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# INVESTIGATION OF CONVECTIVE FLOW OVER AN INCLINED PLATE EMBEDDED IN A DARCIAN POROUS REGIME: LAPLACE TRANSFORM

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**Abstract:** An exact solution of combined effects of radiation and chemical reaction on the magnetohydrodynamic (MHD) free convection flow of an electrically conducting incompressible viscous fluid over an inclined plate embedded in a porous medium is presented. The impulsively started plate with variable temperature and mass diffusion is considered. The dimensionless momentum equation coupled with the energy and mass diffusion equations are analytically solved using the Laplace transform method. Expressions for velocity, temperature and concentration fields are obtained. They satisfy all imposed initial and boundary conditions and can be reduced, as special cases, to some known solutions from the literature. Expressions for skin friction, Nusselt number and Sherwood number are also obtained. The accuracy of the obtained solutions is checked through imposed conditions and graphs. Furthermore, some well-known established results from the literature are obtained as limiting cases from the present solutions. Numerical results for the velocity field, temperature field and concentration field are graphically displayed.
**Keywords:** Inclined Plate, Laplace Transform, Darcian Drag force, Chemical reaction, heat and mass transfer

## 1. INTRODUCTION

The phenomenon of heat and mass transfer occurs as a result of combined buoyancy effects of thermal diffusion and diffusion through chemical species, which plays an important role in geophysics, aeronautics and chemical engineering. Some industrial applications are found in food drying, food processing and polymer production. Hence, a considerable amount of attention has been focused in recent years by various scientists and engineers to study problems involving the conjugate phenomenon of heat and mass transfer either analytically or numerically and the references therein). On the other hand, the studies on the magnetohydrodynamic (MHD) free convection flow with simultaneous effects of heat and mass transfer are encountered in electric power generation, metallurgy, astrophysics and geophysics, solar power technology, space vehicle, nuclear engineering application and other industrial areas (2010, 2010). Rajput and Kumar (2012) studied the MHD flow past an impulsively started vertical plate with variable temperature and mass diffusion. They used the Laplace transform method to find the exact solutions for velocity, temperature and concentration. In a subsequent year Rajput and Kumar (2012) extended Rajput and Kumar (2012) by taking the thermal radiation effect. Oscillatory flow of a viscous, incompressible electrically conducting fluid with heat radiation is analyzed by Singh (2011) and established the closed form solutions. Recently, Turkyilmazoglu and Pop (2012) extended a flow model by introducing a heat source term and by taking two different types of thermal boundary conditions namely prescribed wall temperature and prescribed heat flux. Furthermore, the free convection flow over vertical surfaces immersed in porous media has paramount importance because of its potential applications in soil physics, geohydrology, and filtration of solids from liquids, chemical engineering and biological systems (2012). Osman et al. (2011) studied analytically the thermal radiation and chemical reaction effects on unsteady MHD free convection flow in a porous medium with heat source/ sink. By taking the porous medium effect, Sami et al. (2012) provided an exact analysis to the study of the magnetohydrodynamic free convection flow of an incompressible viscous fluid past an infinite vertical oscillating plate with uniform heat flux. An in other investigation, Sami et al. (2012) studied the MHD free convection flow in a porous medium with thermal diffusion and ramped wall temperature. They obtained exact dimensionless solutions of momentum and energy equations, under Boussinesq approximation using the Laplace transforms. In addition to this, many researchers like Makinde (2012), Khan et al. (2011), Pal and Mondal (2011), Chandrakala (2010), Narahari and Yunus (2011), Narahari and Ishakh (2011), Seth et al. (2011) and Ming and Wang (2010) have discussed different flow situations for different fluid models either with heat transfer or mass transfer or both of them together in the presence of different effects such as MHD, porosity, thermal radiation, chemical reaction etc. Recently, Ziyauddin and Kumar (2010) studied the radiation effects on unsteady MHD natural convection flow in a porous medium with conjugate heat and mass transfer past a moving inclined plate in the presence of chemical reaction, variable temperature and mass diffusion. They used an explicit finite difference method to solve the coupled linear partial differential equations numerically, and the results are graphically displayed. Unfortunately, in this work the plate is not porous as the authors mentioned in the paper. On the other hand the numerical solutions of the free convection problems are more convenient and easy to handle as compare to exact solutions.

