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MATHEMATICAL SHAPING OF THE SUSPENSION SYSTEM OF MACPHERSON TYPE

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Abstract: In the engineering road vehicles, simulating the dynamic behavior is very important because it allows us to study real-world systems without making changes in their actual development, the ultimate goal being to find the ideal configuration. Mathematical models simulate physical tests that allow engineers to see similar results from testing of motor vehicles, but can be obtained repeatedly, safe and much faster than is possible by performing physical tests. We use virtual simulations for the CarSim software, which is a commercial software package that provides efficiency vehicle response to the driver's commands (steering, throttle, brakes or clutch shift) in a given environment (road geometry, coefficients of friction and wind).
Keywords: road vehicles, testing of motor vehicles, CarSim software, MacPherson suspension

1 INTRODUCTION

In the engineering road vehicles, simulating the dynamic behavior is very important because it allows us to study real-world systems without making changes in their actual development, the ultimate goal being to find the ideal configuration. In our experiments we use the virtual simulations offered by the CarSim software (Figure 1), which is a commercial software package that provides efficiency vehicle response to the driver's commands (steering, throttle, brakes, clutch shift) in a given environment (road geometry, coefficients of friction, wind) [1], [2].

To validate simulation models we considered the case of a small car compact coupe class. The feature of this class is the body in two-door, fixed roof and trunk of a sedan shorter than the same model.

The configuration of the Ford Puma is all face 2 + 2-seater, based on the Ford Fiesta platform, but increased track width, aerodynamic bodywork, suspension and stiffer gearbox with shorter ratios.

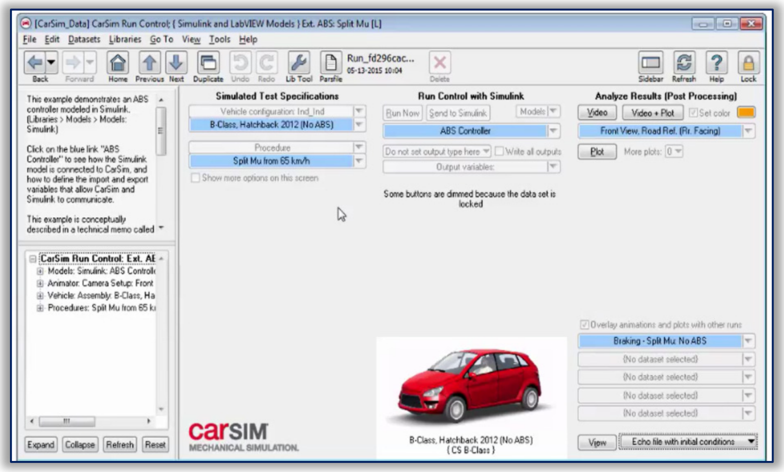

Figure 1. CarSim Interface

2. MATHEMATIC MODEL OF THE MACPHERSON SUSPENSION

For a dynamic simulation of a MacPherson suspension it is necessary to identify its mathematical model; mainly, the MacPherson suspension is composed of a damper and a spring element fitted to a link mechanism (Figure 2) [3], [4]. The sizing mechanism is made for each vehicle individually. The mathematical equations describing the dynamics of the system are the following [5], [6]:

$$\begin{aligned}&(m_s+m_u)\ddot{z}_s+m_u l_c\cos(\theta-\theta_0)\ddot{\theta}-m_u l_c\sin(\theta-\theta_0)\dot{\theta}^2+\\&+k_t\left(z_s+l_c\left(\sin(\theta-\theta_0)\right)-\sin(\theta_0)-Z_r\right)=-f_d\end{aligned} \qquad (1)$$

$$\begin{aligned}&m_u l_c^2\ddot{\theta}+m_u l_c\cos(\theta-\theta_0)\ddot{Z}_s+\frac{c_p b_l^2\sin(\alpha'-\theta_0)\dot{\theta}}{4\left(a_l-b_l\cos(\alpha'-\theta)\right)}+\\&+k_t l_c\cos(\theta-\theta_o)(Z_s+l_c)\left(\sin(\theta-\theta_0)-\sin(-\theta_0)-Z_r\right)-\\&-\frac{1}{2}k_s\sin(\alpha'-\theta)\left[b_1+\frac{d_l}{\left(c_l-d_l\cos(\alpha'-\theta)^{1/2}\right)}\right]=-l_B f_a\end{aligned} \qquad (2)$$

$$a_l = l_A^2 + l_B^2 \quad b_l = 2l_A l_B \quad c_l = a_l^2 - a_l b_l \cos(\alpha + \theta_0) \quad d_l = a_l b_l - b_l^2 \cos(\alpha + \theta_0) \alpha' = \alpha + \theta_0$$

