four various boundary conditions are obtained. Numerical results show the effects of the nonlocal parameter, temperature rise, elastic foundation parameter and boundary conditions on the dimensionless frequency.

For all boundary conditions (SSSS, SCSCS, CCCC, C1CC1C), with the increase value of temperature and nonlocal parameter the dimensionless frequency value decreases.

Note

This paper is based on the paper presented at 13th International Conference on Accomplishments in Mechanical and Industrial Engineering – DEMI 2017, organized by University of Banja Luka, Faculty of Mechanical Engineering, in Banja Luka, BOSNIA & HERZEGOVINA, 26 - 27 May 2017.

References

[1] Bhande, SS., Mane, RS., et al. (2011). A bismuth oxide nanoplate-based carbon dioxide gas sensor. Scripta Materialia, vol. 65, p. 1081-1084.

[2] Chen, X., Hu, J., et al. (2009). Nanoplated bismuth titanate submicrosheres for protein immobilization and their corresponding direct electrochemistry and electrocatalysis. Biosens,Bioelectron, vol. 24, p. 3448-3454.

[3] Chang, ST., Hsieh, BF. (2011). TCAD studies of novel nanoplate amorphous silicon alloy thin-film solar cells. Thin solid films, vol. 520, p. 1612-1616.

[4] Pugno, N. (2005). Non-linear statics and dynamics of nanoelectromechanical systems based on nanoplates and nanowires. Proceedings of the Institution of Mechanical Engineers, Part N: Journal of Nanomaterials, Nanoengineering and Nanosystems, vol. 219, p. 29-40.

[5] Liu, KM., He, XQ,, Wong, CH.(2004). On the study of elastic and plastic properties of multi-walled carbon nanotubes under axial tension using molecular dynamics simulation. Acta Materialia, vol 52, p. 2521-2527.

[6] Shahriar, B., Karamooz Ravari, M.R. Zeighampour H. (2015).Vibration analysis of functionally graded carbon nanotubes-reinforced composite nanoplates using Mindlin strain gradient theory. Composites Structures, vol 134, p. 1036-1043.

[7] Shaat, M., Mahmoud, F.F., Gao,X.L., Faheem, A.F. (2014). Size dependent bending analysis of Kirchoff nano-plates based on a modified couple-stress theory including surface effects. International. Journal of Mechanical Sciences, vol 79, p. 31-37.

[8] Li,Y.S., Cai, Z.Y., Shi, S.Y.(2014). Buckling and free vibration of magnetoelectroelastic nanoplate based on nonlocal theory. Composites structures, vol 111, p. 522-529.

[9] Eringen, A.C., Edelen D.G.B.(1972). On nonlocal elasticity. International Journal of Engineering Science, vol. 10, p. 233-248.

[10] Radić, N., Jeremić, D., et al. (2014). Buckling analysis of double-orthotropic nanoplates embedded in Pasternak elastic medium using nonlocal elasticity theory. Composites Part B: Engineering, vol 61, p. 162-171.

[11] Malekzadeh, P., Setoodeh, A.R., Alibeygi Beni, A. (2011). Small scale effect on the free vibration of orthotropic arbitrary straight-sided quadrilateral nanoplates. Composites structures, vol.93, p.1631-1639.
[12] Radić, N., Jeremić, D. (2106). Thermal buckling of double-layered graphene sheets embedded in an elastic medium with various boundary conditions usinga nonlocal new first-order shear deformation theory. Composites Part B: Engineering, vol.97, p. 201-215.
[13] Zenkour, A.M., Sobhy, M. (2013). Nonlocal elasticity theory for thermal buckling of nanoplates lying on Winkler-Pasternak elastic substrate medium. Physica E: Low-dimensional Systems and Nanostructures, vol.53, p. 251-259.
[14] Satish, N., Narendar, S.,Gopalakrishnan, S. (2012). Thermal vibration analysis of orthotropic nanoplates based on nonlocal continuum mechanics. Physica E: Low-dimensional Systems and Nanostructures, vol.44, p.1950-1962.
[15] Wang, Y.Z., Li, F.M., Kishimoto, K. (2011). Thermal effects on vibration properties of double layered nanoplates at small scales. Composites Part B: Engineering, vol.42, p.1311-1317.

ISSN 1584 - 2665 (printed version); ISSN 2601 - 2332 (online); ISSN-L 1584 - 2665