The aim of the present work is to provide an exact solution for the problem of Ziyauddin and Kumar (2010). More exactly, in this paper we have developed closed form exact solutions for the unsteady MHD free convection flow of a viscous fluid over an inclined plate with variable heat and mass transfer in a porous medium. It also appears from the literature that Muthucumaraswamy and Janakiraman (2006) obtained exact solutions for the MHD flow of viscous optically thin fluid past a vertical flat plate in a non-porous medium. They considered uniform heat and variable mass transfer. Furthermore, their solution of velocity has typo mistake (see Eq. 3.7) and does not satisfy the imposed boundary conditions (see Eq. 3.4). It is worth mentioning that the main focus of the present study is not to reproduce the results of Muthucumaraswamy and Janakiraman (2006). In fact, the present model is more general as it considers the fluid to be optically thick instead of optically thin and takes into account chemical reaction, porous medium, variable temperature at the wall and the plate is inclined at a certain angle with vertical axis.

## 2. FORMULATION OF THE PROBLEM

Let us consider the unsteady flow of an incompressible viscous fluid past an infinite inclined plate with variable heat and mass transfer. The $\overline{\mathrm{x}}$-axis is taken along the plate with the angle of inclination to the vertical and the $\overline{\mathrm{y}}$-axis is taken normal to the plate. The viscous fluid is taken to be electrically conducting and fills the porous half space $\overline{\mathrm{y}} > 0$. A uniform magnetic field of strength $\mathrm{B_0}$ is applied in the $\overline{\mathrm{y}}$-direction transversely to the plate. The applied magnetic field is assumed to be strong enough so that the induced magnetic field due to the fluid motion is weak and can be neglected.

According to Cramer and Pai (1973), this assumption is physically justified for partially ionized fluids and metallic liquids because of their small magnetic Reynolds number. Since there is no applied or polarization voltage imposed on the flow field, the electric field due to polarization of charges is zero. Initially, both the fluid and the plate are at rest with constant temperature $\overline{\mathrm{T}}_\infty$ and constant concentration $\overline{\mathrm{C}}_\infty$. At time $\overline{\mathrm{t}} = 0^+$, the plate is givena sudden jerk, and the motion is induced in the direction of flow against the gravity with uniform velocity $\mathrm{u_0}$ . The temperature and concentration of the plate are raised linearly with respect to time. Also, it is considered that the viscous dissipation is negligible and the fluid is thick gray absorbing-emitting radiation but non-scattering medium. Since the plate is infinite in the $(\overline{\mathrm{x}}, \overline{\mathrm{z}})$ plane, all physical variables are functions of $\overline{\mathrm{y}}$ and $\overline{\mathrm{t}}$ only. The physical model and coordinates system is shown in Figure 1. In view of the above assumptions, as well as of the usual Boussinesq's approximation, the governing equations reduce

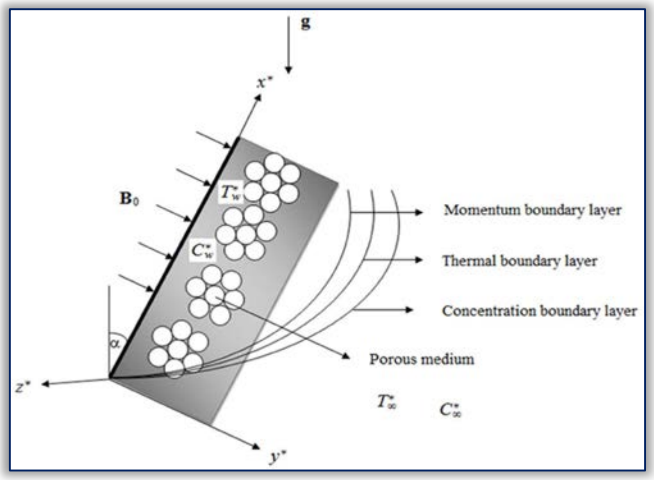

Figure 1: Physical model and coordinates system

$$\frac{\partial \overline{\mathrm{u}}}{\partial \overline{\mathrm{t}}} = \nu \frac{\partial^2 \overline{\mathrm{u}}}{\partial \overline{\mathrm{y}}^2} - \left(\frac{\sigma \mathrm{B}_0^2}{\rho} + \frac{\nu}{\overline{\mathrm{K}}}\right)\overline{\mathrm{u}} + \mathrm{g}\beta\left(\overline{\mathrm{T}} - \overline{\mathrm{T}}_\infty\right)\cos\alpha + +\mathrm{g}\beta_{\mathrm{C}}\left(\overline{\mathrm{C}} - \overline{\mathrm{C}}_\infty\right)\cos\alpha\,, \tag{2.1}$$