The status variables are:: $[x_1\ x_2\ x_3\ x_4]^T = [Z_S\ \dot{Z}_S\ \theta\ \dot{\theta}]^T$

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f_1(x_1, x_2, x_3, x_4, f_a, z_r, f_b) \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = f_2(x_1, x_2, x_3, x_4, f_a, z_r, f_b) \end{cases} \tag{3}$$

where: Z_s is the vertical displacement of the vehicle body weight; $\dot{Z}_s$ is the speed of vertical motion of the vehicle body (bodywork, engine, passenger); θ is the movement of the control arm; $\dot{\theta}$ is the angular speed of movement of the control arm.

The system of nonlinear equations is:

$$\dot{x} = Ax(t) + B_1 f_a(t) + B_2 Z_r(t) + B_3 f_b(t), x(0) = 0 \tag{4}$$

Where: A is the matrix of the parameters of the suspension; B_1 is the matrix of the active force; B_2 is the matrix that takes into account road profile; B_3 is the matrix that takes into account the force applied to the vehicle chassis.

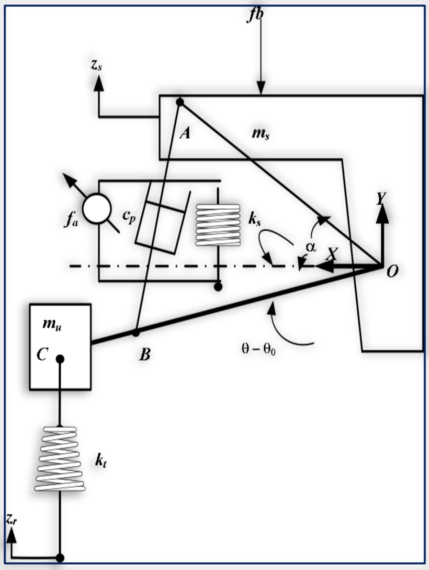

Figure 2. Cinematic diagram

So:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ a_{21} & 0 & a_{23} & a_{24} \\ 0 & 0 & 0 & 1 \\ a_{41} & 0 & a_{43} & a_{44} \end{bmatrix}; \quad B_1 = \begin{bmatrix} 0 \\ \dfrac{l_B \cos(-\theta_0)}{m_s l_c + m_u l_c \sin^2(-\theta_0)} \\ 0 \\ \dfrac{(m_s + m_u) l_c}{m_s m_u l_c^2 + m_u^2 l_c^2 \sin^2(-\theta_0)} \end{bmatrix}; \quad B_2 = \begin{bmatrix} 0 \\ \dfrac{k_t l_c \sin^2(-\theta_0)}{m_s l_c + m_u l_c \sin^2(-\theta_0)} \\ 0 \\ \dfrac{m_s k_t l_c \cos(-\theta_0)}{m_s m_u l_c^2 + m_u^2 l_c^2 \sin^2(-\theta_0)} \end{bmatrix};$$

$$B_3 = \begin{bmatrix} 0 \\ \dfrac{l_c}{m_s l_c + m_u l_c \sin^2(-\theta_0)} \\ 0 \\ \dfrac{m_u l_c \cos(-\theta_0)}{m_s m_u l_c^2 + m_u^2 l_c^2 \sin^2(-\theta_0)} \end{bmatrix}; \quad \begin{aligned} D_1 &= m_s l_c + m_u l_c \sin^2(x_3 - \theta_0) \\ D_2 &= m_s m_u l_c^2 + m_u^2 l_c^2 \sin^2(x_3 - \theta_0) \end{aligned}$$

$$a_{23} = \frac{1}{D_1^2} \left\{ \left[\frac{1}{2} k_s \left(b_l + \frac{d_l}{(c_l - d_l \cos(\alpha'))^{1/2}} \right) (\cos(\alpha' + \theta_0)) - \right.\right.$$

$$\left. -\frac{1}{2}(k_s \sin\alpha' \cos(\theta_0)) \left(\frac{d_l^2 \sin\alpha'}{2(c_l - d_l \cos\alpha')^{3/2}} \right) - (k_t l_c^2 \sin^2(-\theta_0)\cos(-\theta_0) \right] \cdot \qquad ;\ a_{21} = \frac{k_t l_c \sin^2(-\theta_0)}{D_1};$$