$$\frac{\partial \overline{\mathrm{T}}}{\partial \overline{\mathrm{t}}} = \frac{\kappa}{\rho \mathrm{C_p}} \frac{\partial^2 \overline{\mathrm{T}}}{\partial \overline{\mathrm{y}}^2} - \frac{1}{\rho \mathrm{C_p}}\left(\frac{\partial \overline{\mathrm{q}}_\mathrm{r}}{\partial \overline{\mathrm{y}}}\right), \tag{2.3}$$

$$\frac{\partial \overline{\mathrm{C}}}{\partial \overline{\mathrm{t}}} = \mathrm{D}\frac{\partial^2 \overline{\mathrm{C}}}{\partial \overline{\mathrm{y}}^2} - \mathrm{C_r}\left(\overline{\mathrm{C}} - \overline{\mathrm{C}}_\infty\right), \tag{2.3}$$

The initial and boundary conditions are:

$$\begin{Bmatrix} \overline{\mathrm{u}}(\overline{\mathrm{y}},0) = 0\,, & \overline{\mathrm{T}}(\overline{\mathrm{y}},0) = \overline{\mathrm{T}}_\infty\,, & \overline{\mathrm{C}}(\overline{\mathrm{y}},0) = \overline{\mathrm{C}}_\infty & \text{on } \overline{\mathrm{y}} > 0, \\ \overline{\mathrm{u}}(\infty,0) = 0\,, & \overline{\mathrm{T}}(\infty,0) = \overline{\mathrm{T}}_\infty\,, & \overline{\mathrm{C}}(\infty,0) = \overline{\mathrm{C}}_\infty & \text{as } \overline{\mathrm{t}} > 0 \end{Bmatrix} \tag{2.4},$$

where $\mathrm{A} = \mathrm{u}_0^2/\nu$. Following Magyari and Pantokratoras [28], we adopt the Rosseland approximation for radiative flux $\mathrm{q_r}$, namely

$$\begin{Bmatrix} \overline{\mathrm{u}}(0,\overline{\mathrm{t}}) = \mathrm{u}_0 \\ \overline{\mathrm{T}}(0,\overline{\mathrm{t}}) = \overline{\mathrm{T}}_\infty + \left(\overline{\mathrm{T}}_\mathrm{w} - \overline{\mathrm{T}}_\infty\right)\mathrm{A}\overline{\mathrm{t}}\,, \\ \overline{\mathrm{C}}(0,\overline{\mathrm{t}}) = \overline{\mathrm{C}}_\infty + \left(\overline{\mathrm{C}}_\mathrm{w} - \overline{\mathrm{C}}_\infty\right)\mathrm{A}\overline{\mathrm{t}}\,, \ \ \overline{\mathrm{t}} > 0 \end{Bmatrix}$$

$$\mathrm{q_r} = -\frac{4\sigma_0}{3\mathrm{K}_0}\frac{\partial \overline{\mathrm{T}}^4}{\partial \overline{\mathrm{y}}} \tag{2.5}$$

We assume that the temperature differences within the flow are sufficiently small and $\overline{\mathrm{T}}^4$ can be expressed as a linear function of the temperature. This is accomplished by expanding $\overline{\mathrm{T}}^4$ in a Taylor series about $\overline{\mathrm{T}}_\infty$ and neglecting the higher order terms, we get

$$\overline{T}^4 \approx 4\overline{T}_\infty^3\overline{T} - 3\overline{T}_\infty^4 \tag{2.6}$$

Substitution Eqs. (2.5) and (2.6) into Eq. (2.2), yields

$$\frac{\partial\overline{T}}{\partial\overline{t}} = \frac{\kappa}{\rho C_p}\left(1 + \frac{16\sigma_0\overline{T}_\infty^3}{3\kappa K_0}\right)\frac{\partial^2\overline{T}}{\partial\overline{y}^2}, \tag{2.7}$$

Introducing the following dimensionless variables

$$\left\{\begin{array}{c} \bar{u} = uu_0,\ \ \overline{T} = \overline{T}_\infty + \theta(\overline{T}_w - \overline{T}_\infty),\ \ \overline{C} = \overline{C}_\infty + \phi(\overline{C}_w - \overline{C}_\infty), \\ \overline{K} = \dfrac{\nu^2 K}{u_0^2},\ \ y = \dfrac{u_0\overline{y}}{\nu},\ \ Gr = \dfrac{\nu g\beta_T(\overline{T}_w - \overline{T}_\infty)}{u_0^3},\ \ C_r = \dfrac{\nu\overline{C}_r}{u_0^2} \\ Gm = \dfrac{\nu g\beta_C(\overline{C}_w - \overline{C}_\infty)}{u_0^3},\ \ t = \dfrac{\overline{t}u_0^2}{\nu},\ \ Pr = \dfrac{\nu\rho C_p}{\kappa} = \dfrac{\nu}{\alpha}, \\ M = \dfrac{\sigma B_0^2\nu}{\rho u_0^2},\ \ R_a = \dfrac{16\sigma_0\overline{T}_\infty^3}{3\kappa K_0},\ \ Sc = \dfrac{\nu}{D}. \end{array}\right\} \tag{2.8}$$

In view of (2.8), equations (2.1), (2.3) and (2.7) reduce to

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} - Nu + Gr\theta\cos\alpha + Gm\phi\cos\alpha, \tag{2.9}$$

$$Pr\frac{\partial\theta}{\partial t} = (1 + R_a)\frac{\partial^2\theta}{\partial y^2}, \tag{2.10}$$

$$\frac{\partial\phi}{\partial t} = \frac{1}{Sc}\frac{\partial^2\phi}{\partial y^2} - C_r\phi, \tag{2.11}$$

where $N = M + K^{-1}$.

The corresponding initial and boundary conditions (2.4), become

$$\left\{\begin{array}{c} t \le 0: u(y,t) = 0,\ \ \theta(y,t) = 0,\ \ \phi(y,t) = 0,\ \ y > 0 \\ u(0,t) = 1,\ \ \theta(0,t) = t,\ \ \phi(0,t) = t, \\ u(\infty,t) \to 0,\ \ \theta(\infty,t) \to 0,\ \ \phi(\infty,t) \to 0 \end{array}\right\}, \tag{2.12}$$

## 3. METHOD OF SOLUTION

The equations (2.9) – (2.11) represent a set of partial differential equations and thus in order to reduce these into a set of ordinary differential equations in dimensionless form we adopt the Laplace transform to the system of Eqs. (2.9) – (2.11) and using the initial conditions from Eq. (2.12), we obtain

$$\frac{d^2\bar{u}(y,q)}{dy^2} - (q + N)\bar{u}(y,q) = -Gr\cos\alpha\,\bar{\theta}(y,q) - Gm\cos\alpha\,\bar{\phi}(y,q), \tag{3.1}$$

$$\frac{d^2\bar{\theta}(y,q)}{dy^2} = -aq\,\bar{\theta}(y,q), \tag{3.2}$$

$$\frac{d^2\bar{\phi}(y,q)}{dy^2} = -Sc(q + C_r)\,\bar{\phi}(y,q), \tag{3.3}$$

with the transformed boundary conditions

$$\bar{u}(0,q) = \frac{1}{q},\ \ \bar{u}(\infty,q) = 0,\ \ \bar{\theta}(0,q) = \frac{1}{q^2},\ \ \bar{\theta}(\infty,q) = 0,\ \ \bar{\phi}(0,q) = \frac{1}{q^2},\ \ \bar{\phi}(\infty,q) = 0, \tag{3.4}$$

where $\bar{u}(y,q), \bar{\theta}(y,q)$ and $\bar{\phi}(y,q)$ are the Laplace Transforms of $u(y,t), \theta(y,t)$ and $\phi(y,t)$ respectively, and $a = \frac{Pr}{1+R_a}$.

Now solving the system of Eqs. (2.9)–(2.11) subject to the boundary conditions (2.12), one obtains

$$\bar{u}(y,q) = \left\{\begin{array}{c} \dfrac{1}{q}e^{-y\sqrt{N+q}} + a_1\cos\alpha\dfrac{e^{-y\sqrt{N+q}}}{q^2(q - N_1)} + a_2\cos\alpha\dfrac{e^{-y\sqrt{N+q}}}{q^2(q - N_2)} \\ -a_1\cos\alpha\dfrac{e^{-y\sqrt{aq}}}{q^2(q - N_1)} - a_2\cos\alpha\dfrac{e^{-y\sqrt{Sc}\sqrt{C_r+q}}}{q^2(q - N_2)} \end{array}\right\} \tag{3.5}$$