$$\cdot \left[m_s l_c + m_u l_c \sin^2(-\theta_0) \right] + m_u k_s l_c \sin\alpha' \sin(-\theta_o) \cos^2(-\theta_0) \left(b_l + \frac{d_l}{(c_l - d_l \cos(\alpha'))^{1/2}} \right)$$

$$a_{24} = \frac{1}{D_1} \cdot \frac{c_p b_l^2 \sin^2\alpha'}{4(a_l - b_l \cos\alpha')}\ ;\ a_{41} = \frac{-m_s k_t l_c \cos(-\theta_0)}{D_2};\ a_{44} = -\frac{1}{D_2} \cdot \frac{(m_s + m_u) c_p b_l^2 \sin^2\alpha'}{4(a_l - b_l \cos\alpha')};$$

$$a_{43} = \frac{1}{D_2^2} \left\{ \left[\frac{1}{2}(m_s + m_u) k_s \cos\alpha' \left(b_l + \frac{d_l}{(c_l - d_l \cos(\alpha'))^{1/2}} \right) (\cos(\alpha' + \theta_0)) - \right.\right.$$

$$\left. -\frac{1}{2}(m_s + m_u) k_s \sin\alpha' \left(\frac{d_l^2 \sin\alpha'}{2(c_l - d_l \cos\alpha')^{3/2}} \right) + m_s k_t l_c^2 \cos(-\theta_0) \right] \cdot [m_s m_u l_c^2 + m_u^2 l_c^2 \sin^2(-\theta_0)] +$$

$$\left. + \frac{1}{2}(m_s + m_u) m_u^2 k_s l_c^2 \sin\alpha' \sin(-\theta_0) \cos^2(-\theta_0) \left(b_l + \frac{d_l}{(c_l - d_l \cos(\alpha'))^{1/2}} \right) \right\}$$

The software test route is an uneven road which frequency is regular, so the shocks felt by the mechanism of suspension of the car have a sinusoidal distribution. In this paper we will focus on three characteristic parameters: angle of fall coefficient of compression springs and ride height [7].

Table 1. Parameters features

Free running ray [mm]	Effective running ray [mm]	Coefficient of compression spring [N/mm^2]	The compression report of spring / damping race
310	298	232	0.950

3. RESULTS ANALYSIS

The results of the simulation are shown in graphical form as follows:

- Forces on the vertical oscillation (Figure 3)
- Angle of fall according to the damping stroke (Figure 4)
- Damping force (Figure 5)
- Spring compression (Figure 6)