$$\bar{\theta}(y,q) = \frac{1}{q^2}e^{-y\sqrt{aq}}, \tag{3.6}$$

$$\bar{\phi}(y,q)=\frac{1}{q^2}e^{-y\sqrt{Sc}\sqrt{C_r+q}} \tag{3.7}$$

The inverse Laplace transforms of Eqs. (3.5) – (3.6) yield

$$u(y,q)=f_1(y,t,N)-\frac{a_1\cos\alpha}{N_1^2}\begin{Bmatrix} f_1(y,t,N)-e^{N_1t}f_1(y,t,N+N_1)-f_1\left(y\sqrt{a},t,0\right) \\ +e^{N_1t}f_1\left(y\sqrt{a},t,N_1\right)+N_1f_2(y,t,N)-N_1f_3\left(y\sqrt{a},t,0\right)\end{Bmatrix}$$
$$-\frac{a_2\cos\alpha}{N_2^2}\begin{Bmatrix} f_1(y,t,N)-e^{N_2t}f_1(y,t,N+N_2)-f_1\left(y\sqrt{Sc},t,C_r\right) \\ +e^{N_2t}f_1\left(y\sqrt{Sc},t,C_r+N_2\right)+N_2f_2(y,t,N)-N_2f_3\left(y\sqrt{Sc},t,C_r\right)\end{Bmatrix}, \tag{3.8}$$

$$\theta(y,q)=\left(t+\frac{ay^2}{2}\right)\operatorname{erfc}\left(\frac{y\sqrt{a}}{2\sqrt{t}}\right)-\frac{y\sqrt{at}}{2\sqrt{\pi}}e^{-\frac{ay^2}{4t}}, \tag{3.9}$$

$$\phi(y,q)=f_2\left(y\sqrt{Sc},t,C_r\right), \tag{3.10}$$

— Limiting Cases

The following published results are reduced as special cases from the present solutions.

i) By neglecting the thermal and mass transfer effects (when $a_1=a_2=0$), into Eq. (3.8), we get

$$u(y,t)=f_1(y,t,N) \tag{3.11}$$

which is quite identical to the known result obtained from Sami et. al (2012), Eq. (3.7) for $\omega\to0$ i.e. for the impulsive motion of the plate. Furthermore, the solution corresponding to hydrodynamic fluid passing through a non-porous medium are immediately obtained from Eq. (3.11) by neglecting the magnetic and porous effects ($R_a=0$ when M=0 and $K\to\omega$).

ii) Now by taking $C_r\to0$ into Eq. (3.7), we get

$$\phi(y,t)=f_3\left(y\sqrt{Sc},t,0\right) \tag{3.12}$$

similar to the solution obtained by Muthucumaraswamy and Janakiraman (2006), Eq. (3.2).

iii) Finally by substituting $R_a=0$ and $C_r=0$ into Eqs. (2.10) and (2.11), we immediately obtain

$$\theta(y,t)=f_3\left(y\sqrt{Pr},t,0\right) \tag{3.13}$$
$$\phi(y,t)=f_3\left(y\sqrt{Sc},t,0\right) \tag{3.14}$$

which are identical to the solutions obtained by Rajput and Kumar (2012), Eqs. (3.7) and (3.6) respectively. Furthermore, it is worth mentioning that Eqs. (3.8), (3.9) and (3.10), satisfy all imposed initial and boundary conditions. Hence this also provides a useful mathematical check to our calculi.

— Skin-Friction

The skin friction $\tau$ evaluated from Eq. (3.8) is given by

$$\tau=-\left(\frac{\partial u}{\partial y}\right)_{y=0}=-g_1(t,N)+\frac{a_1\cos\alpha}{N_1^2}\begin{Bmatrix} g_1(t,N)-e^{N_1t}g_1(t,N+N_1)-\sqrt{a}g_1(t,0) \\ +e^{N_1t}\sqrt{a}g_1(t,N_1)+N_1g_2(t,N)-N_1\sqrt{a}g_3(t,0)\end{Bmatrix}$$
$$+\frac{a_2\cos\alpha}{N_2^2}\begin{Bmatrix} g_1(t,N)-e^{N_2t}g_1(t,N+N_2)-\sqrt{Sc}g_1(t,C_r) \\ +e^{N_2t}\sqrt{Sc}g_1(t,C_r+N_1)+N_2g_2(t,N)-N_2\sqrt{Sc}g_2(t,C_r)\end{Bmatrix}, \tag{3.15}$$

— Nusselt Number

The rate of heat transfer evaluated from Eq. (3.9) is given by

$$Nu=2\sqrt{\frac{at}{\pi}} \tag{3.16}$$

— Sherwood Number

The rate of mass transfer evaluated from Eq. (3.10) is given by

$$Sh=\sqrt{Sc}\,g_2(t,C_r) \tag{3.17}$$

## 4. VALIDITY

Table 1 provides the comparison of our results for skin friction with those of Ziyauddin and Kumar (2010) corresponding to the cooling of the plate. The results are found quite identical in the integral part. However, the physical behavior of various parameters on the skin friction t in the present work is similar to that of Ziyauddin and Kumar (2010) i.e. skin friction increases with increasing Pr, M and $C_r$, but decreases when $R_a$ is increased.

Table-1: Comparison of skin friction t with Ziyauddin and Kumar (2010)

| Pr | M | $C_r$ | $R_a$ | Present study | Ziyauddin and Kumar (2010) |
|---|---|---|---|---|---|
| 0.71 | 2 | 0.1 | 5 | 3.507153 | 3.507201 |
| 7 | 2 | 0.1 | 5 | 4.107925 | 4.107893 |
| 0.71 | 10 | 0.1 | 5 | 4.829181 | 4.829176 |
| 0.71 | 2 | 0.5 | 5 | 3.805627 | 3.805630 |
| 0.71 | 2 | 0.1 | 15 | 3.416375 | 3.416381 |

In Table 2, we compare our numerical result of Nusselt number for different values of Pr with Turkyilmazoglu and Pop (2012), Table. 6 for N=Q=0. It is interesting to see that the same data as Turkyilmazoglu and Pop (2012) has obtained. This comparison shows the accuracy of our model. Nusselt number increases with the increase of Pr.

Table-2: Comparison of Nusselt number Nu with Turkyilmazoglu and Pop (2012) at t = 0.5 without thermal radiation and heat generation:

| Pr | Present study | Turkyilmazoglu and Pop (2012) |
|---|---|---|
| 0.71 | 0.547134 | 0.547141 |
| 1 | 0.612578 | 0.612601 |
| 3 | 1.052173 | 1.052182 |
| 7 | 1.617354 | 1.617347 |
| 10 | 1.917332 | 1.917349 |

## 5. RESULTS AND DISCUSSION

In this field of study, in order to examine the consequences of velocity field, temperature profile and concentration profiles by allotting numerical values for several arguments. During the numerical computation, the value of the Prandtl number is chosen as Pr=0.71 (air), which is the most encountered fluid in nature and frequently used in engineering and industry. The graphical results are plotted using some built-in functions in the computational software Mathematica 8.0.

Figure 2 illustrates the variation of concentration distribution of the flow field for different values of chemical reaction parameter and time. The physical effect of chemical reaction parameter $C_r$ is seen from this figure, which clearly demonstrates that concentration profiles decrease rapidly when $C_r$ is increased. Moreover, the time parameter has escalated the species of concentration in the concentration boundary layer.

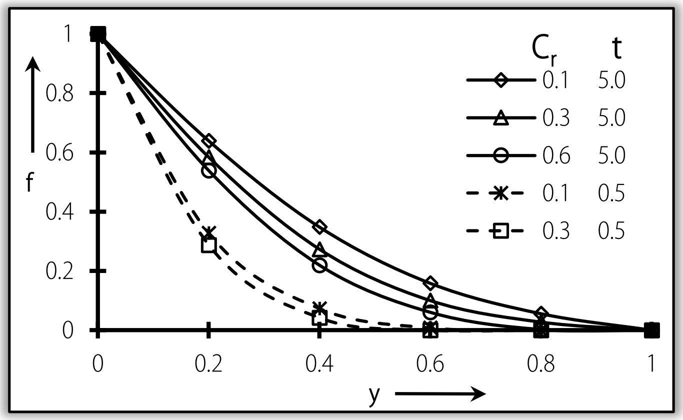


Figure 2: Concentration distribution for t and $C_r$

For various values of $R_a$ (radiation parameter) for both the cases of Pr=0.025 (mercury) and Pr=0.71 (air), the temperature profile is shown in Figure 3 at t = 0.5. It is seen that there is a decrease in temperature in $R_a$ and a similar effect has been observed for Prandtl number. Physically, it is true due to the fact that an increase in Prandtl number increases the viscosity of the fluid, becomes thick and consequently leads to a decrease in the thermal boundary layer.