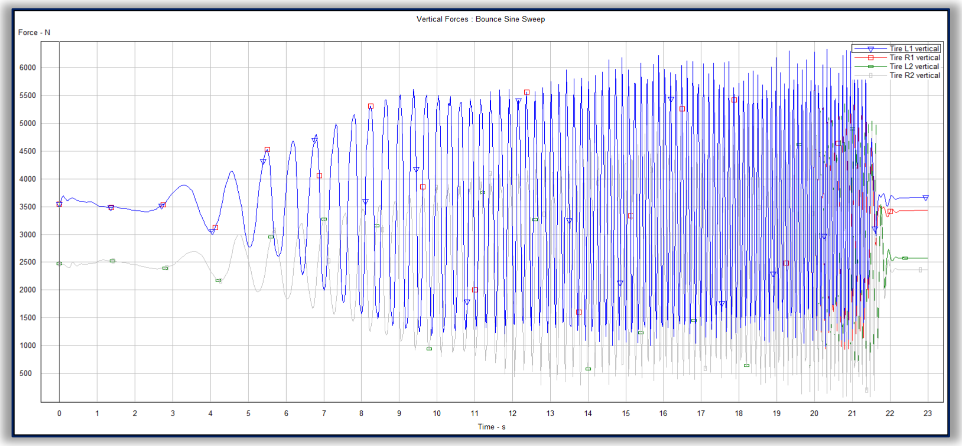

Figure 3. Forces on the vertical oscillation

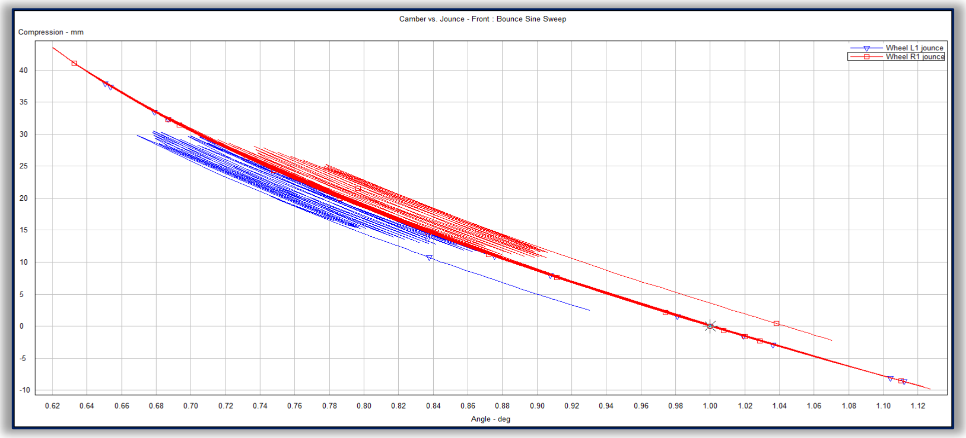

Figure 4. Angle of fall according to the damping stroke

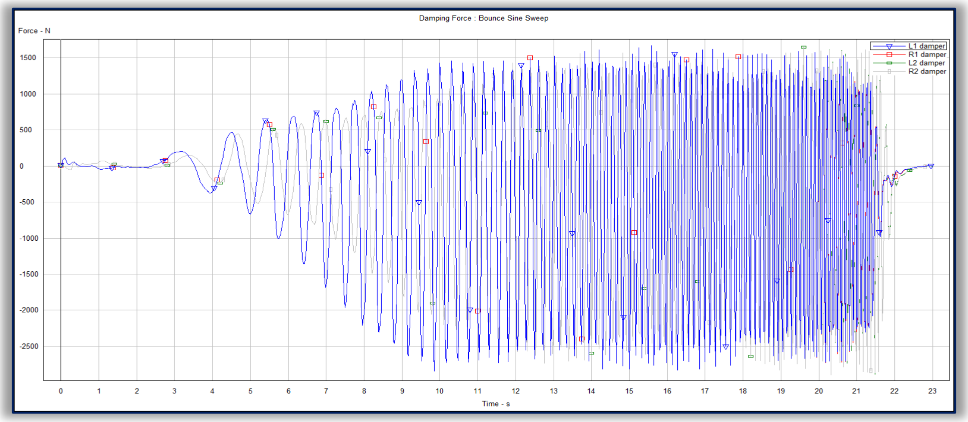

Figure 5. Damping force

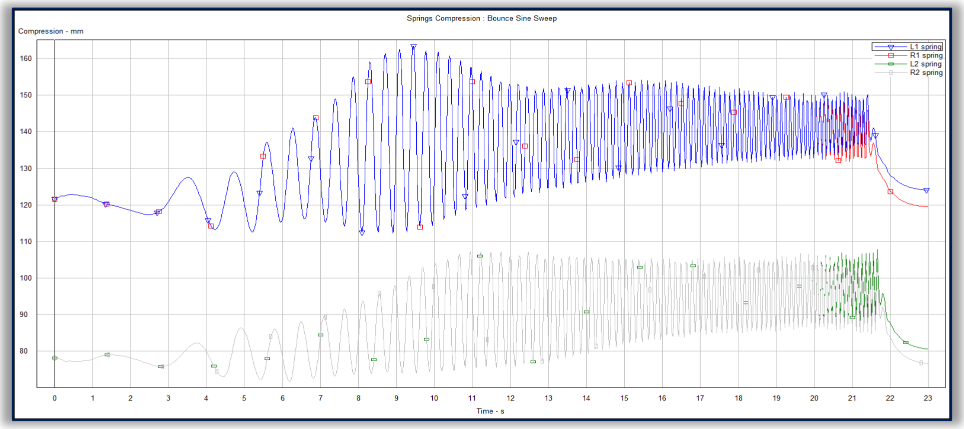

Figure 6. Spring compression

4. CONCLUSIONS

Mathematical models reproduce the behavior of the system at high fidelity. They contain major effects that determine how the tire comes into contact with the road and how the forces resulting from the interaction tire / road transferred through the chassis suspension. However, they do not provide details about transmission connections or compliance structure. The models were validated repeatedly by the manufacturers for the reproduction of motor vehicles, generally the movements necessary for assessing handling (stability, braking and acceleration). On the other hand, these do not include the details necessary for determining the durability of components, fatigue or high-frequency vibration

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