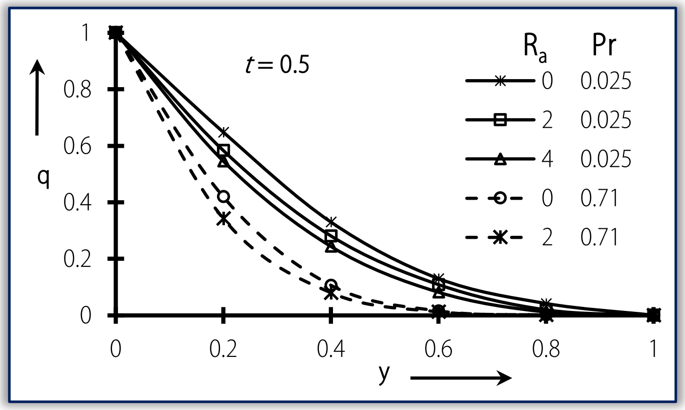


Figure 3: Temperature distribution for $R_a$ and Pr

Figure 4 depicts the influence of porosity parameter (K) and magnetic field (M) on the velocity profiles at t=0.5. The range of magnetic field is taken from 0 to 5. The velocity of the flow field is found to decrease in presence of magnetic field. Physically, it is true due to the fact that the application of a transverse magnetic field to an electrically conducting fluid gives rise to a body force known as Lorentz force which tends to resist the fluid flow and slow down its motion in the boundary layer region. It is evident from Figure 4 that fluid velocity decreases with an increase in the porosity parameter (K). Physically, this refers to the fact that increasing the tightness of the porous medium which is represented by increase in K results in increasing the resistance against the flow.

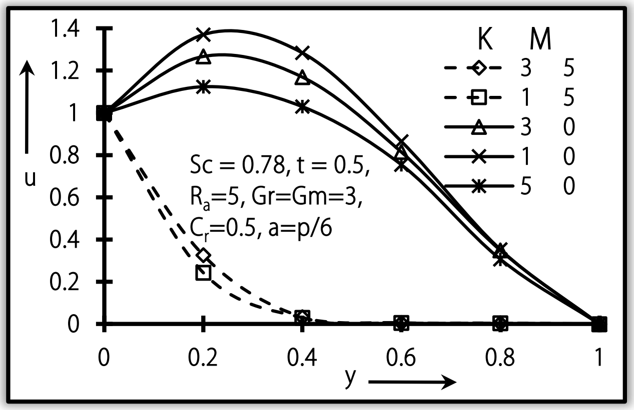


Figure 4: Velocity distribution for K and M

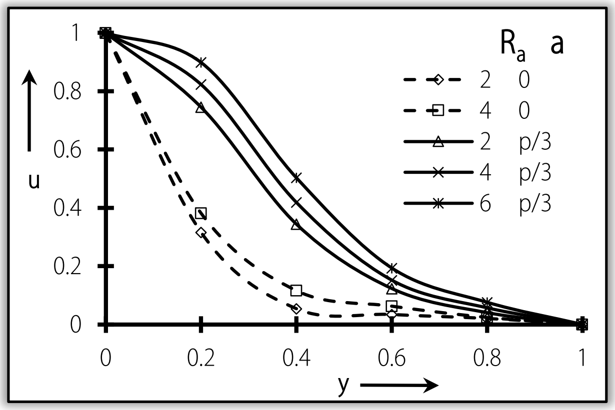


Figure 5: Velocity distribution for $R_a$ and $\alpha$

Figure 5 shows the effect of thermal radiation ($R_a$) and inclination of the plate on the flow velocity. It is found that the flow velocity accelerated as $R_a$ increases. Physically it is true, as higher radiation occurs when temperature is higher and ultimately the velocity rises. As expected, the fluid velocity increases and the peak value is more distinctive due to increase in $R_a$ when $\alpha = \pi/3$. The velocity distribution attains a maximum value in the neighborhood of the plate and then decreases properly to approach the free stream value. Without inclination no peak value has attained near the plate and however, an increase in the inclination enhanced the flow velocity.

## 6. CONCLUSIONS

The analytical solutions of heat and mass transfer in the MHD free convection flow of an electrically conducting incompressible viscous fluid over an inclined plate with variable heat and mass transfer passing through a porous medium are obtained using the Laplace transform technique. The effects of radiation and chemical reaction are also considered and required expressions for skin-friction, Nusselt number and Sherwood number are evaluated. The comparison for the present numerical results of skin-friction and Nusselt number are shown in tables.

The present study brings out the following significant findings:

- Transverse magnetic field produces a type of resistive force which opposes the flow.
- The effects of the permeability and magnetic parameters on velocity are opposite.
- The higher the porosity parameter, the more sharply is the reduction in velocity.
- Temperature decreases with increasing Pr and as well as $R_a$.
- Concentration decreases with increasing $C_r$ and increases with increasing t.
- Skin-friction increases with increasing Pr, M, $C_r$ and decreases if $R_a$ increases.
- Nusselt number increases for increasing Pr.

Appendix:

$$N_1 = \frac{N}{a-1}, \quad N_2 = \frac{N - C_r Sc}{Sc - 1}, \quad a_1 = \frac{Gr}{a-1}, \quad a_2 = \frac{Gm}{Sc-1} \quad \text{for } a \neq 1 \text{ and } Sc \neq 1.$$

$$f_1(v,t,w) = \frac{1}{2}\left\{e^{-v\sqrt{w}}\operatorname{erfc}\left(\frac{v}{2\sqrt{t}} - \sqrt{w\,t}\right) + e^{v\sqrt{w}}\operatorname{erfc}\left(\frac{v}{2\sqrt{t}} + \sqrt{w\,t}\right)\right\},$$

$$f_2(v,t,w) = \frac{1}{2}\left\{\left(t - \frac{v}{2\sqrt{w}}\right)e^{-v\sqrt{w}}\operatorname{erfc}\left(\frac{v}{2\sqrt{t}} - \sqrt{w\,t}\right) + \left(t + \frac{v}{2\sqrt{w}}\right)e^{v\sqrt{w}}\operatorname{erfc}\left(\frac{v}{2\sqrt{t}} + \sqrt{w\,t}\right)\right\},$$

$$f_3(v,t,w) = \left\{\left(t + \frac{v^2}{2}\right)f_1(v,t,0) - \frac{v\sqrt{t}}{\sqrt{\pi}}e^{-\frac{v^2}{4t}}\right\},$$

$$g_1(t,w) = \frac{\partial f_1}{\partial v}(v,t,w)\bigg|_{v=0} = \frac{e^{-wt}}{\sqrt{\pi t}} - \sqrt{w}\operatorname{erf}\left(\sqrt{wt}\right),$$

$$g_2(t,w) = \frac{\partial f_2}{\partial v}(v,t,w)\bigg|_{v=0} = \frac{-2\sqrt{wt}e^{-wt} - \sqrt{\pi}\operatorname{erf}\left(\sqrt{wt}\right) - 2wt\sqrt{\pi}\operatorname{erf}\left(\sqrt{wt}\right)}{2\sqrt{\pi w}},$$

$$g_3(t,w) = \frac{\partial f_3}{\partial v}(v,t)\bigg|_{v=0} = -2tg_1(t,0).$$

Nomenclature:

| | |
|---|---|
| $\bar{u}$ | axial velocity |
| $\bar{T}$ | temperature of the fluid |
| $\bar{C}$ | species concentration |
| $q_r$ | radiation heat flux |
| $\bar{x}, \bar{y}$ | dimensional distances along and perpendicular to the plate |
| $\bar{t}$ | dimensional time |
| $K > 0$ | permeability of the porous medium |
| g | acceleration due to gravity |
| $C_P$ | specific heat at constant pressure |
| D | mass diffusibility |
| $C_r$ | chemical reaction parameter |
| M | magnetic parameter / Hartmann number |
| K | permeability parameter |
| Gr | thermal Grashof number |
| Gm | mass Grashof number |
| Pr | Prandtl number |
| $R_a$ | radiation parameter |
| $C_r$ | dimensionless chemical reaction parameter |
| Sc | Schmidt number |
| $K_0$ | mean absorption coefficient |
| | Greek Symbols |
| $\sigma$ | electrical conductivity |
| $\nu=\mu/\rho$ | kinematic viscosity |
| $\mu$ | viscosity |
| $\rho$ | density of the fluid |
| $\beta_T$ | coefficient of thermal expansion |
| $\beta_C$ | coefficient of concentration expansion |
| $\kappa$ | thermal diffusivity |
| $\alpha$ | inclination of the plate |
| $\sigma_0$ | Stefan-Boltzmann constant |
| | Subscripts |
| w | Wall condition |
| $\infty$ | Free stream condition |

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